REPORT
ON
THE TEACHING OF MATHEMATICS
IN
JAPAN

TOKIO
1912

REPORT

ON

THE TEACHING OF MATHEMATICS

IN

JAPAN

Prepared by the

JAPANESE SUB-COMMISION

OF THE

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

TOKIO

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PREFACE

The utter lack of time at our disposal through unique circumstances which will be explained in the Preface to the SUMMARY REPORT OF THE TEACHING OF MATHEMATICS IN JAPAN prepared by myself and published conjointly with this book, will account for very many shortcomings both in form and content to be found in these Divisional Reports now bound in one volume. It is much to be regretted that our best intentions were forbidden the support of our best energies. Moreover, in the work of translating the Japanese original into English, far greater difficulties were experienced than were anticipated, and indeed to such a degree as would be almost incredible to those who had no hand in it.

For full particulars, the reader is referred to the Preface to the Summary Report mentioned above.

R. FUJISAWA, *Chairman of the Japanese Sub-Commission.*

Tokio, June 1912.

INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS

JAPAN—Delegate: R. Fujisawa (Tokio)

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T[ōjurō] Okada, *Teacher at the Aoyama [Tokio] Normal School.*

CONTENTS

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Being a Series of Reports prepared by the DIVISIONS OF THE JAPANESE SUB-COMMISSION OF THE INTERNATIONAL COMMISSION ON THE TEACHING OF MATHEMATICS.

Article I.——The Teaching of Mathematics in Elementary Schools. Prepared by T. Andō and T. Gotō, *Teachers at the Tokio Higher Normal School,* under the direction and supervision of T. Hayashi, till recently *Professor of Mathematics at the Tokio Higher Normal School* and now *Professor of Mathematics at the Tōhoku Imperial University.*

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TOKIO
1912

CONTENTS.

PART I.

Present Condition of the Elementary Educational System and Method of the Teaching of Mathematics.

PART II.

Recent Tendencies in the Teaching of Mathematics.

THE TEACHING OF MATHEMATICS

IN

ELEMENTARY SCHOOLS.

PART I.

PRESENT CONDITION OF THE ELEMENTARY EDUCATIONAL SYSTEM AND METHOD

OF THE

TEACHING OF MATHEMATICS.

CHAPTER I.

Aim and Kinds of Elementary Schools.

SECTION 1. Aim.

The elementary schools in Japan are established in accordance with State enactments. The principal aim of elementary school education is laid down in the first article of the Ordinance relating to elementary schools, as follows:—

"The principal aim of elementary schools is to afford children the foundations of moral and civic education together with the common knowledge and abilities necessary for ordinary life, care being taken at the same time of their physical development."

The citizens of Japan should receive not only the moral education necessary to them as individuals, but also an education calculated to enable them to realize and fulfil their responsibilities as citizens. Moral and civic education is so

vast in its scope that it cannot be attained to its full extent in the inferior grades and during the limited period given to education in elementary schools. Hence the Ordinance relating to elementary schools states that the principal aim of elementary education is merely to lay the foundations of moral and civic education.

SECTION 2. Kinds of Elementary Schools.

The elementary schools may be classified in various ways according to different points of view.

(1) Classification according to the grade of the curriculum.

The elementary schools are divided into ordinary elementary schools and higher elementary schools, according to the grade of the prescribed curriculum. Those schools, whose curriculum covers both the ordinary and the higher elementary school grade, are called ordinary and higher elementary schools. The ordinary elementary school aims at teaching the subjects essentially required by the nation, and the education therein given is compulsory. The citizens of Japan, therefore, whether male or female, are coerced to receive this education. The school age extends over eight years beginning with the month after a child reaches its sixth year, and ending with the month in which it attains its fourteenth year. Children are required to attend school from the beginning of the school year immediately following the month in which they reach their school age. In most schools, the school year extends over a period from April to March of the following year, but in a few among the 30,000 schools of the Empire, from September to August of the following year. When a child reaches its school age, its parents or guardians should send it to an ordinary elementary school, and they are responsible for its completing the whole course. (The course of study in an ordinary elementary school is six years). It is also a duty for cities, towns or villages to

provide as many ordinary elementary schools as are needed to accommodate the children of school age residing within their limits.

The higher elementary school which has a course of two or three years is for children who have completed the ordinary elementary school course and aims to give a more advanced general education, which is not compulsory.

(2) Classification according to the sources of school expenditures.

Elementary schools are divided into four classes according to the sources of their revenues: (1) elementary schools established by the State, (2) those established by a prefecture, (3) those established by a town, or a village, and (4) those established by individuals.

The elementary schools belonging to the first class are those attached to the four Higher Normal Schools under the direct control of the Department of Education, and the elementary school course of the Peers' School under the control of the Department of the Imperial Household. The elementary schools of the second class are those attached to the normal schools in the prefectures.

(3) Classification according to organization.

Elementary schools are divided, according to their organization of classes, into one class or more than one class elementary schools.

CHAPTER II.

Courses and Subjects of Study in Elementary Schools.

SECTION 1. Courses of Study.

The course of study in the ordinary elementary school

extends over six years, and in the higher elementary school over two or three years.

SECTION 2. Subjects of Study.

The subjects of study in the ordinary elementary school consist of morals, Japanese language, arithmetic, Japanese history, geography, science, drawing, singing, and gymnastics. For girls, sewing is added. According to local circumstances, manual training may be added. The subjects of study in the higher elementary school contain morals, Japanese language, arithmetic, Japanese history, geography, science, drawing, singing, and gymnastics. For girls sewing is added. One or more of such subjects as English, agriculture, commerce, or manual training may also be added.

SECTION 3. Number and Distribution of Weekly Periods.

The distribution of school periods per week for each subject is as follows:—

(1) For the ordinary elementary schools.

Subjects. \ Years.	I	II	III	IV	V	VI
Morals........	2	2	2	2	2	2
Japanese language	10	12	14	14	10	10
Arithmetic	5	6	6	6	4	4
Japanese history, Geography					3	3
Science					2	2
Drawing......			1	1	girls. 1 | boys. 2	girls. 1 | boys. 2
Singing	(4	(4	1	1	1	2
Gymnastics ..			3	3	3	3
Sewing			1	2	3	3
Manual training						
Total	21	24	girls. 28 | boys. 27	girls. 29 | boys. 27	girls. 29 | boys. 27	girls. 30 | boys. 28

Drawing may be taught for one period a week in the first and second year.

Manual training may be assigned one period a week in the first, second, and third year, and two periods a week in the fourth, fifth, and sixth year.

In the above mentioned cases, the principal of the school provides the periods for those subjects by reducing the number of periods for other subjects.

(2) For the higher elementary schools.

<table>
<tr><th colspan="5">(a) Higher elementary schools of a two years' course.</th><th colspan="6">(b) Higher elementary schools of a three years' course.</th></tr>
<tr><th>Years. / Subjects.</th><th colspan="2">I</th><th colspan="2">II</th><th colspan="2">I</th><th colspan="2">II</th><th colspan="2">III</th></tr>
<tr><td>Morals</td><td colspan="2">2</td><td colspan="2">2</td><td colspan="2">2</td><td colspan="2">2</td><td colspan="2">2</td></tr>
<tr><td>Japanese language</td><td colspan="2">8</td><td colspan="2">8</td><td colspan="2">8</td><td colspan="2">8</td><td colspan="2">8</td></tr>
<tr><td>Arithmetic</td><td colspan="2">4</td><td colspan="2">4</td><td colspan="2">4</td><td colspan="2">4</td><td>girls.
3</td><td>boys.
4</td></tr>
<tr><td>Japanese history.
Geography</td><td colspan="2">3</td><td colspan="2">3</td><td colspan="2">3</td><td colspan="2">3</td><td colspan="2">2</td></tr>
<tr><td>Science</td><td>girls.
3</td><td>boys.
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<tr><td>Drawing</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td></tr>
<tr><td>Singing</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td><td colspan="2">1</td></tr>
<tr><td>Gymnastics</td><td colspan="2">3</td><td colspan="2">3</td><td colspan="2">3</td><td colspan="2">3</td><td colspan="2">3</td></tr>
<tr><td>Sewing</td><td colspan="2">5</td><td colspan="2">5</td><td colspan="2">5</td><td colspan="2">5</td><td colspan="2">7</td></tr>
<tr><td>Manual training.</td><td>girls.
2</td><td>boys.
6</td><td>girls.
2</td><td>boys.
6</td><td>girls.
2</td><td>boys.
6</td><td>girls.
2</td><td>boys.
6</td><td>girls
2</td><td>boys.
6</td></tr>
<tr><td>Agriculture</td><td>2</td><td>6</td><td>2</td><td>6</td><td>2</td><td>6</td><td>2</td><td>6</td><td>2</td><td>6</td></tr>
<tr><td>Commerce
English</td><td>2</td><td>6</td><td>2</td><td>6</td><td>2</td><td>6</td><td>2</td><td>6</td><td>2</td><td>6</td></tr>
<tr><td>Total</td><td>girls.
32</td><td>boys.
30</td><td>girls.
32</td><td>boys.
30</td><td>girls.
32</td><td>boys.
30</td><td>girls.
32</td><td>boys.
30</td><td>girls.
32</td><td>boys.
30</td></tr>
</table>

SECTION 4. Number of School Weeks in the Year.

The number of school weeks in the year is about 40.

The school year is generally divided into three terms. The summer vacation extends over 40 days from the latter part of July to the end of August, and the winter vacation, over 14 days from Dec. 25th to Jan. 7th of the following year.

The first term, therefore, covers about 15 weeks from the beginning of the school year to the summer vacation; the second term, about 16 weeks beginning immediately after the summer vacation and ending when the winter vacation begins; and the third term, about 10 weeks from the close of the winter vacation to the end of the school year.

CHAPTER III.

AIM AND SUBJECT-MATTER OF MATHEMATICAL INSTRUCTION.

SECTION 1. Aim.

Section 1 of Article IV of the Rules for Teaching in Elementary Schools, prescribes as follows:—

"The aim of arithmetic is to make children proficient in daily computations, to give them the knowledge necessary for ordinary life, and to train them in accurate thinking."

Thus it may be said that in making children proficient in daily computations and in giving them the knowledge necessary for ordinary life, the so-called substantial culture is aimed at, whilst, in rendering their thinking sound and accurate, formal culture is imparted.

SECTION 2. Subject-Matter.

The extent and arrangement of the subject-matter is summarized as follows:—

(1) Ordinary elementary schools.

Years.	Terms.	Topics of teaching.
I	1	Numeration and notation (up to ten). Addition and subtraction with numbers under ten (mental).
	2	Numeration and notation (up to twenty). Addition and subtraction with numbers under twenty (mental).
	3	Numeration and notation (up to one hundred). Simple multiplication and division (mental).
II	1	Addition and subtraction with numbers under one hundred (mental). Numeration and notation (up to one thousand).
	2	Multiplication (mental).
	3	Division (mental).
III	1	Numeration and notation (less than ten thousand). Addition and subtraction (mental and written).
	2	Multiplication (mental and written).
	3	Division (mental and written).
IV	1	Numeration and notation (less than one hundred millions). Addition, subtraction, multiplication, and division.
	2	Computation of compound numbers.
	3	Numeration and notation of decimals. Addition, subtraction, multiplication, and division of decimals.
V	1	Numeration and notation (in general). Addition, subtraction, multiplication, and division (integers and decimals). Computation of decimal compound numbers. Mensuration.
	2	Computation of compound numbers. Mensuration.
	3	Metric system of weights and measures. Foreign systems of weights and measures. Mensuration.
VI	1	Numeration and notation of fractions. Addition, subtraction, multiplication, and division of fractions.
	2	Ratio. Percentage.
	3	Review.

(2) Higher elementary schools.

Years.	Terms.	Topics of teaching.
I	1	Integers. Decimals. Compound numbers: Metric system. *Shaku-kan* system. Foreign systems. (Weights and measures.) Time, money.
	2	Fractions: Common measures. Common multiples. The four rules of fractions. Percentage: Percentage. Taxes. Public loans. Stocks. Simple and compound interest.
	3	Proportion: Direct proportion. Inverse proportion. Proportional parts.
II	1	Proportion: Direct proportion. Inverse proportion. Compound proportion.
	2	Summary review: Four rules of integers, decimals and fractions. Computation of compound numbers and mensuration. Arcs, angles, latitude, longitude, and standard time.
	3	Summary review: Proportions. Percentage.
III	1	Mensuration: Rectangle, triangle, square root. Parallelogram. Rhombus. Trapezoid, polygon.
	2	Mensuration: Circumference and diameter of circle. Rectangular parallelopiped, cube root, prism, circular cylinder, pyramid, circular cone, frustum of pyramid, frustum of circular cone.
	3	Review: Four rules. Proportions. Percentage. Interest. Arithmetical progression. Geometrical progression.

SECTION 3. Text-Books.

For use in all the elementary schools in the Empire the Department of Education compiles and publishes the text-books in accordance with the apportionment of subject-matters prescribed in the Ordinance relating to elementary schools. Those on arithmetic are divided into two kinds, one for teachers and the other for pupils. The former consist of one volume for each year, while among the latter those for the first and the second year of ordinary elementary schools are lacking. The text-books for the pupils contain a summary of the subject-matter and exercises. The books for the teachers have two columns on each page, the one containing exactly the same matter as the text-books for pupils, and the other containing important helps for teaching, methods of explanation, suggestions to teachers, and practical problems, similar to those in the pupils' books, thus helping the teachers in their work.

All the elementary schools in the Empire are obliged to use these text-books and the pupils must possess them. However, according to circumstances, each school is at liberty to omit some of the subject-matter contained in these text-books, provided the main scheme is adhered to.

SECTION 4. Details of Teaching.

Each school, observing the provisions of the Ordinance relating to elementary schools, prepares a plan of teaching by apportioning the details of the subjects contained in the state text-books, to their respective periods. We call this the details of teaching. These details of teaching are not uniform in all the schools, but the items contained are the topics of teaching and the number of periods assigned to them. In addition, suggestions for teaching, teaching appliances, reference-books, and items connected with other subjects are also recorded therein. By way of example we insert here the details of teaching in arithmetic in the

Elementary School attached to the Tokyo Higher Normal School.

First Year of the ordinary course.

First Term. Estimated teaching hours: 44.

Numeration and notation of numbers under ten; their addition and subtraction.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
1	I Colloquial counting by calling *hitotsu* (one), *futatsu* (two).	about 2 hours.	Pebbles, balls.
	(1) from one to five.	(1)	Counters, beans.
	(2) from one to ten.	(1)	Flowers, dumb-bells.
2	N. B.—(1) Begin by asking the children their own age, the age or number of the members of their family. Teach them to count by means of objects, charts, or sounds, and perfect the children's ability of counting already possessed previous to the teaching of arithmetic proper. (2) This method gives the children some vague notion of numbers under ten. Begin first with counting objects, then lead children to count independently of objects. (3) Ten pieces of counters are necessary for the use of each child.		
	II To add one to numbers under nine.	about 1 hour.	
	III To add two to numbers under five.	about 2 hours.	
	(1) To add two to one, two, three.	(1)	Bells,
	(2) To add two to four, five.		"otedama" (small bags filled with beans used in a girls' game), &c.
	(3) Exercises in the above.	(1)	
	IV To add three to numbers under five.	about 2 hours.	
	(1) To add three to one, two, three.	(1)	Eggs, boxes, &c.
3	(2) To add three to four, five.		
	(3) Exercises in the above.	(1)	

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	V To add four to numbers under five.	about 2 hours.	
	(1) To add four to one, two, three.	(1)	
	(2) To add four to four, five.		
	(3) Exercises in the above.	(1)	
	VI To add five to numbers under five.	about 2 hours.	Dolls, rings, &c.
4	(1) To add five to one, two, three, four.	(1)	
	(2) To add five to five.		
	(3) Exercises in the above.	(1)	

N. B.—(1) As among all additions with numbers under ten, children find it most difficult to add four to five and five to four, it is necessary from the outset to give a very clear conception regarding it.

(2) As adding five to five is most frequently needed in various computations, children should be trained to become familiar with it.

(3) Children should be led indirectly to understand that in adding the sum does not vary when the arrangement of the numbers to be added is changed in any way.

Topics of teaching.	Hours.	Teaching appliances and reference-books.
VII Formal counting by calling one, two (with suffixes).	2 hours.	Thick paper. Ordinary paper.
(1) from one to ten.	(1)	Boards, pencils, slate pencils, chopsticks, &c.
(2) Exercises in the above.	(1)	

N. B.—(1) At first teach children to count by means of those things that are to be counted by sheets or pieces, and later teach them to count unaided by objects.

(2) This should be done in close connection with the previously practiced method of colloquial counting as *hitotsu, futatsu*.

(3) Attention is to be called to the fact that when the unit names of things are attached to numbers, there sometimes occurs a phonetic change in the reading of the resulting concrete

numbers. For example, when we say one piece, six pieces or ten pieces, we should say for the sake of euphony, "ippon," "roppon," "jippon," instead of saying "ichi hon" (one piece), "roku hon" (six pieces) or "jū hon" (ten pieces).

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	VIII To add two to numbers	about	
5	above six.	$1\frac{1}{2}$ hours.	Pictures,
	(1) To add two to six, seven, eight.	(1)	picture cards, pens.
	(2) Exercises in the above.	(.5)	Drawings. Flags.
	IX To add three or four to numbers above six.	$1\frac{1}{2}$ hours.	
	(1) To add three to six, seven.	(1)	
	(2) To add four to six.		
	(3) Exercises in the above.	(.5)	
	X To add six, seven, eight, or nine.	about 3 hours.	
6	(1) To add six to one, two, three, or four.	(1)	
	(2) To add seven to one, two, three.		
	(3) To add eight to one, two.	(1)	
	(4) To add nine to one.		
	(5) Exercises in the above.	(1)	
	XI To denote numbers under ten in figures.	about 5 hours.	
	(1) To denote 1 and 2.	(1)	
7	(2) ,, ,, 3 ,, 4.	(1)	
	(3) ,, ,, 5 ,, 6.	(1)	
	(4) ,, ,, 7 ,, 8.	(1)	
8	(5) ,, ,, 9 ,, 10.	(1)	
	N. B.—10 should be taught merely as figure.		
	XII To compare two numbers in the relation of greater-and-smaller.	about $\frac{1}{2}$ hour.	

N. B.—(1) Children are to be prepared for exercises in subtraction by acquiring an accurate conception of the relation of numbers as greater-and-smaller.

(2) Let children be informed of the serial order of numbers from one to ten, so that they may easily recognize where each number stands.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	XIII To subtract one, two.	about 2½ hours.	
	(1) „ „ one.	(.5)	
	(2) „ „ two.	(1)	
	(3) Exercises in the above.	(1)	
	N. B.—This instruction should always be given in connection with exercises in addition.		
	XIV To subtract three.	about 2 hours.	
9	(1) To subtract three from ten, nine, eight, seven, six, five or four.	about 1½ hours.	
	(2) Exercises in the above.	(.5)	
	XV To subtract four.	about 2 hours.	
	(1) To subtract four from ten, nine, eight, seven, six, five.	about 1½ hours.	
	(2) Exercises in the above.	(.5)	
	XVI To subtract five.	about 1½ hours.	
10	(1) To subtract five from ten, nine, eight, seven, six.	(1)	
	(2) Exercises in the above.	(.5)	
	XVII To subtract six.	about	Text-books.
	(1) To subtract six from ten, nine, eight, seven.	1½ hours.	Note-books.
	(2) Exercises in the above.	(.5)	Picture-books, &c.
	XVIII To subtract seven, eight, nine.	about 2 hours.	
11	(1) To subtract seven from ten, nine, eight.		

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	(2) To subtract eight from ten, nine.	(1.5)	
	(3) To subtract nine from ten.		
	(4) Exercises in the above.	(.5)	
	XIX Zero, i. e., the difference between two equal numbers.	about 1 hour.	
12	XX Chinese characters denoting numbers from one to ten.	about 3 hours.	
	(1) To denote one, two, three, four.	(1)	
	(2) „ „ five, six, seven.	(1)	
	(3) „ „ eight, nine, ten.	(1)	
	N. B.—The conception of nought should be explained and its symbol 0 be taught.		
13	XXI Review.	about 6 hours.	

Second Term. Estimated teaching hours: about 48.

Numeration and notation of numbers under 20; their addition and subtraction.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
1	I Chinese figures denoting numeration and notation of numbers from 11 to 19.	about 2 hours.	
	(1) Numeration of numbers from 11 to 19.	(1)	
	(2) Chinese characters representing these numbers and exercises in the above.	(1)	
	N. B.—(1) In teaching the numeration of numbers, care should be taken to develop in the minds of children the fundamental idea of numeration.		

(2) It is not good to urge children to be quick in inverse computation. For, if the idea of the serial order of natural numbers be clearly given by means of direct computation, it will be easy to commit it to memory.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
2	II To add one, two, three.	about 2 hours.	
	(1) To add one to eleven.... eighteen.	(1)	
	(2) To add two to eleven.... seventeen.		
	(3) To add three to eleven.... sixteen.		
	(4) Exercises in the above.	(1)	

N. B.—(1) Lead children to assimilate the previously acquired conceptions, and to understand and remember them clearly.

(2) Let the process of computation be made as follows:—

$$15+2=5+2+10.$$

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	III To add four, five, six, seven, eight.	about 3 hours.	
	(1) To add four to eleven.... fifteen.		
	(2) To add five to eleven.... fourteen.	(1)	
	(3) To add six to eleven.... thirteen.		
	(4) To add seven to eleven.... twelve.		
	(5) To add eight to eleven.	(1)	
	(6) Exercises in the above.	(1)	
3	IV To add numbers above eleven.	about 2 hours.	
	(1) To add eleven to one.... eight.		
	(2) To add twelve to one.... seven.		
	(3) To add thirteen to one.... six.	(1)	

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	(4) To add fourteen to one.... five.		
	(5) To add fifteen to one.... four.		
	(6) To add sixteen to one.... three.		
	(7) To add seventeen to one.... two.		
	(8) To add eighteen to one.	(1)	
4	V Notation of numbers from ten to nineteen.	about 2 hours.	
	(1) Notation of 1....19.	(1)	
	(2) Exercises in the above.	(1)	

N. B.—Children should first be taught the notation of numbers from ten to about thirteen, and then be led to know the rest by induction.

(2) Children's attention should be called to the method of denoting the number ten by putting the figure one as the higher digit, and nought as the lower digit.

Let them compare with this such numbers as 12, 13.

(3) As figures stand for different values, according to their positions, care should be taken in the notation of numbers greater than ten, and the figures standing in the unit place should be properly arranged. Also in the notation of more than two numbers in succession, the children should be taught to leave a regular space between every two numbers.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	VI To compare two numbers in the relation of greater-and-smaller.	about 1 hour.	

N. B.—The aim of comparing two numbers in the relation of greater-and-smaller is to lay the foundation for subtraction by affording the conception of the said relation.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	VII To subtract one, two, three.	about 2 hours.	
	(1) To subtract one from nineteen....twelve.		

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
5	(2) To subtract two from nineteen....thirteen.	(1)	
	(3) To subtract three from nineteen....fourteen.		
	(4) Exercises in the above.	(1)	
	N. B.—Let the process of computation be made as follows:— $15-2=5-2+10$.		
	VIII To subtract four, five, six, seven, eight.	about 2 hours.	
	(1) To subtract four from nineteen....fifteen.		
	(2) To subtract five from nineteen....sixteen.	(1)	
	(3) To subtract six from nineteen....seventeen.		
	(4) To subtract seven from nineteen, eighteen.		
	(5) To subtract eight from nineteen.		
	(6) Exercises in the above.	(1)	
	IX To find the difference between two numbers.	about 2 hours.	
6	(1) Difference between one and two....nineteen.		
	(2) Difference between two and three....nineteen.		
	(3) Difference between three and four....nineteen.		
	(4) Difference between four and five....nineteen.		
	(5) Difference between five and six....nineteen.		
	(6) Difference between six and seven....nineteen.	(1)	
	(7) Difference between seven and eight....nineteen.		
	(8) Difference between eight and nine....nineteen.		

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	(9) Difference between nine and ten....nineteen.		
	(10) Exercises in the above.	(1)	
	N. B.—(1) Children should first be taught to find the difference of two numbers, by comparing them with the help of counters or objects. When they have learned this, then they should be drilled with abstract numbers. (2) By showing the series of numbers, children should be drilled in finding the difference between any two numbers selected from the series. (Diagram showing the serial order of numbers).		
7	X Addition in which the result of adding a simple number to nine or eight becomes a number above eleven.	about 4 hours.	
	(1) To add two, three, four, five to nine.	(1)	
	(2) To add six, seven, eight, nine to nine.	(1)	
	(3) To add three, four, five, six to eight.	(1)	
	(4) To add seven, eight, nine to eight.	(1)	
	XI Addition in which the result of adding a simple number to seven, six becomes a number above eleven.	about 3 hours.	
8	(1) To add four, five, six, seven, eight, nine to seven,	(1)	
	(2) To add five, six, seven, eight, nine to six.	(1)	
	(3) Exercises in the above.	(1)	
	N. B.—Children should be taught to express in words the process of computation. For example, in expressing $9+5=14$, they should say: "We separate the five into one and four, then add the one to nine, and get ten. By adding the four to this ten, we obtain fourteen."		

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
9	XII Addition in which the result of adding a simple number to five, four becomes some number above eleven.	about 2 hours.	
	(1) To add six, seven, eight, nine to five.	(1)	
	(2) To add seven, eight, nine to four.	(1)	
	XIII Addition in which the result of adding a simple number to three, two becomes some number above eleven.	about 2 hours.	
10	(1) To add eight, nine to three.	(1)	
	(2) ,, ,, nine to two.	(1)	
	(3) Exercises in the above.		
	N. B.—The process of computation is the same as in adding to a simple number any smaller number than the above.		
11	XIV Subtraction in which the result of subtracting two, three becomes a simple number.	about 1 hour.	
	XV Subtraction in which the result of subtracting four, five becomes a simple number.	about 2 hours.	
	(1) To subtract four from eleven ....thirteen.	(1)	
	(2) To subtract five from eleven ....fourteen.	(1)	
12	XVI Subtraction in which the result of subtracting six, seven becomes a simple number.	about 3 hours.	
	(1) To subtract six from eleven ....fifteen.	(1)	
	(2) To subtract seven from eleven....sixteen.	(1)	
	(3) Exercises in the above.	(1)	

N. B.—Children should be taught to express in words the process of computation.

Weeks.	Toipcs of teaching.	Hours.	Teaching appliances and reference-books.
13	XVII Subtraction in which the result of subtracting nine, eight becomes a simple number.	about 3 hours.	
	(1) To subtract eight from eleven....seventeen.	(1)	
	(2) To subtract nine from eleven ....eighteen.	(1)	
	(3) Exercises in the above.	(1)	
14	XVIII To subtract numbers greater than eleven.	about 4 hours.	Models of ships, warships. Picture post-cards.
	(1) To subtract eleven from nineteen....eleven.	(1)	
	(2) To subtract twelve from nineteen....twelve.		
	(3) To subtract thirteen from nineteen....thirteen.	(1)	
	(4) To subtract fourteen from nineteen....fourteen.		
	(5) To subtract fifteen from nineteen... fifteen.	(1)	
	(6) To subtract sixteen from nineteen....sixteen.		
	(7) To subtract seventeen from nineteen....seventeen.		
	(8) To subtract eighteen from nineteen....eighteen.		
	(9) To subtract nineteen from nineteen.		
	(10) Exercises in the above.	(1)	
	XIX Numeration and notation of twenty and computation in connection therewith.	about 2 hours.	
	(1) Numeration and notation of twenty.	(.5)	

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	(2) Computation concerning twenty.	(1.5)	
	N. B.—Let the process of computation be made as follows:— $19-15=19-10-5$.		
15	XX Review.	about 4 hours.	
	(1) "Text-book on Arithmetic." p. 38.	(2)	
	(2) "Text-book on Arithmetic." p. 39.	(1)	
	(3) "Text-book on Arithmetic." p. 40.	(1)	

Third Term. Estimated teaching hours: about 36. Numeration and notation of numbers under one hundred. Fundamental conception of addition, subtraction, multiplication and division.

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	I Numeration and notation of tens and computation concerning them.	about 4 hours.	
1	(1) Numeration and notation of tens.	(1)	
	(2) To add tens to tens.	(1)	
	(3) To subtract tens from tens.	(1)	
	(4) Exercises in the above.	(1)	
	II Numeration and notation of numbers consisting of tens and units, together with their computation.	about 4 hours.	
2	(1) Numeration and notation of numbers consisting of tens and units.	(1)	
	(2) To add units to tens.		

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	(3) Subtraction in which units are subtracted from tens and units, so that tens remain.	(1)	
	(4) To add tens to units.		
	(5) To subtract tens from tens and units.	(1)	
	(6) Exercises in the above.	(1)	
3	III Numeration and notation of one hundred.	about 3 hours.	
	(1) Numeration and notation of one hundred.	(1)	
	(2) Computation concerning one hundred.	(2)	
	IV To count numbers directly and inversely.	about 2 hours.	
	(1) To count directly.	(1)	
4	(2) „ „ inversely.	(1)	
	V To double, to treble.	about 6 hours.	
	(1) To double one, two, three, four, five.	(1)	
5	(2) To double six, seven.	(1)	
	(3) „ „ eight, nine, ten.	(1)	
	(4) To treble one, two, three, four.	(1)	
	(5) To treble five, six.	(1)	
	(6) Exercises in the above.	(1)	
	N. B.—(1) As this lesson aims at giving children the fundamental conception of multiplication, it should be taught in connection with successive addition. (2) The multiplication sign and multiplication table should not be given as yet.		
6	VI To find how many times a number is contained in another.	about 4 hours.	
	(1) The inverse of doubling.	(2)	

Weeks.	Topics of teaching.	Hours.	Teaching appliances and reference-books.
	(2) The inverse of trebling.	(2)	
7	VII To divide into equal parts.	about 4 hours.	
	(1) To divide into two equal parts.	(1.5)	
	(2) To divide into three equal parts.	(1.5)	
	(3) Exercises in the above.	(1)	
8	VIII Review.	about 9 hours.	
	(1) Text-book. p. 60.	(1)	
	(2) ,, p. 61.	(1)	
	(3) ,, p. 62.	(1)	
	(4) ,, p. 63.	(1)	
	(5) ,, p. 64.	(1)	
	(6) ,, p. 65.	(1)	
	(7) ,, Miscellaneous problems.	(3)	

(End of the details of teaching for the First Year)
(The rest is omitted)

CHAPTER IV.

Method of Teaching.

SECTION 1. In General.

In the Regulations for carrying out the Imperial Ordinance relating to elementary schools, chap. I, sec. 1, art. 4, item 5, we read :—

"In teaching arithmetic, efforts should be made to set the children free in application by rendering them accurate in understanding and proficient in operation. At the same time, care should be taken to enable them to explain correctly the methods and reasons of different operations and to render them proficient in mental arithmetic."

Accordingly, in teaching the rules of computation, we should not proceed in a merely mechanical way, but should attempt to make the children practice computations after understanding fully the reason for them and to have those reasons explained by the children themselves. In computations, children should strive after exactness and not after mere speed. By repeated practice they will unconsciously acquire speed of operations. In teaching the solution of applied problems, we ought first to impart clear and accurate conceptions of the facts involved and of their relations; then proceed to give a clear understanding of the numbers and numerical relations involved in those facts, and finally come to the solution of the problems. There are, however, some exceptional cases wherein the rules for computation and solution are to be taught mechanically. This is the case whenever the principle of solution cannot be understood by the children although the practical formula is indispensably necessary.

Exercises in mental computation are limited to numbers below 100, and special emphasis is laid on fundamental computations and their processes. What are here called the fundamental computations include adding a simple number to another simple number and the reverse, moreover finding the product of two simple numbers and the reverse. The process of computation is intended to show how the computation is carried on. For example, in $35-18$, first analyzing 18 into 10 and 8, we have $35-10=25$. Next we have $25-8=17$. The number of hours for practising mental computations is greatly limited in the classes above the third year of the ordinary elementary schools on account of the exercises in written arithmetic and in "soroban" calculation being added. But whenever opportunity offers, mental calculation is practiced. This exercise is limited to the numbers below 100, and the most difficult computations are selected for this drill.

Even with numbers above 100, simple computations are selected for mental exercise. This is because mental arithmetic is especially useful and convenient in ordinary life.

The general outline of the method of teaching in each year is given below :—

SECTION 2. First Year of the Ordinary Elementary School.

(1) The chief aim in the present year is to lay the foundation for computations in addition and subtraction. The numbers to be given in the first term do not go beyond ten.

The work begins with the numeration and the computation of objects. Then digits and symbols of operation are taught, and children are led to compute either by sight or mentally by hearing problems orally proposed. The numbers to be given in the second term go up to twenty, and in the third term, besides the review exercises on what has been given in the first and the second term, some very simple computations with numbers less than one hundred are assigned.

(2) Computations with numbers up to ten.

As children have some notion of numbers previous to entering school, we examine at first, how far their knowledge about numbers goes in order to begin the work in the most appropriate way. As a result of this investigation we find that children in cities generally know how to count the numbers less than ten, while those in the country do not, and many of them know no more than to mention their own age.

As thus children already know how to count some numbers and have some vague notions of number, this is taken as the starting point of instruction, and they are taught how to count the numbers less than ten by objects presented to their eyes.

The objects used in teaching numeration are such things as hats, balls, flags, pencils, pieces of chalk or china, note-books,—things which are easy to handle and which the children will naturally be inclined to count as they are accustomed to handle them. In selecting these objects care is taken, of course, to have variety both in kind and in shape. These shapes are to include solids, planes, and straight lines. The different kinds of objects are selected so as to comprise the greatest possible variety in the names of concrete numbers counting by means of the following suffixes: *hon* (pieces), *mai* (sheets), *hiki* (animals), *ha* (birds), *satsu* (volumes), *ji* (letters), *nin* (men), *ha* (bundles), *hyō* (bags), *ken* (houses), *sō* (vessels), *kago* (baskets), *fukuro* (sacks), *hako* (boxes), *nichi* (days), *kumi* (sets), *sen* (cents), *ka* (things), *gyō* (columns), *retsu* (lines), &c.

(3) Addition with numbers under ten.

This begins by adding one and proceeds successively, by adding two, three, &c. Children are taught intuitively to understand the idea of addition and see its result, by showing them real objects. (object computations).

Then they are taught by means of a rough drawing on the black-board or a slate, or on paper, and are led to recognize the result. (computations by the aid of diagrams).

Last of all they are required to find out the result mentally, on hearing the problems proposed orally by the teacher. (mental computation). In this grade of teaching, it is usual to let children have real objects, such as pebbles, shells, beans, &c., which are easily handled and preserved.

(4) Figures under ten.

The idea of different numbers having been imparted by means of objects, diagrams, and words, and figures being then presented, children are led to know that these represent the former and gradually a mechanical connection is formed

between them. Not more than two figures are given in one hour and thereafter the children are drilled in reading and writing these figures. They are also practiced in recognizing these figures occurring in problems, or to express in figures the numbers that are orally given so as to comfirm the connection between the notion of numbers and the figures.

The reason for the peculiar notation of 10 is to be given after the notation of numbers above 11 has been taught.

After figures are given, computations by figures are added to computations by objects, and by diagrams, and to mental computation.

(5) Subtraction with numbers under ten.

The method is the same as that of addition above stated.

In teaching subtraction by means of objects, if the remainder is not intuitively recognized, the computation by way of "counting off," is applied.

(6) Numeration and notation of numbers from 11 to 19.

Children are first taught to obtain the numbers from 11 to 19 together with their numeration, by adding to 10 the numbers from 1 to 9. They are then taught to understand the position of tens and of units, by means of a computing apparatus based upon the notion of positions or a diagram on the black-board constructed on the same principle. They are further taught to understand that when the number in the units-place amounts to ten, it should be transferred over to the tens as one group, and that denoting this 10 by means of a mark in the tens-place and also recognizing the marks still remaining, if any, in the units-place, a number is to be formed out of the marks in both places, e. g., if there is one mark in the tens-place, and there are two marks in the units-place, the number is 12. And after this children are taught, that the left hand 1 of 12 denotes

one group of ten units and the right hand 2, two of one unit. The teaching of the notation of numbers up to 19 is based upon this principle.

The reason for the notation of 10 is here to be explained. Namely, children are taught to understand that, as there is one group in the tens-place, 1 is noted as the mark representing it, and, as there is none in the units-place, simply 0 is noted, and in this way 10 is duly expressed.

(7) Addition in which the sum becomes more than 10, and subtraction which is the reverse of this addition.

a. Computation wherein there occurs no change in the number ofdig its.

For example, in computing $12+5$ the children are taught to operate as follows:

First find the sum of $2+5$, which is 7, and then add this 7 to 10, which is 17. The reverse process $17-5$ is taught like this: $7-5=2$ and $10+2=12$. As children are already drilled in such computations as $2+5$ or $7-5$, what they are specially taught to comprehend now and to keep in mind is the method of analysis and synthesis of numbers. Therefore, special care is taken to make the process of computation perfectly clear. In this grade of instruction, the analysis and synthesis of numbers are at times concretely shown by means of counters. Children are also asked to show clearly the process of computation either by means of counters or by words.

b. Computation wherein there occurs a change in the number of digits.

For example, in the computation of $7+5$, it is shown how we may take 3 from 5, then add the 3 to 7 to make 10, and then we obtain 12 by adding the remaining 2 to 10; and for the reverse process $12-5$, it is explained that we first subtract 5 from 10, and obtain 5, and that we

obtain 7 by adding 2 to 5. For this subtraction there is another method of computation, by which we first subtract from 12 only 2 contained in 5, and obtain 10, and then we take 3 from 10 to get 7. However, this method seems to be less frequently followed than the other.

Computations of this kind together with the addition and subtraction of numbers under ten given in the first term, form the foundation of addition and subtraction in general. Therefore, when this is first taught, children are made to comprehend it by means of the analytical computation mentioned above, and even in exercises the same process is insisted on. However, as the children advance in their exercises, they are required to recall at once, without repeating the intermediate process, that $7+5$ is 12 or that $12-5$ is 7.

When the fundamental computations in addition and subtraction are sufficiently practiced problems are to be given sometimes in systematical arrangement, and at other times not in systematical arrangement, both in due proportion. In regard to the problems given in words or in figures, care is taken that neither shall be unduly preferred. Moreover, as the children gradually advance in the practice of computation, they are enjoined more difficult problems and are urged to exert more of energy in their solution.

(8) Numeration and notation of numbers up to one hundred.

The numeration and notation of numbers exceeding 10 are taught by making use of a contrivance usually called counting-board and diagrams which are so constructed as to show clearly the characteristics of decimal notation. Calculations are limited to simple addition and subtraction within the limits of numbers not exceeding 10. Thus it will not be necessary to point out special method of teaching in this regard.

(9) Counting-board and number-chart.

The counting-board which has been long and extensively used is the Russian one. Besides this, there have been invented various other kinds of apparatus, quite similar and consequently not worth mentioning specially. The above mentioned counting-board seems to be very convenient for the purpose of teaching the characteristics of decimal notation, though being of the latest invention, it is not yet widely prevalent. In its structure, it is apparently similar to the Russian instrument. The rods, on which the counters are strung, being arranged vertically, render it easy to show the respective position of units, tens, and hundreds. Moreover, the frame around the apparatus being so constructed as to allow the apparatus to take any position, it may also be used in the same way as the Russian *tschotü*. Further, if the rods are put up vertically, and a cross-bar be attached it may serve in lieu of our *soroban*. It is, therefore, not only serviceable in teaching the decimal notation, but it is also of great help in teaching *soroban*-calculation, and to keep up the connection between *soroban*-calculation and written arithmetic.

The number-chart is used merely in place of real apparatus. It is not utilized to impress the idea of numbers and to facilitate their analysis and synthesis. Consequently the relative merits of the chart of numbers in series and that of numbers in groups, have not been much investigated.

SECTION 3. Second Year of the Ordinary Elementary School.

(1) The chief aim in the present year is to train the children in mental arithmetic with numbers under one hundred and to establish the foundation of multiplication and division, by making them familiar with the multiplication of two simple numbers and the reverse operation. In the first term, addition and subtraction with numbers under

one hundred are assigned, and towards the end of the term, the numeration and notation of numbers under one thousand are given. In the second term, the fundamental computations of multiplication are taught, i. e., the multiplication table with its simple applications. In the third term, the reverse of fundamental multiplication is chiefly taught, i. e., division in which both the divisor and the quotient are simple numbers. Also simple division in which the quotient is a number of two or three digits, and division by ten or hundred are given.

(2) To add a simple number, and its inverse.

A) Cases in which the each partial sum is less than 10.

As a preparatory step in teaching this addition, the computation of finding the sum of two simple numbers that does not exceed ten is assigned, together with that of adding a simple number to a number of two digits so that the sum of the units does not exceed ten. What is to be insisted on with special clearness before the children's eyes, is the process of computation. For example, $35+4$ is taught as $5+4=9$ and $30+9=39$, therefore the required answer is 39.

In assigning problems, the children are first taught to compute by seeing arithmetical expressions written on the black-board or in their note-book. Then they are asked to solve problems given orally.

B) Cases in which the partial sum of the first order goes over to the second order.

By way of preparation there are assigned additions of two simple numbers in which the sum exactly amounts to ten, together with those in which ten is added to numbers of some tens. Then, children are taught that the solution depends upon a computation similar to that of case *A*).

For example, in $34+6$, it is shown that $4+6=10$, and $30+10=40$, therefore, the answer is 40.

The remarks concerning the assignment of problems in case *A*), hold good here, and need not be repeated.

C) Cases in which the partial sum of each order exceeds ten.

By way of preparatory work, there are reviewed additions with two simple numbers in which the sum exceeds ten. The same process of computation as in cases *A*) and *B*) is to be followed also in this case. For example, in $29+7$, it is shown that $9+7=16$, and $20+16=36$, therefore the required answer is 36.

D) The reverse of case *A*).

By way of preparatory work, subtraction of numbers under ten is reviewed together with cases in which a simple number is subtracted from a number of two digits involving no process of borrowing from the next higher digit. The process of computation is as follows :—

Example : $39-2$.

$9-2=7, \quad 30+7=37.$

$\therefore \quad 39-2=37.$

E) The reverse of case *B*).

To serve as preparatory work, there is a review of subtractions in which a simple number is subtracted from ten or twenty. The process of computation is as follows :—

Example : $30-4$.

$10-4=6, \quad 20+6=26.$

$\therefore \quad 30-4=26.$

F) The reverse of case *C*).

By way of preparatory work, there are reviewed subtractions in which a simple number is subtracted from a number of tens and units, the remainder being a simple number. The process of computation is as follows :

Example: $34-6$.
$14-6=8, \quad 20+8=28.$
$\therefore \quad 34-6=28.$

(3) Additions with numbers of two digits and the reverse.

A) Additions in which tens are added.

Preparatory work: Of additions in which tens are added to tens, those are given in which the sum does not attain three digits. The process of computation is as follows:—

Example: $39+20$.
$30+20=50, \quad 50+9=59.$
$\therefore \quad 39+20=59.$

B) Additions in which tens and units are added.

a. Additions performed by adding such numbers to a simple one that the sum of the units does not overlap into the second digit.

Preparation: additions of two simple numbers in which the sum does not make ten; also additions in which a simple number is added to tens.

Process of computation:—

Example: $2+47$.
$2+7=9, \quad 40+9=49.$
$\therefore \quad 2+47=49.$

b. Additions in which by adding the tens and units to a simple number, the sum of the units overlaps into the tens.

Preparation:— additions of two simple numbers in which the sum attains or exceeds ten; also additions in which ten or ten and units are added to tens.

Process of computation:—

Example: $9+28$.
$9+8=17, \quad 20+17=37.$
$\therefore \quad 9+28=37.$

c. Addition of the previously mentioned numbers to numbers of two digits.

Preparation: additions of simple numbers in which the sum grows by tens, together with those in which some tens or ten and units are added to tens.

Process of computation:—

Example: $19 + 28.$

$10 + 20 = 30,$

$9 + 8 = 17,$

$30 + 17 = 47.$

$\therefore \ 19 + 28 = 47.$

C) The reverse of case *A*).

Preparation: subtraction in which tens are subtracted from tens.

Process of computation:—

Example: $59 - 20.$

$50 - 20 = 30,$

$30 + 9 = 39.$

$\therefore \ 59 - 20 = 39.$

D) The inverse of case *B*).

Preparation: subtraction in which some units are subtracted from tens and units, together with those in which simple numbers are subtracted from ten and units, leaving a remainder of some simple number.

Process of computation:—

Example: $47 - 28.$

$47 - 20 = 27,$

$27 - 8 = 19.$

$\therefore \ 47 - 28 = 19.$

(4) Multiplication table.

By way of preparatory work, successive addition is practiced, and the result of the succesive additions is called out as it stands in the multiplication table. It is shown, for example, that, since $2 + 2 + 2 + 2 + 2 = 10$, if 2 be added

five times in succession, the result amounts to 10, and put in the abbreviated form $2 \times 5 = 10$, it is to be called in the Japanese fashion, "five two ten." The multiplication table itself is first practiced both progressively and retrogressively, but in accordance with the systematic arrangement. Later on, it is assigned regardless of the systematic arrangement.

At the beginning of the practice, the euphonic character of the table is appealed to, and each table is repeatedly exercised. But later on, the exercise in euphonic utterance is dispensed with, except in difficult tables.

(5) Division.

By way of preparatory work, the multiplication table is exercised in progressive and retrogressive order both in accordance with the systematic arrangement. And then, the meaning of contra-division being explained, a dividend and a divisor are introduced with the purpose of showing that, in order to find how many times the latter is contained in the former, some multiple of the latter is computed by means of the multiplication table, in order to be contracted with the former so as to find the answer, or quotient. In order to come upon this very quotient, the multiplication table is repeatedly exercised till it can be recited without hesitation either in progressive or in retrogressive order. Only after contra-division is mastered equi-division is entered upon.

The classification of contra-division and equi-division is made in accordance with the ordinary terminology of children. Thus they are discriminated as follows :—

Contra-division is the division answering the question : how many each.

Equi-division is to divide a number into so and so many parts.

How contra-division with concrete numbers is done may be seen from the following example :

Example: If two sheets of paper are given to each child and there are ten sheets, for how many children will there be two each?

Solution: $10^{\text{sheets}} \div 2^{\text{sheet}} = 5.$

$\therefore$ for 5 children.

SECTION 4. Third Year of the Ordinary Elementary School.

(1) The chief aim in this year is to teach written addition, subtraction, multiplication and division with numbers less than ten thousand, and in order to acquire thorough familiarity with these operations the first term is given over to addition and subtraction; the second, to multiplication; and the third, to division.

The teaching of written arithmetic, is based, as a matter of course, upon the mental arithmetic learned in the preceding two years. Besides, mental arithmetic itself is constantly exercised in order to make it thoroughly familiar. The exercise in mental arithmetic given for the sake of written arithmetic is mostly in fundamental computations, and is reviewed at the beginning of the teaching hours in written arithmetic. For the exercise in mental arithmetic itself, five or ten minutes are usually assigned at the end of the hours for written arithmetic. According to circumstances, however, one whole hour may sometimes be given to exercises in mental arithmetic.

As the school year advances the practice of analysis for computations in mental arithmetic becomes less frequent. For example, $27+36$ is to be computed as $27+30=57$ and $57+6=63$; $63-36$, as $63-30=33$ and $33-6=27$.

(2) Written addition.

The very first lesson in written addition should not only explain the method but also demonstrate its superiority—even though but slightly. In selecting examples, therefore, for the teaching of written addition, such are taken as

are rather difficult for mental addition; e. g., two numbers of three digits or four of two digits. Further, in computing them in accordance with the method of written addition, those are to be selected, if possible, which contain the least difficulty; e. g., a set in which the partial sums of each digit do not exceed ten.

There is another consideration showing the need of selecting for the first lesson in written addition, sets in which the partial sums of each digit do not exceed ten. In order to make clear that in both written and mental addition the principle of computation is the same, it is necessary to begin addition with more digits in the former than in the latter. If, however, the partial sum of any digit should exceed ten, there would arise a difficulty about putting down the result.

After the utility as well as the method of written addition is thus made clear, an example is assigned in which the partial sum of one place exceeds ten, with a view to making the children compute it by starting from the higher digits. But they soon recognize the inconvenience attached to this way of computing, as it necessarily involves erasure and correction, and they naturally feel the need of computing by starting from the lower digits.

As for the tens that go over to the next higher place they are not allowed to be marked by means of either dots or figures, but they are immediately to be added to the first number of the next place. To take as an illustration 358 + 217, the computator says to himself, 8, 15, and puts down 5; then he says, 1, 6, 7, and puts down 7; then again he says, 3, 5, and puts down 5; and thus he gets the answer, 575.

$$\begin{array}{r} 358 \\ \underline{217} \\ 575 \end{array}$$

The operation sign is commonly not allowed to be attached to the operation form. The same holds good also in regard to written subtraction and multiplication.

(3) Written subtraction.

What is to be attended to in regard to the connection between mental and written subtraction is the same as in the case of addition.

There are three methods of computation in case the subtrahend of a certain digit is greater than, and consequently not to be directly subtracted from, the minuend of the same place. First: one unit is taken from the minuend of the next higher place; it is added as 10 to the minuend of the place in question; and then the subtraction is performed. Second: the subtrahend is immediately subtracted from the 10 brought down from the minuend of the next higher place; and then the remainder is added to the minuend of the place in question. Third: the subtrahend is subtracted from 10; the remainder is added to the minuend of the place in question; and then the subtrahend of the next higher place being increased by one is to be subtracted from the minuend of that place. Of the three, the first seems to be most prevalent.

(4) Written multiplication.

As far as the connection between mental and written multiplication is concerned, the same holds good as in the case of addition, likewise the prohibition to mark the tens to be carried over by dots or figures. The following example is given by way of illustration.

Example: $256 \times 3 = 768.$

$$\begin{array}{r} 256 \\ 3 \\ \hline 768 \end{array}$$

(5) Written division.

Of the methods of written division at present used in Japan, the following two are the leading ones:

Example: $143 \div 13$.

```
(A)  13 ) 143 ( 11        (B)          11
          13                        ______
         ----                  13 ) 143
           13                        13
           13                       ----
         ----                         13
            0                         13
                                    ----
                                       0
```

Method (B) is convenient for emphasizing the places of the quotient, but is inapplicable to cases in which the dividend happens to be the result of addition, subtraction, and multiplication.

Method (A) is applicable to all cases, but is inconvenient for beginners to ascertain the places of the quotient.

On account of the reasons just given, although (B) is employed in the first lessons of written division in the third year, yet (A) should be adopted as the ordinary form of written division given in the fourth year, but for convenience' sake both (A) and (B) are used.

In regard to the two kinds of division, they are named respectively division into equal parts and division by a series of subtractions. In the second year, they were distinguished by their colloquial appellations in order not to puzzle the learners by difficult technical names whose meaning is not understood. These popular designations, tedious as they are, had to be put up with. At this stage, however, the subject-matter ought to have become sufficiently clear so that the technical terms may be adopted.

SECTION 5. Fourth Year of the Ordinary Elementary School.

(1) The chief aim in this year is to practice written addition, subtraction, multiplication, and division with numbers less than one hundred millions, and also to teach by way of repetition an easy method of computation with compound numbers and decimal fractions. In the first term, written addition, subtraction, multiplication, and division

constitute the main work to which are added some easy computations with compound numbers, together with those in which certain numbers are divided by two, three, four, five, ten, as well as those in which the half, two-thirds, three-fourths, four-fifths, three tenths, etc., of certain numbers are to be obtained. In the second term, compound numbers are mainly given; in the third, decimal fractions in addition, subtraction, multiplication and division. Decimal multiplication and division, however, are rather reserved for the fifth year.

(2) Written addition, subtraction, multiplication and division.

As regards written addition, subtraction, multiplication and division, the method of computation has been taught in the third year; so, in the present year, the art of computation is to be thoroughly acquired by repeated exercises.

Computations with large numbers are seldom assigned. The main exercises in computation to be taken, are:

In addition, sums consisting of several numbers of five or seven digits.

In subtraction, corresponding to addition.

In multiplication, sums whose multipliers consist of numbers of two or three digits.

In division, dividends whose divisors and quotients both consist of two or three digits.

The best exercises should not be either too complex or too simple.

In regard to the method of division, method (A), i. e., the method most generally applicable, is given in the present year as was mentioned under that heading in the third year, for it is indispensable in the solution of applied problems as well as in compound numbers.

(3) To find the fractional parts of numbers.

In order to teach, for example, how to find $\frac{2}{3}$ of 9, first

it is explained how "'3 equal parts' is to be contracted into '3 parts,'" and then two or three exercises are assigned. Next it is shown how "'2 times the result of 3 parts' is to be contracted into 'twice one third,'" and then this is exercised by similar examples.

Fractions proper are to be taught in the sixth year.

(4) Compound numbers.

The first-step in the teaching of compound numbers is taken in the second year. By the end of the third year, the following relations among various units are to be completed:—

Length (*jō, shaku, sun*).
Distance (*ri, chō, ken, shaku*).
Weight (*kan, monme, kin*).
Capacity (*koku, to, shō, gō*).
Time (week, day, hour, minute, second).
Area (*chō, tan, se, bu*).
Money (*yen, sen, rin*).

And yet, as the aim is simply to make clear the relations between different units, there seldom occurs a compound number having more than two different units. Therefore in the present year, the general method of computation of compound numbers is to be built solidly on the basis of these acquired relations between different units.

With regard to compound units, those that serve as standards are caused to be clearly conceived either by inspection or by actual measurement. For the others, they are rather left to be conjectured from the previously noted relations between different units.

For compound computations, the numbers containing less than three different units are chiefly adopted.

In regard to the method of reducing compounds, the normal method is explained first, and later on the more convenient one.

If, for example, 2 *ri* are to be reduced to *chō*, method (R) is normal, and method (S) convenient.

(R)	36$^{ch\bar{o}}$			(S)	2^{ri}
	2				36
	72				72 (*chō*)

(5) Mensuration of area.

The mensuration of a rectangle is given.

In order to teach the mensuration of a piece of land three *ken* long and two *ken* wide, we first consider a strip of the full length, but only one *ken* wide, then two such strips to make up the width, and finally the computation is made, 3 *tsubo* × 2 = 6 *tsubo*, which is the answer.

(6) Decimal fractions.

The decimal fractions taught in this year do not go beyond the third place. In the first lesson decimals of one place are taught, and exercises are assigned for their computation; in the second lesson, decimals of two places are taught, with their computation exercises; in the third lesson, decimals of three places and their computation exercises, and so on. In this way the places of decimals are gradually extended.

In regard to notation, it is taught that, in case a decimal lacks an integral part, 0 is put to the left of the decimal point. It is also taught that the decimal point is read "point."

Of the decimal computations in the four rules, multiplication and division with decimal multiplier and decimal divisor are not given in the present year.

SECTION 6. Fifth Year of the Ordinary Elementary Schools.

(1) The chief aim in the present year is to teach children the computation of integers, decimals, and compound numbers, to make them proficient in the solution of applied problems

as well as in some easy mensuration, and to inform them of the outlines of the metric and foreign systems of weights and measures. In the first term, exercises in the previously acquired four rules with integers and decimals are assigned, then the multiplication and division with decimal multipliers and divisors are taught, and finally the computation of decimal compound numbers together with the mensuration of rectangular parallelopiped is given. In the second term, the computation of non-decimal compound numbers is practiced, and the mensuration of triangles, polygons, and circles is explained. In the third term, the metric and foreign systems of weights and measures together with their computation are taught, and also the mensuration of parallelograms, trapezoids, prisms, circular cylinders and spheres.

(2) Multiplication with decimal multipliers.

The method of computation is explained as below:

Example: 7×0.3.

The pupils are first reminded that if 0.3 be multiplied by 10, it becomes 3, and that, if the multiplier be multiplied by ten the product also is multiplied by ten. In the present case, 7 is first multiplied by 3, 0.3 being temporarily regarded as 3, and 21 is obtained. As this 21 is, from the nature of the case, ten times as much as the product sought, it is, accordingly, divided by ten in order to obtain the answer 2.1.

(3) Division with decimal divisors.

In case of division, attention is first called to the fact that, if both the divisor and dividend be multiplied by the same number, the quotient is not altered. The pupils are then instructed to multiply the decimal divisor by 10, 100, etc., in order to change it into an integer, and that consequently the dividend also must be multiplied by the same

decimals, before the operation is carried on.

(4) Modulus of circumference (ratio of circumference to radius).

As the modulus of circumference, 3.1416 is given.

The children are told dogmatically that "the circumference of a circle is some 3.1416 times as great as its diameter," and actual measurements are made by way of verification. Each child is further required actually to measure the circumference and diameter of some circular utensil which may be found at home and to ascertain clearly, by way of division, that the circumference of a circle is some 3.1416 times as great as its diameter.

(5) Metric system of weights and measures.

As reference articles for teaching the metric system, a model of the standard meter, a gram balance, a liquid measure of one liter, etc., are provided. And it is shown by actual measurement that one meter is equal to three *shaku* and three *sun*, that fifteen grams are equal to four *monme*, and that one liter is nearly equal to five *gō* and five *seki*. On the basis of these relations, other units of the metric system are also explained.

As in the physical examination of children their heights and weights are measured according to the metric system, the results are utilized for the computation of problems.

(6) Foreign system of weights and measures.

Mainly English and American systems are explained. As reference articles for teaching a foot rule and an ounce balance are provided. And it is shown by actual measurement that one foot is nearly equal to one *shaku*, and that one pound is nearly equal to one hundred and twenty *monme*. For the other units, the children are required to compute the relations between them and the corresponding units in

our system, e. g., one mile is nearly equal to fourteen *chō* and forty five *ken*, and one knot is nearly equal to seventeen *chō*.

For convenience in teaching, postal stamps (one inch long), nails, ribbons, collars, news-papers (all being measured by inches), inkstands, bundles of Berlin wool (one ounce in weight), oil-cans (containing five gallons), etc., are provided.

(7) Mensuration.

As regards the mensuration of rectangles, triangles, polygons, parallelograms, trapezoids, and rectangular parallelopipeds, the reasons for the method of computation are brought home to the children's understanding. But, in regard to the mensuration of circles, circular cylinders, pyramids, circular cones, spheres, etc., it is well-nigh impossible to explain the principles on which the computation is based so as to be commonly understood. Therefore, the formula is stated dogmatically, and verified by actual measurements to show the correctness of the method adopted in the computation in question.

SECTION 7. Sixth Year of the Ordinary Elementary School.

(1) The chief aim of this year is to give some simple computations in fractions and percentage as well as exercises reviewing the whole of the arithmetic hitherto taught. In the first term, fractions are taken; in the second, computations in ratio and percentage; and in the third, a general review takes place.

(2) Method of finding the greatest common measure.

In this year, the method of finding the greatest common measure is mainly conjectural as is exemplified in the following instance.

Example:

To find the greatest common measure of 4, 8, 12.

First, all the measures of 4 are put down below that num-

ber, then it is examined whether or not they are also measures of 8 and 12, and the common measures are determined. Finally the greatest of them is selected, and thus the answer is obtained.

For this year also the method of finding the least common multiple is similar to the finding of the greatest common measure.

(3) Multiplication with fractional multipliers.

Multiplication with fractional multipliers is effected in accordance with principles of multiplication with decimal multipliers.

Example: $\frac{5}{7} \times \frac{3}{4}$.

If $\frac{5}{7}$ be multiplied by 3, there results a product, 4 times as great as it ought to be. Therefore, if the product is divided by 4, the correct answer is obtained. That is, $\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4}$.

(4) Division with fractional divisors.

The method of computation is shown below:—

Example: $\frac{5}{7} \div \frac{3}{4}$.

If $\frac{5}{7}$ be divided by 3, there results a quotient equal to $\frac{1}{4}$ of the quotient sought. Therefore, that quotient being multiplied by 4, the answer is obtained, i. e., $\frac{5}{7} \div \frac{3}{4} = \frac{5 \times 4}{7 \times 3}$.

(5) Remarks concerning the teaching of fractions.

Generally speaking, simple computations are mostly assigned so that they may even be computed mentally. For, the computations needed in every day life are all comparatively simple. Moreover, if the method of computation is thoroughly acquired in simple cases, then also complex cases are easily understood.

SECTION 8. First Year of the Higher Elementary School.

The subject-matter of instruction in the higher elementary school consists chiefly of what is taken in the ordinary elementary school, the grade, however, being somewhat higher, with the addition of ratio and book-keeping for daily use. In the first year of the higher elementary school, there are taught at first fractions and percentage of a more advanced grade; later on, ratio and proportion. To come to details, there are given first the general method of finding the greatest common measure and the least common multiple, then exercises in more or less complex fractions and percentage, and lastly the computation of direct proportion, inverse proportion, proportional parts, etc.

In teaching the general method of finding the greatest common measure and the least common multiple, it is first thoroughly practiced and then explaind in principle. For, if it were explained theoretically from the very beginning, this would merely complicate matters and render comprehension difficult.

SECTION 9. Second Year of the Higher Elementary School.

In this year, the computation of proportion previously learned is reviewed, the computation of compound proportions is given, and computation in connection with bookkeeping is assigned. Then follow review exercises in the matter of the previous year.

SECTION 10. Third Year of the Higher Elementary School.

In this year, the lessons of all the preceding years are reviewed and supplemented.

There is a repetition of the methods of mensuration previously learned to which are added the mensuration of the pyramid, circular cone, frustum of pyramid, frustum of circular cone as well as the extraction of square roots and cube roots; and finally a review of the four rules, pro-

portion and percentage, together with arithmetical and geometrical proportion.

The rules for finding the volume of pyramids and circular cones are given dogmatically and verified by actual measurements.

In regard to the extraction of square roots and cube roots, the method is given first, and after this is understood, the principle is explained by means of simple problems.

In case ordinary book-keeping be taught, a simple and well-defined case is selected. The subject-matter is usually taken from domestic economy; or else it is one of the practical problems that come up in connection with the administration of a school, a village, a town, or a small corporation. The instruction consists of some method of entry and computation as well as of some general knowledge concerning these problems.

SECTION 11. Applied Problems.

Applied problems are proposed from the first year of the ordinary elementary school to the third year of the higher elementary school; first such as are very simple and easy, and then others gradually increasing in complexity and difficulty, so that the children may be led not only to understand the solution of real problems, but also to acquire perspicacity and accuracy.

Further, in the construction of problems, attention is paid to the following points.

(1) Of various concrete numbers, none is to be used too frequently.

(2) Of similar problems, those are to be presented first that appeal to the actual experience of the children, then such as contain facts they have experienced in the past, and finally such as are to be understood by conjecture and comparison.

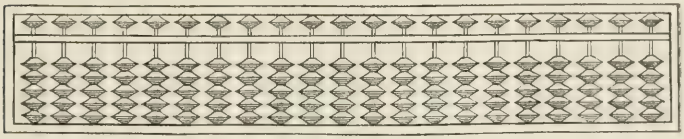
(3) Of the following various conditions of life, none is to be treated preferentially in the choice of problems.

(*a*) Items relating to the home.
(*b*) Items relating to the school.
(*c*) Items relating to the native province.
(*d*) Items relating to the state and society.

SECTION 12. Soroban-Calculation.

(1) The structure of the *soroban* and the way to represent numbers on it.

Soroban-calculation is the computation which is effected by means of a *soroban*. As is seen in the following illustration, it consists of vertical rows of six counters each.



Each of the five lower counters represents a unit of its respective place, and the upper one, five units of the same place. Places are determined by adopting any column for the units of integers.

To represent 3 on a *soroban* we have but to push up three lower counters in the row adopted for the units of integers. In order to represent 5 we push down the upper counter of the same column. To represent 6, we add thereto one counter below. In this way we shall proceed up to 9. To represent 10, we have to push up one lower counter in the column of the second digit of integers. The other numbers of the second digit are represented as mentioned before.

In the arrangement of digits the relation between the higher and lower ones is the same as in the arabic notation, and the left is considered as higher.

(2) Relation between *soroban*-calculation and written arithmetic.

In regard to the principle of computation in the four rules there is no difference between *soroban*-calculation and written arithmetic. In the method of operation, however, the existence of more or less difference cannot be disregarded. To point out a deviation, there exists for *soroban*-calculation a division table. It consists of the quotients and the remainders of all the divisions between any two digits. For example, if 3 be divided by 4, the quotient is 0.7, and the remainder 0.2. This result is expressed in a few words, and is announced, without expressing the particular relation, as 4, 3 : 7, 2.

Concerning the relative merit and demerit of *soroban* and written calculation, there has been much discussion among mathematicians in Japan, and it would hardly be possible to decide which to prefer. As, however, *soroban*-calculation has been in use in Japan from olden times, and as it is being used in everyday computations among the people, and further, as it is, in the computation of addition and subtraction, so easy to understand and so quickly performed, that no written arithmetic can ever expect to cope with it, it is assigned together with written arithmetic in most of the elementary schools in Japan. Yet, as is shown in the regulations relating to teaching in the elementary schools, written arithmetic is essential, *soroban*-calculation being used in addition, according to local circumstances. Therefore, even in the elementary school where *soroban*-calculation is given, it is taught only in addition to written arithmetic, beginning with the fourth year after the rudiments of written arithmetic have been given.

(3) Teaching of *soroban*-calculation in each year.

In the fourth year of the elementary school, where *soroban*-calculation is to be taught, first the names of the parts of the *soroban*, then the representation of numbers are taught, and finally the computation of easy additions and subtractions by this instrument is assigned. The order of com-

putation is as follows:

1. Addition and subtraction requiring no use of *pentaroids*.

2. Addition in which the partial sum of a certain place amounts to five, and the reverse process of subtraction.

3. Addition and subtraction requiring the use of *pentaroids*.

4. Addition in which the partial sum of a certain place amounts to ten, and the reverse process of subtraction.

5. Addition in which the partial sum of a certain column exceeds ten, and the reverse process of subtraction.

6. Other difficult cases.

In the fifth year of the ordinary elementary school, in addition to the exercise of addition and subtraction given in the previous year, compound addition and subtraction are given. The order is shown below.

A) Decimal compound numbers.

The method is similar to the one followed in the teaching of addition and subtraction with decimal numbers in the fourth year.

B) Non-decimal compound numbers.

1. Representation of non-decimal compound numbers.

2. Addition and subtraction of the same.

a) For cases in which the partial sum of each place does not overlap into the next higher order, and in which the partial subtraction of each place is possible.

b) For cases in which just one unit is carried over, and cases in which but one minuend is smaller than the subtrahend.

c) For cases in which more than two units are to be carried over, or in which more than two minuends are smaller than the subtrahends.

d) Other difficult cases.

In the sixth year of the ordinary elementary school, *soroban* multiplication and division are given. The order is

as follows:

A) Multiplication.

1. To multiply by integers of one digit.
2. To multiply by integers of two digits.
3. To multiply by integers of three digits.
4. To multiply by integers of more than four digits.
5. To multiply by one decimal.
6. To multiply by two decimals.
7. To multiply by three decimals.
8. To multiply by mixed decimals.
9. Reduction of compound numbers.

B) Division.

1. To divide by integers of one digit.
2. To divide by integers of two digits.
3. To divide by integers of three digits.
4. To divide by integers of more than four digits.
5. To divide by one decimal.
6. To divide by two decimals.
7. To divide by three decimals.
8. To divide by mixed decimals.
9. Numeration of compound numbers.
10. To multiply or divide compound numbers.

In the higher elementary school, *soroban*-calculation is taught by assigning exercises of computation in the four rules and urging the children to carry them out promptly and accurately. In the school in which ordinary book-keeping is given, the incidental computations are also performed by means of this instrument, thereby securing its practical utility.

In regard to *soroban* computation,—especially for quickness of operation, there prevailed, in ancient times, a method whereby one person vocalized the numbers and another one fingered the implement. This being assuredly inconvenient and uneconomical, another method has been adopted in modern times in which the computer handles the instru-

ment whilst looking at a series of written numbers,—and this is the case also in school exercises.

CHAPTER V.

EXAMINATIONS.

SECTION 1. No Periodical Examinations.

In the Regulations for carrying out the Imperial Ordinance relating to elementary schools, chap. I, sec. 1, art. 23, declares: "In order to ascertain the competency or grade of each class in elementary schools, the decision should be made, not by means of special examinations, but by weighing the daily achievements of the children." Accordingly, in the elementary schools of Japan, no examination takes place deciding the relative merits of attainments by means of written papers presented by the children,—that is to say, examinations for which dates are specially assigned at the end of a term or year and in which a number of questions in each course of study are set and answered by all the children in question.

SECTION 2. Method of Estimating the Pupils' Standing.

In spite of the fact mentioned in the last section, if a teacher deems it necessary, he may at times assign questions and ask the children to present written papers. Besides, as the daily work of children is mostly ascertained by examining their exercise books in arithmetic, he takes them as partial evidence. Or again, at the end of each lesson he may note in his class-memorandum the number of correct exercises of each child and take this also as partial evidence.

Considering all this together with his personal observa-

tions, he decides the merit of attainment in each child, and determines whether or not it deserves to be promoted.

SECTION 3. Selection of Themes.

The selection of themes to be assigned for ascertaining the attainments of the children is made with due regard to the efficaciousness of these themes in making clear whether or not the children are as proficient as is demanded by the aim of arithmetical teaching in the elementary schools, and if not, at what point the defect lies; that is to say, whether there is any defect in their idea of numbers, and their computation, whether there is any defect in their knowledge pertaining to every day life, whether there is any defect in the solution of applied problems, or whether there is any defect in their perceptive faculty, especially in regard to accuracy. In accordance with these points both the themes are selected, and the daily merits of attainment are estimated.

SECTION 4. Marking of Results.

The results indicating the merit of attainment in each subject are marked not by means of percentages, but by means of scales "A, B, C, D," or "First, Second, Third," etc. They are to be got not by giving marks and taking their average or by some such process, but from general and broad considerations aided by common sense judgment.

Although a table of merit is prepared in each class showing the marks of all the children, it is meant simply to afford a general idea of their relative superiority, and not to decide the real order of merit.

Morever, as this table is nothing more than material for reference intended for the teacher and the principal, it is never allowed to be seen by any child. The attainments of

each child, however, being estimated once or twice a term, the results are reported to its legal guardian by means of a report-book. It is needless to say that home and school ought to have a thorough understanding concerning the children in order to co-operate efficaciously in their education.

PART II.

RECENT TENDENCIES IN THE TEACHING OF MATHEMATICS.

CAPTER I.

RECENT TENDENCIES CONCERNING THE SCHOOL SYSTEM.

SECTION 1. Prolongaton of the Period cf Compulsory Education.

Prior to 1907, a four years' course in the ordinary elementary school had been prescribed as the minimum of compulsory education in Japan. The reform, however, that was introduced into the educational system in the said year, has added two years, and consequently a six years' course in the ordinary elementary school has since become obligatory. And yet there remains much that is not quite satisfactory,—especially if we consider the future development of the Empire. Government authorities in education, learned men, pedagogical and practical experts, all agree as to the necessity of further prolongation of the period of compulsory education in the near future in accordance with national progress and the general conditions of the people.

SECTION 2. Increase of Institutes for Supplementary Education.

In recent years, there has been a gradual increase of institutes providing supplementary education for those who have completed the six years' course in the ordinary elementary school and are preparing to enter the world of business.

SECTION 3. The Problem of Co-Education.

In relation to the merits and demerits of educating both sexes in the same class room, quite a discussion is rife. Nevertheless, in our country, separate education has been adopted as a fundamental principle. In the instruction promulgated by the Department of Education, we are told that, if the education of both sexes be given as much as possible in conformity with their different natures and habits as well as their different needs in life, it not only makes male education reveal its real value, but also enables the female education to be most suitable to the tender sex.

According to the present regulations, if the number of girls in one school year of the ordinary elementary schools or the numder of girls in all the classes of a higher elementary school is enough to organize one class, they are to be separated from the boys. In teaching arithmetic, it is in the fifth or the sixth year of the ordinary elementary school, and upwards, that the need of separate education is specially felt. At this stage the mental ability of girls, excepting, of course, a few special cases, reveals its inferiority to that of boys. Experts all agree in acknowledging this.

CHAPTER II.

Recent Tendencies Concerning the Aim and Subject Matter of Teaching Mathematics.

SECTION 1. Tendencies Concerning the Aim.

There is nothing which deserves special notice.

SECTION 2. Tendencies Concerning Subject-Matter.

In recent years, as all things in society are gradually

developing, various problems present themselves, many of which can not be solved at all without the aid of mathematical conceptions. Consequently the importance of teaching arithmetic in the elementary schools has been increasingly realized, and its subject-matter has been gradually increased so as to become too minute and too complex. Take for example, the systems of weights and measures: it is no longer sufficient to know the traditional system alone; the more important foreign systems must also be thoroughly mastered. And yet there is a limit to the time of teaching and to the ability of the children, being unable to cope with limitless needs. Therefore a reactionary tendency is gradually prevailing, manifesting itself in the desire to reduce the subject-matter.

CHAPTER III.

Recent Tendencies Concerning the Method of Teaching.

SECTION 1. Simultaneous and Separate Treatment of the Rules of Arithmetic in Teaching the First Steps.

More than twenty years ago, there prevailed the principle of the separate treatment of the four rules. From then until some ten years ago the principle of the simultaneous treatment of the rules was followed, and a many-sided treatment was the result. In recent years there has been adopted the principle of mediate treatment, an adjustment of the two, which is neither exclusively separate nor exclusively simultaneous in the teaching of the four rules. To illustrate: at first addition with numbers less than 10 is taken, and then corresponding subtraction; and again some easy cases of addition with numbers less than 20, and then corresponding subtraction; then again more or

less difficult cases of addition in the same limits, and its corresponding subtraction; and so on.

SECTION 2. Mental and Written Arithmetic.

That during the first year's computation depends upon mental arithmetic is a matter of course. In later years, however, written arithmetic is taught and nearly all the hours of arithmetic teaching are occupied by it. Consequently children naturally tend to avoid mental arithmetic even in cases of simple computations, and before graduation their mental computing ability declines exceedingly. In recent years, however, this defect has been generally acknowledged, and there have appeared many who advocate the claim that the most important of every day computations depend, not upon written arithmetic, but upon mental arithmetic. At any rate, mental arithmetic has in recent years come to be regarded with much favor.

SECTION 3. Soroban-Calculation.

In olden times *soroban*-calculation used to be the essential part of our arithmetic. After western civilization was imported into Japan, written arithmetic was pushed to the fore with great esteem for its importance, whilst *soroban*-calculation was shoved to the rear with a mistaken contempt for it, as being but a barbarous art of computation, and it barely managed to subsist,—owing to its necessity in the practical life of the people. In recent years, however, the specific merits of this form of calculation came to be acknowledged; for, in regard to speed of computation, there is nothing that surpasses it, and if it is taken as a base of computation, thorough proficiency in mental arithmetic is easily attained, and even a union between it and written arithmetic is not very difficult of attainment. Therefore, the day for the revival of *soroban*-calculation is surely dawning.

SECTION 4. Computation Exercises.

The multiplicity of matters to be taught has greatly reduced the hours for exercises in computation, and the result has regrettably affected the computation ability of children. In order to remedy this defect the following points have been considered with special attention.

1. Fundamental computations.
2. Difficult points in computation.

The so-called fundamental computations are nothing but the tables for addition, subtraction, multiplication, and division, compiled from the respective relations between any two of the simple numbers, or digits. Thorough proficiency in these tables makes computation even with larger numbers exceedingly easy. Moreover, as the difficult points in each computation are always found in some particular place, if, in that computation, this very place is specially attended to, thorough proficiency is found to be attainable in a relatively short period of time. The didactic application of the above principle is more and more extensively prevailing from day to day.

SECION 5. Fractions and Decimals.

Although, at the time when fractions and decimals are first taught, there is no reason to suspect any special difficulty either in understanding or handling them, yet before long there crops up among the children a tendency of becoming muddled. This may possibly be the result of raising the grade and retrenching the exercises too much. Therefore, in recent years, special attention has been bestowed in order to ground the children firmly in the computation of fractions and decimals which are exceedingly simple, the computation of which is practiced repeatedly.

SECTION 6. Measurement and Construction.

The teaching of weights and measures, time, angles, and

temperature has made it necessary to require actual measurements made by the children. The measurements required are of two kinds, namely, actual measurements and rough estimates. The rough estimates are, for instance, by stepping-off or visual-measurement in the case of distances. Also, in teaching weights and measures as well as angles, children are required to construct objects representing certain quantities. For instance, the form of one *shō* vessel of actual size is to be made of thick paper; coloured paper is to be cut two *sun* square, or a circle of one *shaku* in diameter is to be cut of a newspaper, and the method of computing its area recorded on it, &c.

SECTION 7. Applied Problems.

In former days, problems were exclusively taken from the text-book or from collections of problems. In recent years, however, teachers very often frame problems out of the materials presenting themselves to the children, with a view to bring home to them the practical application of arithmetic.

Also both for solution of problems and their drill, the preparation has become increasingly thorough. In regard to the former, the degree of complexity of the actual relations of numbers involved in the problems is considered, and in regard to the latter, a sample problem is selected for each mode of solution, thereby affording, on the one hand, a standard for the same kind of solution, and on the other hand, material for repeated exercises.

CHAPTER IV.

Examination.

Except what has been mentioned on this subject in the last chapter of the first part, nothing is to be specially noted.

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Article II.—The Teaching of Mathematics in Middle Schools. Prepared by N. Nishikawa, *Professor at the Tokio Higher Normal School*, under the direction and supervision of T. Hayashi, till recently *Professor at the Tokio Higher Normal School* and now *Professor at the Tōhoku Imperial University.*

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CONTENTS.

CHAPTER I.

ENACTMENT RELATING TO MIDDLE SCHOOLS.

By Imperial Ordinance, the enactment relating to middle schools was promulgated in Japan, and by it the aim, establishment, organization and other principal items concerning middle schools were prescribed.

In accordance with the said Ordinance, the Minister of State for Education enacted the regulations relating to the curriculum, to the grade of each branch, to organization, accommodations, admission and dismissal in the middle schools together with other regulations for carrying out the Ordinance relating to middle schools, and prescribed the arrangement of each branch, the number of recitation hours, materials of teaching, etc. in the syllabus of teaching making this obligatory upon all middle schools.

The present Ordinance relating to middle schools, the regulations for carrying out the Ordinance relating to middle schools and the syllabus of teaching in middle schools have taken effect in April, 1899, April, 1901, and April, 1902, respectively. Two or three times after that, necessary revisions were introduced into the regulations for carrying out the Ordinance relating to middle schools, in order to keep abreast with the times. In July, 1911, in order to remedy the deficiencies acknowledged to exist in the educational formation due to the middle and upper classes of our people, a great many new revisions have been made in the regulation for carrying out the Ordinance relating to middle schools as well as in the syllabus of teaching, and both are expected to take effect in April, 1912.

CHAPTER II.

AIM AND ORGANIZATION OF MIDDLE SCHOOLS.

SECTION I. Aim of Middle Schools.

The aim of the middle school is plainly stated in the respective enactment as follows:

"The aim of a middle school is to give the pupils a higher common education, namely, the middle school is an institute for giving boys a higher common education and for training them to form the chief support of the nation, not an institute for giving a preparatory education." However, it is quite plain that those who have received a higher common education only and have no special knowledge or ability in some specific science or art are not particularly fit for practical business. Therefore, there are very few among the graduates of middle schools, who are engaged in practical business, and most of them prefer to enter schools of higher grade.

But recently many technical schools of middle school grade have been established in each prefecture for the purpose of imparting teachnical education in a three or four years' course, to those who have finished their elementary education. Thus, many of those who intend to engage in practical business having received a few years' special education, enter those technical schools, and most of the pupils who enter middle schools hope to advance into the imperial universities after passing through the high school, or to enter some higher special school or technical school.

However, the number of higher special schools and their equipment are not sufficient to admit all the graduates of middle schools, so each higher school selects its matriculates

by an entrance examination. Therefore, the pupils of middle schools are very anxious to pass the great barrier of these selective examinations and people generally value a middle school according to the number of graduates who have entered schools of higher grade, and disregard the character formation, the development of personality and the physical training which these middle schools may afford. Accordingly, the middle schools themselves have become accustomed to be a kind of preparatory schools and often seem to neglect their original aim of giving the pupils a higher common education.

Lately the Department of Education introduced some revisions into the regulations for carrying out the Ordinance relating to middle schools perfecting the chief topics of instruction, changing the arrangement of recitation hours of the different subjects, and increasing the number of school days. Moreover industrial training is newly added to the curriculum, and according to local circumstances elementary agriculture, commerce or manual training is prescribed. This shows that the middle school is an institute not for giving a merely preparatory education, but for the character formation of the middle and upper classes of the nation and for the acquisition of the ordinary attainments required in actual life. The authorities are going to close the high schools preparing for the Imperial Universities and to establish new higher middle schools for the purpose of imparting to those who have finished the middle school course a higher common education more thorough than ever before. Notwithstanding all the efforts made by the authorities, the trend of the time makes the graduates of middle schools still more desirous to enter schools of higher grade, and it is hardly to be expected in reality that the middle schools will discard their present character as preparatory institutions and accomplish their original mission of giving a higher common education.

SECTION II. Organization of Middle Schools.

The establishment and organization of middle schools are prescribed by the Ordinance as follows:

In each prefecture one or more middle schools should be established according to local circumstances and requirements.

The Minister of Education can order each prefecture to increase the number of middle schools in case he finds it necessary. A *gun*, city, town, or village, as well as a town or village school corporation may establish a middle school, provided it be found necessary and not a hindrance to the maintenance of the elementary school education.

Private individuals may establish middle schools in accordance with the prescriptions of the respective Ordinance.

The course of the middle school should be five years.

The number of pupils of a middle school should be less than four hundred. Under special circumstances it may be go up to six hundred.

Classes should be formed of pupils of the same school age.

The number of pupils of one class should be less than fifty.

In 1909 the population of Japan was estimated at fifty two millions, and the number of middle schools, their classes and their pupils was as follows:

Schools	304,
Classes	3,237,
Pupils	121,970.

SECTION III. Curriculum and Number of Recitation Hours in Middle Schools.

The curriculum of the middle school is prescribed in the regulations for carrying out the Ordinance relating to middle schools; it consists of morals, Japanese and Chinese classics, some foreign languages, history, geography, mathematics, natural history, physics and chemistry, law and economics, drawing, music and gymnastics.

The foreign languages are English, German or French. Law and economics, as well as music, may be omitted.

In the revised regulations, commerce, agriculture, and manual training are added, and provision is made to establish, according to local circumstances, one or more of these subjects. As the supply of teaching accomodations and the condition of local finances make it difficult to enforce these regulations at once, they may be omitted for some time, or even in case that they have taken effect these subjects may be made optional. The number of recitation hours is also prescribed in the regulations, and the number of recitation days should be more than 200 days, (according to the revised regulations more than 220 days); the number of days for examinations and school excursions not being included.

The number of recitation hours per week is prescribed as follows:

Subjects \ Years	1st Year	2nd Year	3rd Year	4th Year	5th Year
Morals	1	1	1	1	1
Japanese and Chinese classics..	7	7	7	6	6
Foreign language ...	6	6	7	7	7
History .... / Geography	3	3	3	3	3
Mathematics	4	4	4	2	4
Natural history....	2	2	2	1st term 2 2nd term 2 3rd term 1	
Physics and chemistry				1st term 2 2nd term 3 3rd term 4	4

Years / Subjects	1st Year	2nd Year	3rd Year	4th Year	5th Year
Law and economics					2
Drawing	1	1	1	1	
Music............	1	1	1		
Gymnastics	3	3	3	3	3
Total	28	28	29	30	30

In schools not teaching law and economics or music the respective periods are to be appropiately distributed among some other subjects.

In addition to the foregoing schedule an hour in drawing may be assigned in the fifth year at the option of the pupils.

In gymnastics, the periods of the foregoing schedule may be increased by not more than two hours. If necessary the recitation hours of the foregoing schedule may be increased by two or less than two hours. In the revised regulations they are as follows:

Years / Subjects	1st Year	2nd Year	3rd Year	4th Year	5th Year
Morals	1	1	1	1	1
Japanese and Chinese classics..	8	7	7	6	6
Foreign language..	6	7	7	7	7
History .... / Geography	3	3	3	3	3

Years / Subjects	1st Year	2nd Year	3rd Year	4th Year	5th year
Mathematics	4	4	5	4	4
Natural history ..	2	2	2	2	
Physics and chemistry				4	4
Law and economics					2
Industrial training.				(2)	(2)
Drawing	1	1	1	1	1
Music...........	1	1	1		
Gymnastics	3	3	3	3	3
Total	29	29	30	31 (33)	31 (33)

Besides, practice in agriculture may be assigned.

In the schools where law and economics or music is omitted, their periods shall be appropriately distributed among other subjects.

Weekly periods in industrial training may, with the Minister's permission, be supplied from the recitation hours of other subjects.

If the weekly periods in industrial training are to be increased or to be assigned from the third year upward, with the Minister's permission, they may be supplied by decreasing recitation hours in some other subjects.

Gymnastics may be increased by three hours. But the total number of weekly periods should not be more than 34 hours.

CHAPTER III.

SCHOOLS IN CONNECTION WITH MIDDLE SCHOOLS.

SECTION I. Applicants for Admission into Middle Schools.

Concerning applicants for admission the Ordinance relating to middle schools and the regulations for carrying out the Ordinance relating to middle schools prescribe as follows:

The applicants for admission into middle schools must either be male graduates of ordinary elementary schools, not less than twelve years of age, or must have attainments equal or superior to those of the graduates of ordinary elementary schools.

Of those applying for admission to the first year class such as have completed the ordinary elementary school are given preference over other applicants.

If there are more applicants having completed the ordinary elementary school than can be admitted, they should be selected by examination.

All the graduates of ordinary elementary schools are qualified to enter middle schools, but since the number of applicants always exceeds the vacancies most of the applicants admitted are such as have studied for one or two years more in higher elementary schools. For example, the number of applicants for admission and of applicants admitted in 1909 is given below:

Applicants for admission	57,553;
Applicants admitted	31,221;
Percentage	54,24%.

Among the above mentioned 31,221 applicants admitted, there were 8,830 (28.28%) who entered the middle school right after their graduation from ordinary elementary schools and the others have been studying one year or more in

higher elementary schools. Consequently the minimum age of the applicants admitted is twelve, and the maximum thirteen or fourteen.

SECTION II. Graduates of Middle Schools.

Either the graduates of middle schools or those whose attainments are equal or superior to those of graduates are qualified to enter high schools, higher special schools of various kinds and industrial special schools. But as there are more applicants than vacancies, only a small part of them are admitted into those schools, and the others have to undergo preparatory studies in order to pass the entrance examination the year after. However some of them are unable to accomplish their objects in one or two years after their graduation, and many of them change their long-cherished purpose and become officials, teachers, or else enter business.

The following table shows the condition of the graduates of middle schools for 1909.

Graduates of middle schools ..	14,595;
Students in high schools	1,087;
Studnts in governmental, public and private schools	4,099;
Government officials and teachers ..	1,277;
Engaged in business..	1,495;
Deceased	51;
Others	6,586.

CHAPTER IV.

AIM OF MATHEMATICS IN THE MIDDLE SCHOOLS.

SECTION I. General Aim.

In order to discuss the aim of the teaching of mathematics we must consider its value from the standpoint of higher

common education. Though the value of mathematics may present manifold aspects, all may be reduced to the following three:

1. Imparting knowledge necessary to daily life.

Closely related to daily life and indispensable to it is knowledge of the national language, and next to that, of the art of computation and of ordinary mathematical quantities. This is universally acknowledged.

2. Giving fundamental knowledge.

For various sciences, arts or industry, which have advanced material civilization, mathematical knowledge provides a foundation, and the advancement and development of these achievements more and more demand mathematical knowledge. And thus, for all who are to be in touch with present-day civilization and to form the middle and upper class of the people the study of mathematics is indispensable.

3. Cultivation of mental powers.

Mathematics is the groundwork of accurate science. It renders reasoning accurate and thinking precise. And the influence and effect which it exerts upon the cultivation of our mental powers can not be compared with any other subjects in ordinary education. Moreover, the training of mental powers as such is of itself very necessary for all who are to study any kind of science or art or to engage in any kind of business in future. Therefore, the value of mathematical training for general education is not inferior to the direct usefulness of mathematical knowledge.

The teaching of mathematics in the middle schools ought to be in thorough accord with the points stated above, and ought to aim at giving, first, practical knowledge, and secondly, formal culture, and it ought to attach importance equally to both, and never be partial to either.

Concerning the teaching of mathematics, the regulations for carrying out the Ordinance relating to middle schools also prescribe the following:

The chief aim of the teaching of mathematics is to make clear the relation of mathematical quantities, to make computation thoroughly familiar and to make thinking accurate.

In the revised regulations this is supplemented as follows:

The chief aim of mathematics is to give the pupils knowledge concerning mathematical quantities, to make them thoroughly familiar with computation, to render them skilled in its application as well as to train them in accurate thinking.

Thus, the teaching of mathematics in middle schools, though its aim is not clearly stated, tends to put too much stress on the cultivation of mental powers and to lack not a little in giving knowledge of the practical sort. The cultivation of mental powers, however, can be accomplished not by the teaching of mathematics only, but by every subject of study; modern psychologists unite in acknowledging this. Of course, also in the teaching of mathematics, this must be zealously striven for, but the direct aim of the teaching of mathematics must be to impart knowledge concerning mathematics and skill in its application.

SECTION II. Aim of Each Branch.

The general aim of mathematics has been stated above. But if we come to discuss each branch, circumstances are not always the same. Each branch has its own special aim. Consequently, we must understand the aim of each branch and strive to attain it at the same time, trying to accomplish the general aim by acting in concert and helping one another.

I. Aim of arithmetic.

The aim of arithmetic in middle schools is as follows:

(1.) To give pupils the knowledge of arithmetic necessary for daily life and to make them understand its application.

(2.) To make pupils familiar with computation.

(3.) To make pupils understand the meaning of computation and the reasons of rules, and to ground them in mathematical reasoning.

(4.) To prepare for other branches of mathematics.

Of the whole of mathematics the knowledge of arithmetic has the most important connection with our daily life. So it must be the aim in teaching arithmetic to give pupils whatever is most necessary in daily life, skill in its application, in general, knowledge and ability adapted to practical life.

Though much work and practice are spent in elementary schools to make the pupils familiar with computation, the result in applicants admitted to middle schools is found to be unsatisfactory. When pupils try to solve arithmetical or algebraical problems they often fail although they know the theory and method of solution. These unsatisfactory results in the teaching of arithmetic and algebra are owing to the fact that pupils are not thoroughly familiar with computation. In the teaching of arithmetic, it is found that though the pupils in the lower classes are interested even in mechanical exercises of numerical computations, the pupils of the higher classes gradually cease to be interested in such exercises. Hence, in the lower classes, pupils should be made thoroughly familiar with computation in arithmetic.

Most of the arithmetic taught in middle schools has already been learned in elementary schools. Thus, there is no need of teaching the rules of computation, etc. But the more fundamental knowledge, as to the meaning of computation, the reasons of rules, etc., even if it is touched upon in elementary schools, it is only in the form of examples, and certainly not fully understood. It would be

unfair to demand so much of elementary school teaching, but in the middle schools, as the pupils' reasoning faculty develops with age and as at the same time algebra is also taught, the pupils must be made to understand these fundamentals and thus make the best use of their algebraic knowledge. As, in addition to arithmetic, algebra, geometry and trigonometry are also taught in middle schools, the necessary preparation for the study of these branches must be made in teaching arithmetic by keeping in mind its connection with them. Many inconveniences are experienced in the teaching of other branches of mathematics simply because in the teaching of arithmetic the connection with other branches has been lost sight of.

II. Aim of algebra.

The special aim of algebra is as follows:

1. To make pupils familiar with computation.

2. To give pupils explanations concerning the rules of computation of various kinds.

3. To give pupils the elementary notion of function.

To make pupils familiar with computation must be one part of the aim of the teaching of algebra for the same reasons as hold good in arithmetic.

As the reasons for the various methods of arithmetical computation can not be thoroughly given in arithmetic, they must be explained in algebra to make the pupils understand them definitely.

To give the pupils an elementary conception of function is most important as a direct aim in teaching algebra. All mathematical quantities found in daily life and in several sciences are variable in nature, and those variations necessarily accompany the variation of other related quantities. Unless we have clear conceptions concerning these quantities and their relations, we can never understand nature and have control over her. Algebra is the science relating to these quantities. The main aim of algebra is to investigate

the nature of function. This is the reason why accurate sciences like physics, etc. can not exist apart from the knowledge of algebra. And this is the reason why algebra is included in the curriculum of higher common education.

III. Aim of geometry.

The principal aim of the teaching of geometry in middle schools is as follows:

1. To give pupils knowledge concerning space and to make them understand its application.

2. To cultivate the mental powers.

Geometry treats of the nature of space, and knowledge concerning space is necessary to every one. So it must be one of the chief aims of the teaching of geometry to give pupils knowledge concerning space and its application.

The reasoning usually adopted in the teaching in middle schools is based upon analogy and is seldom by way of direct proof. In algebra and trigonometry, the proofs are given by way of changing expressions, so that these branches are much inferior to geometry in cultivating the reasoning faculty. Of so many subjects taught in middle schools, geometry is the fittest for the purpose of cultivating the reasoning faculty. This point, therefore, should be emphasized in teaching geometry.

IV. Aim of trigonometry.

The aim of trigonometry is:

1. To make pupils know the solution of triangles and its application.

2. To establish connection and unity among various branches of mathematics.

In middle schools not many hours can be applied to teaching trigonometry. Therefore, we must be satisfied with affording pupils the solution of triangles and its application. And, at the same time these two points must be looked upon as the chief aim of the teaching of trigonometry.

In teaching mathematics in the middle schools, various

branches are taught extending over several years, and though they are taught with special reference to their mutual connection, it is very hard to harmonize them. But in trigonometry many splendid opportunities for this purpose present themselves, because diagrams taught in geometry and various rules of computation taught in arithmetic and algebra are frequently applied in trigonometry. This unity in the teaching of mathematics should be made one of the aims of the teaching of trigonometry.

CHAPTER V.

SUBJECT MATTER OF MATHEMATICS.

SECTION I. Subjects, their Distribution, and Number of Recitation hours.

The subjects, their distribution and the number of recitation hours in the teaching of mathematics in middle schools are prescribed in the regulations for carrying out the Ordinance relating to middle schools as well as in the syllabus of teaching.

The branches of mathematics taught in middle schools are arithmetic, algebra, geometry and trigonometry, and the following table shows their distribution and recitation hours per week.

Branches \ Year	I	II	III	IV	V
Arithmetic	4	2			
Algebra		2	2	2	
Geometry			2	2	2
Trigonometry					2

The recitation hours stated in the foregoing table are too few in comparison with the matter prescribed in the syllabus of teaching, and many inconveniences and difficulties are experienced in teaching them, and it is quite impossible to accomplish the real object of the teaching of mathematics. So it was generally hoped for some time that the recitation hours in mathematics would be increased. Later on, some parts of the regulations for carrying out the Ordinance relating to middle schools were revised, and it was permitted to apportion to mathematics the recitation hours of law and economics or music, in case these were omitted. Moreover, it was also permitted, if necessary, to increase the total number of recitation hours per week by not more than two hours. Thus not only those schools in which law and economics or music are omitted, but also those in which they are taught, all increased the recitation hours in mathematics. The number of recitation hours which were added in different schools were not the same, but in most of them one hour per week was added, as follows:

Years / Branches	I	II	III	IV	V
Arithmetic	5	2			
Algebra		3	3	2	1
Geometry			2	3	2
Trigonometry					2

In the revised regulations which will be put into force next year, one hour in the third year only is added, and in case law and economics or music are omitted, their recitation hours can be expediently apportioned to some other branch, but outside of this, it is not permitted to add any recitation hours.

As the result of the revision, the number of recitation hours after next year will be as below. Even at present, however, law and economics or music is omitted in many schools, and the whole or a part of the recitation hours in these subjects are apportioned to mathematics.

<table>
<tr><th>Branches \ Years</th><th>I</th><th>II</th><th>III</th><th>IV</th><th>V</th></tr>
<tr><td>Arithmetic ......</td><td>4</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Algebra ....</td><td></td><td>4</td><td rowspan="2">5</td><td rowspan="2">4</td><td></td></tr>
<tr><td>Geometry ....</td><td></td><td></td><td rowspan="2">4</td></tr>
<tr><td>Trigonometry ....</td><td></td><td></td><td></td><td></td></tr>
</table>

It is to be regretted that the number of recitation hours in mathematics now prevailing in many schools is not adopted in the revised regulations. However, in the third year class one hour per week is added, and at the same time the revised syllabus of teaching provides for taking arithmetic and algebra together, and for omitting the theory of permutation and combination, and the binomial theorem from algebra, and thus the inconveniences and difficulties in the teaching of mathematics caused by the lack of recitation hours may be considerably reduced.

SECTION II. Grade and Distribution of Each Branch.

The grade and distribution of each branch in the course in mathematics are prescribed by the Syllabus of Teaching for middle schools.

Syllabus of Teaching for Middle Schools.

Mathematics.

(*1*) *Arithmetic.*

First Year .. .. 4 hours a week.

Introduction.

Numeration ; notation ; decimals.

Integers and decimals.

Addition, subtraction, multiplication and division.

Compound numbers.

Time ; metric system of weights and measures ; shaku-kwan system of weights and measures ; Japanese money ; foreign systems of weights and measures as well as of money ; reduction and numeration of compound numbers ; addition, subtraction, multiplication and division in compound numbers ; English system of weights and measures as well as of money ; in addition to these, complex non-decimal compound numbers may conveniently be given with or after fractions.

Properties of integers.

Divisibility ; prime numbers ; factors ; greatest common factor ; least common multiple.

Fractions.

Principal properties of fractions ; (reduction to the lowest terms ; reduction to the common denominator) ; reduction of fractions to decimals ; addition, subtraction, multiplication and division of fractions.

Ratio and proportion.

Ratio ; proportion.

Second Year .. .. 2 hours a week.

Ratio and proportion continued.

Chain rule ; proportional parts ; alligation.

Percentage.

Preliminary remarks ; percentage ; interest ; daily computations relating to percentage.

Powers and roots.

Square and square roots ; cube and cube roots ; mensuration.

(2) *Algebra.*

Second Year .. .. 2 hours a week.

Introduction.

Definition of symbols; algebraical expressions; associative law; expression of definitions; negative numbers.

Integral expressions.

Addition, subtraction, multiplication, and division.

Equations.

Linear equations with one unknown quantity.

Third Year .. .. 2 hours a week.

Equations continued.

Linear simultaneous equations with more than one unknown quantity.

Integral expressions continued.

Formulae relating to distributive law; factors; greatest common factor; least common multiple.

Fractional expressions.

Fundamental properties of fractions (reduction to the lowest terms; reduction to the common denominator); addition, subtraction, multiplication, division of fractions.

Equations continued.

Equations with one unknown quantity reducible to linear equations; quadratic equations with one unknown quantity; equations with one unknown quantity reducible to quadratic equations; simultaneous equations containing quadratic equations; explanations relating to the solution of equations.

Fourth Year.

Irrational expressions.

Extension of the definition of exponent; irrational numbers; rational numbers approximating to irrational numbers.

Ratio and proportion.

Cases with pure numbers; cases with quantities (com-

mensurable quantities, incommensurable quantities); surds.

Progressions.

Arithmetical progression; geometrical progression.

Permutation and combination.

Binomial theorem.

Case with integral exponents.

Logarithm.

Fundamental properties of logarithms; logarithmic tables.

(3) *Geometry.*

Third Year .. .. 2 hours a week.

Introduction.

Straight line.

Angles; parallel lines; triangles; parallelograms.

Circles.

Fundamental properties of circles; angles at the center; chords; angles in the segment; tangents; contact of two circles.

Fourth Year .. .. 2 hours a week.

Circle continued.

Inscribed and circumscribed figures.

Area.

Linear figures; equality and congruence of areas.

Proportion.

Definition of equal ratios; general theorems derived from this definition.

Application of proportion.

Proportional lines; similar figures.

Fifth Years .. .. 2 hours a week.

Application of proportion continued.

Areas; loci.

Planes.

Straight lines and planes; solid angles.

Polyhedrons.

Prism ; pyramid ; regular polyhedrons.

Curved solids.

Sphere ; circular cylinder ; circular cone.

(4) *Trigonometry.*

Fifth Year .. .. 2 hours a week.

Method of measuring angles.

Sexagesimal method.

Circular functions.

Circular functions of acute angles ; mutual relations among circular functions ; change of the sign and magnitude of circular functions ; circular functions of negative angles ; circular functions of supplementary angles.

Formulae relating to the sum of angles.

Circular functions of the sum and difference of two angles ; circular functions of the multiple or division of angles ; formulae relating to the product of circular functions ; formulae relating to the sum and difference of circular functions.

Relation between sides and circular functions of angles of triangles.

Use of logarithmic tables.

Solution of triangles.

Methods of surveying height, distance, &c, and practical exercises.

REMARKS AS TO TEACHING.

1. In teaching mathematics, always use exact words in stating or proving rules and propositions, and try to make them accurately understood.

2. In arithmetic, do not simply give computation rules, but always put stress on computation practice, and try to train pupils to compute promptly and accurately.

3. In computation, make the verification if possible, and thus strengthen the pupils' conviction.

4. In selecting arithmetical problems, try to select such

as are most adapted to daily life, and in assigning problems relating to percentage or to other computations in daily life, give special attention to explaining the items involved in those problems.

5. In arithmetic, if the reason for a certain rule cannot be understood by the pupils, begin to deal with the rule as such, and defer its rigorous proof to algebra.

6. In teaching linear equation in algebra, do not limit it to one place, but, often make use of its simpler cases so as to awake pupils' interest in algebra.

7. In teaching geometry, attach importance to the strictness of reasoning. In teaching proportion, for instance, neither omit nor leave ambiguous any part of it in order to make it easier. It is not objectionable, however, to postpone this till later on.

8. Give geometrical problems an adequate place, viewing them from the standpoint of proofs.

9. It is preferable to give different forms of proofs or the relation among proofs after some progress has been made rather than at the very beginning.

10. In trigonometry, the measuring of heights, distances, &c. together with practice therein should be given as early as possible so as to appeal to the pupils' interest.

11. In regard to circular functions, it is preferable to use their true values at first and then their logarithms.

12. Reference articles for teaching should be the following:

A sun-dial, a watch, a compass, a meter, comparative measures of three kinds, a Japanese balance, metric weights, a plat-form scale, a liquid measure, a dry measure, metric measures, foreign linear measures, a blackboard compass, blackboard rules of various kinds, a model of vernier, a theodolite, a tape measure, Gunter's chain, flag-staffs, &c., figures of Japanese coins, figures of foreign coins, models of geometrical solids.

The above-mentioned syllabus of teaching for middle schools was put into force in 1902, and has since been in practice up to this date. In accordance with it mathematics as a subject of instruction, proves to be so difficult for both teachers and pupils notwithstanding its special importance that complaint has constantly been heard from both as well as from the public about the result. Indeed, the problem how to remove the great difficulties from the teaching of mathematics in middle schools and how to secure its proper influence has for many years been constantly discussed among educators, some having tried to investigate the causes of the difficulties and others to contrive methods of teaching. But in spite of all the effort expended no satisfactory solution has as yet been reached.

It seems that the cause of inconveniences and difficulties in the teaching of mathematics is the imperfect distribution of each branch and poor arrangement of its materials in the present syllabus of teaching. These problems can not be solved unless this fundamental arrangement is revised. The course of teaching is, however, prescribed by the syllabus of teaching in middle schools and the teachers must follow it and can not freely change it. So their investigation can never touch the fundamental points.

In the middle school attached to the Tokyo Higher Normal School, there has been a long investigation as to the causes of the inconveniences and difficulties in the teaching of mathematics, and finally it was found that the present syllabus of teaching is not adapted to the pupils, with regard to their psychological development and with reference to the relations of the different branches of mathematics. A new plan of apportionment of each branch and a new arrangement of the subject matter was drafted and put in force since the year 1905. It has been found that it is very convenient and profitable in teaching mathematics. Recently, in many middle schools this good effect has been heard of

and has aroused the desire of imitation. But as this arrangement is against the presen tsyllabus of teaching and as no text books have been written according to this new plan, it is, alas, quite hard to put it in force.

However, in the revised syllabus of teaching exactly the same view has been adopted as to mathematics and the new plan will be widely followed by middle schools in general. Thus the long-discussed questions have been solved.

Now, let us mention the main topics of the teaching of mathematics adopted in the middle school attached to the Tokyo Higher Normal School, explain how it differs from the ordinary one, and mention also the revised syllabus of teaching.

Main Topics in the Teaching of Mathematics adopted in the Middle School attached to the Tokyo Higher Normal School and main Reasons therefor.

I. Arithmetic and Algebra.

According to the present syllabus of teaching in middle schools, arithmetic is assigned to the pupils below the second year class, and it is quite difficult to make them thoroughly understand the reasons concerning computation, and to make them familiar with its use. Not only those who are engaged in teaching mathematics meet with many inconveniences and difficulties, but graduates of middle schools always feel uneasy about their own knowledge in arithmetic and algebra.

Now the explanation of the rules of computation in arithmetic is mostly based upon the theory of algebra, and algebra is reciprocally based upon arithmetic. But these intimate relations are overlooked in the present syllabus of teaching, and those items which can be explained only in algebra, or can be more easily explained in algebra, are all taught compulsorily in arithmetic, and in the teaching

of algebra the items taught in arithmetic are overlooked, and thus the two collide. Arithmetic is taught in the first two years when pupils are still young and later on in algebra what relates to arithmetic is almost entirely disregarded. Thus the knowledge of algebra and geometry is not utilized in the teaching of arithmetic, and many inconveniences and difficulties accrue in teaching arithmetic, nor can its complete purpose be accomplished.

Indeed we can not understand why arithmetic and algebra are separated, and why arithmetic is assigned to the first and the second year, while algebra is in the third year and upward. It is not in accord with the state of the pupils' psychological development and the relations between arithmetic and algebra. This is the reason why the present syllabus of teaching is not adhered to and why a new arrangement of the subject matter has been drafted.

Let us mention the outlines of the new plan and explain the difference between it and the present syllabus of teaching.

Apportionment of the main topics of arithmetic and algebra.

First Year .. .. 4 hours a week.

Numeration, notation, decimals.
Compound numbers (a part).
Addition, subtraction, multiplication, division.
Multiples, factors, fractions.
Compound numbers (a part).
Ratio and proportion.
Compound proportion (an outline).
Proportional parts (an outline).
Percentage (an outline).
Miscellaneous problems in ratio and proportion.

Second Year .. .. 4 hours a week.

Negative numbers.
Addition, subtraction, multiplication and division in algebra.

Linear equations (containing no letters in co-efficients).
Simultaneous linear equations (,,).
Factors (a part).
Greatest common factor.
Least common multiple.
Fractional expressions.
Literal equations (of the first degree).

Third Year .. .. 2 hours a week.

Powers and roots (a part).
Quadratic equations.
Factors (a part).
Equations of higher than the second degree (those that can be reduced to quadratic equations).
Fractional equations.
Simultaneous quadratic equations.

Fourth Year. .. .. 2 hours a week.

Powers and roots (a part).
Irrational equations.
Inequalities.
Ratio and proportion.
Compound proportion.
Chain rule.
Proportional parts.
Alligation.
Miscellaneous problems in proportion.
Progressions.

Fifth Year .. .. 1 hour a week.

Logarithms.
Percentage.
Interest.
Annuities.
Permutation.
Combination.
Binomial theorem.
Mathematical induction.

On numbers.

On equations.

The number of recitation hours is insufficient to complete the lessons of arithmetic and algebra by the end of the fourth year. It is more convenient to teach such arithmetical items as percentage, interest, &c. which relate to daily life, in the fifth year. And further, it is of great benefit to give algebra together with trigonometry and solid geometry so as to make possible thereby some systematic unity among the various branches of mathematics. Accordingly the syllabus in hand adds in the fifth year one hour for arithmetic and algebra to what is at present prescribed.

In the first year, arithmetic is to be taught, not by taking a certain definite portion of the first part, but by selecting from all parts simple materials as well as those items which constitute the basis of mathematics. And such items which are more difficult than the rudiments of algebra, or which can be better understood in connection with similar items of algebra, are to be taught from the second year upward on suitable occasions in algebra. Consequently, from the second year upward, there is absolutely no separate apportionment of hours for arithmetic and for algebra.

The pupils have already learned in elementary schools the meaning and rules of computations relating to addition, subtraction, multiplication, and division, compound numbers and fractions in arithmetic, at least in their outlines. Consequently, in the first year, they should mainly be assigned problems and drilled in them rather than be taught by beginning with the definitions of these items. And while they are thus drilled, they should be strengthened in whatever knowledge they have previously acquired, their deficiencies should be supplemented, the rules of computation should be unified and rendered thoroughly familiar. The syllabus in hand indicates the materials to be taught not in their order, but merely in the proper amount.

In regard to the proportion and percentage in arithmetic, the knowledge the pupils have acquired in elementary schools is far from being complete. Therefore, these points must be accorded special treatment. In the first year, simpler cases are given in connection with ratio and proportion, compound interest, and annuity in algebra. At the time when proportion is given, it is hinted how numbers might be expressed by means of letters. And while applied problems are given, the solution by means of equations is at times alluded to, so as to clear the way for algebra.

Arithmetical square roots and cube roots are taught in connection with algebraical square roots and cube roots in quadratic equations. A thorough understanding of mensuration is unattainable without the help of geometry. And, in geometry, when theorems relating to area and volume are given, applied problems requiring computation should be given, so as to make their meaning thoroughly understood, and these problems are therefore to be omitted in arithmetic.

At the beginning of the second year, negative numbers are given before algebra proper be understood, and pupils are made thoroughly proficient in computation. Thus they are thoroughly prepared for the free use of all numbers, positive or negative, before they are introduced to the method of expressing numbers by letters, and finally to algebra proper.

In teaching algebra, the corresponding items in arithmetic should always be kept in mind. In teaching addition, subtraction, multiplication and division in algebra, their foundation in arithmetic should be explained. The greatest common factor and the least common multiple in algebra, to give another instance, are taught in connection with the corresponding arithmetical problems. Moreover, by mixing numerical exercises in algebraic computation, or, by inserting large numbers in the applied problems in equation, the computing ability which was exercised in the first year, is kept up or further improved.

In the ordinary text book, equations with literal coefficients are given from the very beginning. But the thorough understanding of their solution is impossible apart from the knowledge relating to factors and fractions. Accordingly the syllabus in hand puts literal equations (of the first degree) after fractions. Until then numerical equations alone are to be treated.

The theories relating to fractional equations are rather of an advanced nature, so that the meaning of multiplying or dividing both sides of equations by an expression containing an unknown quantity can not be made thoroughly clear unless it is explained that an equation may have two or more than two roots. Accordingly what concerns fractional equations is to be given after quadratic equations. Also, as the proper arrangement ought to give irrational equations after surds are given, the syllabus in hand has adopted this order.

Computations relating to inequality can never be effected mechanically. If the attention relaxes, it is hardly possible to escape errors. Pupils are thus helped to become cautious and trained in mathematical reflection. Therefore these computations are adopted by the new syllabus.

Even though the measurement of quantity is taught in geomtery, it is to be restricted to simple numerical computations. In algebra, even if the domain of numbers is gradually enlarged by taking in negative numbers, irrational numbers, imaginary numbers, &c. the explanation for them can not be given sufficiently, and consequently the pupils' ideas concerning numbers are generally confused. In regard to equations, the same holds true; pupils have no unified knowledge relating to the different solutions of various equations. Even if particular attention is had in teaching, the matter given to beginners, being extended over several years, cannot but be disconnected bits of knowledge. By summarizing, and systematizing however, some comprehensive

algebraic knowledge can for the first time be brought home to their minds. Accordingly the syllabus in hand at the end shows the heading: "On numbers and on equations", meaning a general summary.

II. Geometry and Trigonometry.

The middle school geometry ought to aim mainly at cultivating the reasoning faculty, and to promote the understanding of logical treatment. Moreover the matter treated in geometry being the properties of space, the knowledge of which is so necessary for every one, and so broad in its applications, this study should never be neglected. The syllabus now in force, however, is too brief in this respect, and it is difficult to discern its view about it. If we look at the text books compiled in accordance with the syllabus now in force, those that are rigorous in logic are deficient in practical applications, those that are satisfactory in that respect are lacking in logical rigour, and none fulfil both requirements.

Among the teachers of mathematics in middle schools, there are many who consider the middle school geometry as too dry, and too difficult for the pupils to understand, and prefer to teach the so-called rudiments of geometry, or prefer to give fundamental geometrical conceptions by way of intuition and not by means of rigorous definitions, or prefer to increase the number of axioms, or to prove theorems by way of experiment. But, after all, these methods simply deteriorate the logical faculty, impede the training in reasoning, leave bad effects injurious to later teaching in rigorous logic in geometry, and are not at all adequate for the teaching of geometry. According to the experience in middle schools, if the teaching is adequate there seems to be no difficulty in giving geometry by a rigorously logical method from the third year upwards. On the contrary, the pupils' interest may thereby be commanded. And there is no need

of giving the so-called rudiments of geometry.

The present syllabus attaches importance to mental training as well as to practical knowledge, and appeals from the very beginning to rigorously logical methods, at the same time trying to promote understanding by taking into consideration the difficulty of the different subjects and their mutual connections.

In middle school trigonometry, it is proper to attach importance to its practical side rather than its theoretical side. As, however, trigonometry deals with the application of the properties of figures given in geometry as well as of the rules of computation given in arithmetic and algebra, special attention must here be paid to the interrelations between various branches of mathematics.

Below is given the schedule of the new syllabus and its principal topics:

(1) Arrangement of main topics of geometry.

Third Year .. .. 2 hours a week.

Introduction.
Angles.
Triangles.
Parallel straight lines.
Linear figures.
Parallelograms.
Fundamental properties of circles.
Circular arcs and chords.
Circular segments.

Fourth Year .. .. 2 hours a week.

Tangents.
Two circles.
Inscribed figures.
Circumscribed figures.
Relations among theorems.
Loci.
Ratio and proportion.

Areas.
Similar figures.

Fifth Year .. .. 2 hours a week.

Similar figures (continued).
Parallel planes and straight lines.
Normals.
Dihedral angles.
Polyhedral angles.
Polyhedrons.
Right circular cylinder, right circular cone.
Sphere.

(2) Arrangement of main topics of trigonometry.

Fifth Year .. .. 2 hours a week.

Circular functions of an acute angle.
Solution of a right-angled triangles.
Circular functions of any angle.
Circular functions of the sum and the difference of two angles.
Circular functions of the multiple and division of an angle.
Relations between the sides and the angles of a triangle.
Area, circumscribed circle and inscribed circle of a triangle.
Solution of a triangle.
Surveying of distance and height.
Outline of surveying.

Triangles are given previous to parallel straight lines. For, closed figures such as triangles are for beginners much easier to understand than the others.

The relations among theorems are given separately when occasion offers for their explanation. And for the proofs relating to loci, accurate knowledge concerning the relations among theorems is indispensable. Therefore they are to be given once more before the general definition of locus and

the method of its proof are given.

In regard to locus, the general difinition is avoided at first, and its definition is given in particular cases. In time when pupils have nearly completed the circle, they come to grasp, by way of induction, what locus means. Taking this opportunity, therefore, the general definition is derived and the method of its proof is given.

Theorems relating to the ratio and proportion of quantities are discussed with their numerical values, and are explained by way of reducing them to the corresponding items given in algebra. And the case of ratios having irrational values is not much touched upon. Also, if the items relating to areas were given prior to ratios, as is prescribed in the syllabus actually in force, it would be inconvenient to make clear their application by way of assigning computation problems. So the syllabus in hand has put all the items relating to areas under proportion.

Constructions and computations of areas and volumes are made, not to be given all at once, but to be distributed in various places so that they may conveniently be taught in connection with different theorems. Also, in constructions, connections are contrived to be made with drawing by way of giving simpler methods by means of rules, compasses and measures in addition to the geometrical method.

In the measurement of angles, the right angle is too big as the unit and is inconvenient to give accurate conception concerning them. So, even in geometry, the sexagesimal method is given. As the method of circular measure is to be appropriately explained in connection with the number π of the circle, it is also to be taught in geometry.

In trigonometry, the solution of triangles and its application have been made the chief aim. Accordingly, no great importance has been attached to general angles, &c. And the items relating to simpler trigonometrical equations are given, inserted in problems.

The syllabus of teaching for middle schools, which was revised this year and which is going to be put into effect, is given below.

Revised syllabus of teaching for middle schools.

Mathematics.

In mathematics, though the items to be taught are divided into arithmetic, algebra, geometry and trigonometry, and are thus apportioned to each year, the mutual relations among various branches must be considered, and especially complex items relating to arithmetic must be taught in connection with algebra and geometry.

First Year .. .. 4 hours a week.

Arithmetic.

Try to keep the connection with the same in the elementary school and assign supplementary exercises and reviews in integers, decimals, compound numbers, fractions, and percentage. Also teach proportion.

Second Year .. .. 4 hours a week.

Algebra.

Negative numbers.

Integral expressions.

Addition, subtraction, multiplication and division; linear equations; factors and multiples.

Fractional expressions.

Reduction to the lowest terms; reduction to the common denominator; addition, subtraction, multiplication, and division; fractional equations.

Third Year .. .. 5 hours a week.

Algebra.

Evolution.

Square root; cube root; quadratic equations; irrational expressions.

GEOMETRY.

Straight lines.

Angles; parallel lines.

Linear figures.

Triangles; parallelograms.

Circles.

Fourth Year .. .. 4 hours a week.

ALGEBRA.

Proportion.

Ratio; proportion; compound proportion; proportional parts; alligation.

Progression.

Arithmetical progression; geometrical progession.

GEOMETRY.

Proportion.

Proportional lines; similar figures.

Fifth Year .. .. 4 hours a week.

ALGEBRA.

Logarithms.

Percentage.

Percent; interest.

GEOMETRY.

Planes.

Planes and straight lines; dihedral angle; solid angle.

Polyhedron.

Prism; pyramid.

Curved solids.

Circular cylinder; circular cone; sphere.

TRIGONOMETRY.

Trigonometrical functions.

Trigonometrical functions of an acute angle; trigonometrical functions of any angle; trigonometrical functions of the sum and the difference of two angles.

Solution of triangles.

Simple surveying.

Remarks.

1. In mathematics, it is necessary not only to make pupils understand accurately, but also to make them proficient in computation and familiar with its application.

2. In arithmetic, it is not objectionable to assign *soroban*-calculation in addition to mental and written arithmetic.

3. In geometry, give the locus, geomerical construction, area and volume in their proper places according to opportunity.

SECTION 3. Modification of the Subject-Matter in the Course of Mathematics.

If in the middle school, the elements of differential and integral calculus, and analytical geometry, could be taught in addition to arithmetic, algebra, geometry, and trigonometry, the aim of teaching these latter courses might be attained more thoroughly, as is very desirable. But for this purpose the number of recitation hours would have to be increased, and moreover similar desires exist not only in the mathematical course, but also in almost all the others; so, this hope can not easily be realized, when the matter is considered from the general point of view of middle school education.

In the actual practice the hours of teaching are insufficient even for the matter indicated in the existing syllabus and every school has increased its recitation hours to meet the necessity. But, as was mentioned above, in the remodeled regulation one hour is added, but in the third year only, and the total number of recitation hours is less than what most schools apportioned to this course heretofore.

This being the present condition, it is almost hopeless to increase in the near future the number of recitation hours

for the course in mathematics. Accordingly, there is no other way to meet the demand than to attempt to make the most of the number of recitation hours by omitting comparatively unnecessary matter and improving the method of teaching.

In the remodeled syllabus, it is noticeable that every effort was used to economize in the number of recitation hours, by maintaining a close connection between the course in arithmetic of the middle school with that of the elementary school, by teaching the more complicated matter in arithmetic together with algebra and geometry, by omitting the chain method in arithmetic, and permutation, combination and the binomial theorem in algebra. It prescribes also to teach locus, geometrical construction, area, and volume in geometry in their appropriate place, not simultaneously, and to connect the application of the logarithmic tables in trigonometry with the calculation of logarithms in algebra, etc. Hence, if the proper method of teaching is pursued, the recitation hours will not be as insufficient as they were before, and possibly there many even be some surplus.

But this surplus of hours produced as a result of these efforts, is still inadequate to introduce new matter. The only thing that we can do is to raise the standard in the most necessary subjects.

Even within these limits it is desirable to give the graphical representation of functions. In the solution of equations, it will arouse great interest in the pupils, if besides algebraical calculation, a similar solution of problems by means of the graphical representation of equations is taught. To go further and teach how to represent algebraical functions, trigonometrical functions, etc., by means of graphs is very necessary not only because it enables students to recognize the notion of function clearly, but because it is very often applicable in physics, chemistry, and other sciences, as well as in practical life.

The matter indicated in the remodeled syllabus is the minimum to be taught in the middle school, and nothing of it ought to be omitted.

SECTION 4. Connection between branches.

In the course in mathematics, every body recognizes the necessity of keeping up the connection between the several branches. But in the actual teaching of mathematics in the middle school, we must confess with regret, this was not satisfactorily practiced heretofore.

Teachers simply give the lesson from the text-books they use, and they have no opportunity to consider the relations between the several branches and to harmonize them; for, in our country, the distribution of subjects together with the selection and arrangement of materials, is fixed by regulations, and the text-books compiled in accordance with them must be used. Moreover, since very few text-books for all branches are compiled systematically by the same author, there is no wonder if no relation between branches is found.

These conditions entail inconveniences for both teachers and pupils and the educational authorities have long since endeavoured to remove these disadvantages. By the recent remodeling of the syllabus embodying the results of the investigation carried on in the Middle School attached to the Tokyo Higher Normal School, arithmetic and algebra are united and the distribution of subject matter is made in accordance with their intimate relation to each other. In geometry and trigonometry, it is not shown clearly how to connect them, but the syllabus indicates merely the main points, and the whole matter is left to the teachers without laying down any regulation about the arrangement and treatment of minor points. This procedure will gradually solve the problem mentioned above, and when the remodeled syllabus is carried into effect the former defects can be amended.

Next we shall discuss the problem how to connect each branch in the course of mathematics.

With regard to arithmetic and algebra the syllabus adopted in the Middle School attached to the Tokyo Higher Normal School, is most suitable. In algebra and geometry it tends to apply the knowledge of arithmetic and algebra in teaching geometry, and to utlize in teaching algebra the matters taught in geometry, and thus to establish a close connection between them. As to the teaching of proportion after the Euclidian method, it is not only not suited to the understanding of students, but it cannot give the relation of geometrical and algebraical proportions. There should also be a connection between algebra and geometry, by applying the method of algebraical solution to problems of geometry, giving problems of calculation related to theorems about area and volume in teaching geometry and explaining ratio and proportion of quantities by numerical examples, etc.

As to the connection between algebra, geometry and trigonometry, it will suffice to pay special attention to it whilst teaching, for by their very nature they are closely related. It is desirable to begin trigonometry in the latter part of the fourth year, although in the remodeled syllabus, it is prescribed in the fifth year only. To lengthen the term of teaching by beginning as soon as possible, is to improve the understanding, to deepen the impression and to make the teaching more effective. And this can be done without increasing the total numbers of recitation hours. However, as trigonometry requires knowledge of geometry, we cannot begin with it arbitrarily at any time. But in the latter part of the fourth year, the theory of similar figures is taught in geometry, so we ought to begin to teach trigonometry at the same time, and try to keep a close relation between them, and to attain the good results mentioned above.

CHAPTER. VI. EXAMINATIONS.

SECTION 1. Aim of Examinations.

The primary purposes of examinations are as follows:

1. To investigate how the matter taught has been understood by the pupils, and to use the results as references in teaching.
2. To judge whether pupils have learned their lessons well or not.

Although the followings are but concomitants of examinations, they are very often considered as of much more importance, and are added to the aims of examinations.

3. To encourage students to study.
4. To systematize and regulate their acquirements.
5. To accustom students to write answers within a limited time.
6. To make them familiar with examinations.

SECTION 2. Kinds of Examinations.

There are three kinds of examinations in the middle school: occasional examinations, term examinations and yearly examinations. They are held to afford data for estimating the attainments of students. There is no special examination at the completion of the whole course of the middle school.

The occasional examinations are to take place once, twice or thrice in each term. Usually less than one hour is assigned for each examination, the time being taken from the recitation hours. Sometimes notice is given several days beforehand, but not always.

The term examination is held, according to a previously arranged schedule at the end of both the first and the second term to test the attainments in the matter of the respective term. The time needed for each examination is one or two hours.

The yearly examination is held at the end of the school year in order to test the attainments in the matter of that year. The time needed for each examination is one or two hours.

SECTION 3. Methods of Examination.

Examinations in the middle schools are usually held for all the students of the same class or the same year at the same time, giving the same questions to them all. Written answers are required, and no oral examination has been applied to the course in mathematics.

SECTION 4. Desirability of Examinations.

There are diverse opinions among educators concerning the value and desirability of examinations. But in reality there is no school where examinations are not held, even if now and then there are some middle schools without periodical examinations. The main reason for examinations is the fact that the regulations for the enforcement of the Ordinance relating to middle schools prescribe that the promotion or graduation of pupils is to be in accordance with the merits of daily achievements and the results of examinations. It is also due to the fact stated in the previous chapter, that graduates of middle schools mostly wish to enter higher schools, and these schools select their matriculates by examination, that there is no school higher than the middle school grade where no examination takes place, that the qualification as teachers, judges, procurators, lawyers, ordinary and higher civil officers, etc. is attained by examination, and that when private companies engage employes the decision mostly depends upon an examination testing their abilities, or upon the result of their graduation examination at school. All these facts indicate the importance generally attributed to examinations in our country. And thus great importance has come to be attached to them

in the middle schools also. But recently in every high school, higher commercial school, higher technical school some of the matriculates are selected without examination, provided the principals of the middle schools recommend their conduct and superior scholarship. This method is most adapted to select first-class students, for the decision as to their scholarship is given not simply according to one examination, but by the judgement of middle school principals who have been well acquainted with them for several years. And it seems as if the tendency to put too much stress upon examination is moderated in this way. And yet, as there is a limit to the number of matriculates and not all pupils who have been recommended by each school can be accepted, it becomes necessary to select the best ones. Consequently, each principal finds it necessary to indicate the order of merit in scholarship and conduct of graduates to be recommended, and thus again the attainments of all the graduates must be ascertained in order to select the most proficient.

But it is not by any means as easy to ascertain the exact attainments of so many students as it would be to measure their stature. Although from the educational point of view, it is necessary to grade the students' ability, yet it is not required to determine the order of their attainments by the percentage method. And, even if there be some necessity to do so, and the indication of proficiency in each branch by percentage be possible, these very percentages are to be regarded not as indicating the order of real attainments but only as a formal approximation. The only correct way of indicating the degree of proficiency is to go by some scale, e.g. of five, and to find to which category in the scale a certain student belongs. If this method be adopted, special examinations can be omitted in mathematics, for the attainments of students can be ascertained during recitation hours. But if the percentage method be adopted, some

mechanical system must naturally be applied, in order to give percentages in the examination papers by judging them according to a certain standard.

Though up to the present there is no school without examinations, yet it has generally come to be admitted that various evils accompany the method of weighing proficiency exclusively by the result of a few examinations; and consequently there has been an endeavour more and more to take into consideration the daily standing of pupils,—attaching much importance to it,—in addition to the result of examinations, most frequently graded by a scale of five degrees. The enforcement of the new regulations, in accordance with which various higher schools take some matriculates without examination, has put an end to the tendency just mentioned,—a result presumably not anticipated by the authorities and yet proving injurious to middle school education.

Athough there are various opinions concerning the value of examinations, no one would oppose them, if their evils were removed and their advantages realized. And the condition of our country being at present as mentioned above, examinations can not practically be dispensed with. Thus it is not the time to discuss the desirability of examinations, but we must investigate how to remove the evils and realize the advantages.

SECTION 5. Propcsal for the Improvement of Examination Methods.

The following evils accompany examinations:

1. Pupils tend to neglect their daily lessons, and to cram for examinations. This is injurious, especially, in a systematic subject such as mathematics.

2. The result of the examinations does not necessarily indicate the scholarship of pupils. They come to look for success by chance.

3. They arouse fear, or help to create ignoble aspirations and low cunning.

4. They do not arouse genuine interest in the subject.

5. They produce a spirit of mean rivalry.

These are the evils very likely to accompany examinations, yet they are not essential to examinations, but rather consequences arising from mismanagement. In other words, they are apparently but the result of the method aiming at establishing the relative order of the pupils' proficiency exclusively in accordance with the result of examinations held infrequently. Consequently they may be removed, if the following improvements be introduced:—

1. Dispense with fixing the order of the pupils' proficiency.

2. Weigh the merit of attainments both by the daily standing and by the result of examinations.

3. Increase the number of examinations.

To put it more minutely, the judgment as to the merit of attainment should aim not at comparing the relative superiority of pupils, but at deciding the pupils' attainments in reference to a fixed standard, and consequently the determination of the relative order of attainments must be replaced by expressing the grade of proficiency. There need not be many grades; five will suffice: A indicating the highest grade, B the next, C the grade absolutely required in the class in question, D somewhat defective, and E unsatisfactory. This may put an end to mean rivalry, and may sufficiently incorporate the daily standing in its results. Moreover, if importance be attached to the daily standing and if at the same time the frequency of examinations be increased, holding exmination whenever a topic is completed and another is to be introduced, so that they may occur at least once or twice a month, or four or five times a term, then daily negligence, fear, etc., may be repressed.

Examination answers must be used not only as material

for judging attainments, but also as references for investigating the effectiveness of teaching. If the defects and mistakes are marked for reconsideration, if defects and mistakes common to many students are referred to in the next lesson, and if the pupils are thus acquainted with the results of the examination, the following benefits may be attainable :—

1. To encourage daily study,
2. To give exact and systematic knowledge,
3. To promote accuracy of expression,
4. To give familiarity with examinations.

CHAPTER. VII. Method of Teaching.

SECTION 1. Method of Teaching in General.

1. Form of teaching.

In regard to the form of teaching, the Herbartian theory had for a time been actively advocated in Japan, and its formal steps in teaching had been extensively adopted in elementary and middle schools. In case, however, these steps are adhered to, there results the need of spending much time in teaching, and great difficulty in covering the matter assigned. On account of these defects they have not become extensively prevailing. Moreover the result of recent research into pedagogical methods has made clear that the Herbartian formal steps can not be taken as the standard form for teaching, and they have now become almost obsolete.

The real aim of teaching is the systematic and exact acquirement of arts and sciences. Especially in the teaching of such sciences as mathematics, the method attaching much importance to the psychological basis or depending exclusively on formal logic, is not effective in attaining the real aim of teaching. Taking into consideration the nature of science and the principles of cognition, improvements must be made

in the method of teaching. In this sense, the three steps of the so-called standard form of Sallwürk are most adequately applicable to the teaching of mathematics.

2. Mode of teaching.

In regard to the mode of teaching of mathematics in the middle school the developmental and the training teachings are combined, and the teaching of new subjects is effected for the most part, by questions and answers thus encouraging preparation and review. But, the dialogue method is based mostly on the result of psychological investigation and can not transcend the current method neglecting the result of logical investigation. It either leaves too much to the mental activity of the students or falls into mere formalism and regrettably lacks much in developing mental activity, and in cultivating the critical and constructive faculties.

In order to make students acquire accurate ideas, clear conceptions and definite rules, it is very necessary to improve the dialogue method, and thereby to reveal its true value in accordance with logical principles.

In the lessons in which conversation and dialogue are mainly used the pupils' eyes, ears and mouths are much in requisition, but not their hands. For instance, in case a certain formula is derived by way of algebraic transformation, the teacher usually asks a pupil to mention the outlines of the process whilst he himself writes it on the black-board. In this case, however, it would be better to make all the pupils speak in response to his questions rather than to point out a pupil or two to recite. Or again in case a geometrical theorem is taught, the teacher usually draws the figure and outlines the proof on the black-board, and show them to the pupils. But, in this case also, it would be better to begin by making the pupils draw the figure or part of the figure from his dictation. As for the additional

figures needed in the demonstration, it is preferable that not only the teacher at the black-board, but also the pupils should draw them and write out the demonstration each one for himself so that no pupils are left unengaged. Looking at it either as affording diverse corresponding forms of mental activity or as giving accurate impressions, it would prove not a little beneficial to the teaching to make use of every appropriate occasion to call into activity not only the pupils' eyes, ears and mouths but their hands also.

3. Preparation and review.

In order to attain the full effect of examinations efforts should be made to use the method not only of direct instruction but also of indirect guidance by way of supplementary study. It is certainly most advantageous to put stress on preparation and repetition as practical applications of the latter method. Concerning the usual method of prepraration, however, there are many points to be considered, especially in reference to mathematics. When problems for exercise and application are assigned in mathematics, it is certainly very important to afford the pupils an apportunity to prepare and solve these problems by themselves. However, whenever a new rule or theorem is given, it is not only unnecessary but even hurtful to make pupils prepare it from the text book, because thereby the effect of teaching is diminished, nay the very aim of teaching is frustrated. The teacher of mathematics ought not to be satisfied with mere instruction on the subject in hand or explanation of the text books; he ought at the same time to insist on the proofs, on the rules to be derived from them and on their correct formulation, that is, he should aim at training the reflective faculty, cultivating the constructive faculty and developing the power of expression. But, as text books may be looked upon as being of the nature of explanatory notes or manuscripts of lectures having

been carefully written, ordinary pupils can easily understand the proofs for the theorems, the rules given, the reasons explained, etc., all at first reading. If text books be assigned for preparation, pupils may without effort understand the proofs and memorize the rules, and no room is left for the teacher for an adequate scheme to train the pupils in reflection, construction and expression and there may be no other result of his teaching except to examine the preparation gone through, to guard against carelessness in making it, and to aid the understanding of a few backward pupils. It is regrettable that almost all schools encourage text book preparation, and forget the loss thereby entailed in true teaching as well as the great hindrance put in the way of cultivating the pupils' real ability.

On the other hand, matters not explained in the text book, especially problems for exercise and application, are exceedingly effective for instruction as they afford the pupils occasion for indepedent thought and are accordingly very appropriate for home-tasks.

In any case repetition is indispensable, and in all middle school it is generally assigned. In most cases it is left entirely to pupils to work at it independently out of recitation hours, and only now and then there is an investigation as to its actual performance. It is seldom assigned for recitation hours under the teachers' guidance. However, as pupils in middle schools are young and lack self-control, especially those in the lower classes simply repeat the matter as it was given and lack ability to regulate and systematize the knowledge already acquired; therefore it seems indispensable to spend a part of the recitation hours for a review to be performed under the teacher's guidance.

4. Method of problem exercises.

Exercise in problems occupies a great part of mathematical

teaching and the effect of the teaching depends much on the method of assigning exercises.

The exercises in problems consist of two kinds, one to be assigned in the class room and the other for home work. The former aims at cultivating the pupils' ability to solve problems under the teachers' guidance, the latter at making them proficient in the solution of problems as well as at enabling them to write out the solution correctly. Simple problems are actually assigned for class room exercise, and difficult ones for home-tasks. Sometimes difficult ones are assigned for class room exercise, but only when they have been well prepared at home. This method is apparently effective in affording pupils an opportunity to reflect fully and to be drilled in many problems. But if this method is generally adopted, there is no opportunity for the pupils, on the one hand, to receive the teacher's direction as to what points ought to be noticed or how to find the solution, and for the teacher, on the other hand, to find what points are difficult for the pupils or what defect there is in their reasoning, etc. In short, it impedes the teacher in giving adequate guidance, and so to speak, leaves the pupils in the situation of the self-taught.

Thus after all this way cannot cultivate the ability to solve problems and consequently cannot attain the true aim of problem exercises.

Teachers ought to propose new problems of different kinds for class room exercise and try to afford the pupils adequate guidance by way of suggestions. They ought also to present at times many problems for all the pupils and go about inspecting the work here and there,—paying special attention to weaker pupils, and giving them the necessary hints. After all, problem exercises in teaching mathematics require much of the teacher's ingenuity. Much depends on the teachers tact, and on his painstaking preparation. As for those teachers who leave the work entirely to the

pupils and idly go about the class room as if they were inspectors, thus wasting the whole hour, they may well be said to understand nothing of the real meaning of problem exercises.

For the home-task problems similar to those assigned in the class room are mainly to be selected; also problems are to be added, of which the solution may easily be written, for they are to be worked out by the pupils' own ability. The latter are to be written in the home-task copy book and are to be inspected. The former are to be matter for examination during the following lesson. According to the nature of the problems, some of them should be written on the black-board for the critical and constructive investigation of the whole class. But the uniform adoption of this method frequent in middle schools, is due to the ignorance of time-economy. There are not a few cases where according to the nature of the problem it is enough to examine the correctness of the answer, or to ask orally or in writing for the outline of the solution to be followed. Recitation hours are precious to give the pupils the direct guidance of their teacher. So care should be taken to make the best use of these hours, to afford the pupils a benefit to be secured in no other way, and to make them appreciate the great value of the teacher's direct guidance.

SECTION 2. Various Problems Concerning the Teaching of Mathematics.

1. Systematic teaching in geometry.

In view of the difficulty of giving geometry in middle schools systematically and according to rigorous method from the very beginning, it seems prevalent in Europe and America to begin with experimental geometry by way of preparation; but in our own country this need is rarely felt. Some ten years ago, the rudiments of geometry were given as preparatory instruction, but contrary to anticipation it was found

that not only the difficulty in teaching geometry remained the same, but that the pupils were deprived of the demand for rigorous reasoning as well as of the genuine interest in and due regard for scientific methods. As geometry was first added to the middle school curriculum at that time, this defect was perhaps partly due to the lack of suitable text-books and adequate methods of teaching. But the difficulty in teaching geometry does not really arise from the geometrical theorems themselves, but rather from the logical method of proof. And to begin with experimental geometry by way of preparation cannot, of course, deliver geometry from the difficulty inherent in the teaching of logical reasoning. It must rely on the pupils being more mature in years and attainments. Accordingly, in our country, geometry is prescribed from the second to the third year,—the pupils by that time averaging above fourteen years of age, and systematic geometry is to be taught from the very beginning. The result of experience up to date goes to show, that by an efficient teacher, geometry can be taught by rigorous logical method from the very beginning without any special difficulty.

Thus no need is recognized of giving experimental geometry in order to get rid of difficulties connected with systematic geometry. If, however, in the first and the second year, before systematic geometry is taught, there is an appropriate treatment of experimental geometry, it may afford the pupils some necessary preparatory knowledge which is not without profit. But this may partly be done in the course of drawing, and there is no need of giving to this preparatory instruction part of the precious hours assigned for teaching mathematics.

2. Order in teaching algebra and geometry.

As above mentioned, middle school algebra should be given in connection with arithmetic, so that complicated

matters in arithmetic are taught in connection with the corresponding matter in algebra. And if in teaching geometry the preparatory instruction in experimental geometry is dispensed with, and systematic geometry begun immediately, it will be sufficient to start geometry in the third year. And if algebra is given in connection with arithmetic, the problem concerning the precedence of algebra or geometry in the middle school is settled without further discussion.

3. Historical teaching in mathematics.

The historical steps in the development of mathematics cannot be adopted as steps in teaching. Reference to the history of mathematics will be found useful in making the pupils acquainted with the course of development of mathematical investigation as well as with the circumstances under which new discoveries were made, in fostering the spirit of investigation, in stimulating interest, etc. But we cannot acknowledge the need of devoting much time to this. It will suffice to indicate the circumstances relating to the discovery of important points occurring in the subject matter or to mention so much of the historical development of mathematics as will prove beneficial for cultivating the reflective and constructive faculties.

4. Practical application and experimental teaching.

To make the pupils find the properties and relations of mathematical qualities and geometrical figures by way of experiment, then to lead them to understand the logical reasoning in the matter of proofs and finally to make practical applications of the knowledge is in accord with the best principles of teaching, and very effective for awakening interest, and for promoting a closer understanding and more lasting impressions. And yet, experimental teaching and practical application require much equipment and time, and are not easy to put into practice. Consequently they

are not yet much introduced in toour middle schools, though their need has become increasingly felt, and they have gradually come to be regarded as of much importance. The experimental teaching and practical application actually followed in ordinary middle schools are: in arithmetic, the use of weights and measures, estimating distances by the eye and by stepping off, the measurement of weights and contents; in geometry, the measurement and computation of surfaces and volumes; and in trigonometry, practice in the measurement of distances, angles, heights and areas. No middle school ever trained its pupils in the use of section paper, paper-folding or the slide rule. Even the laboratory for physics and chemistry is found in few middle schools. Much more, we must wait for many years to come to see the establishment of mathematical laboratories.

SECTION 3. Text-Books, Collections of Problems, and Reference Books.

The regulation concerning the selection of text-books for a middle school is the following. With the sanction of the local governor, the principal shall select the text-books from among those that have the approval of the Minister of State for Education. The number of approved text-books for mathematics amounts to several dozen, of which the following are most extensively used.

ARITHMETIC.

RIKITARO FUJISAWA.—Elementary Text-Book of Arithmetic.

TSURUICHI HAYASHI.—New Text-Book of Arithmetic.

HISASHI TERAO AND KŌKURŌ YOSHIDA.—Mathematical Text-Book for Middle Schools: Arithmetic.

TEIJI TAKAGI.—Text-Book of Arithmetic for Elementary Education.

SEITŌ KABA.—New Text-Book of Arithmetic.

KAMENOSUKE NAGASAWA.—New Text-Book of Arithmetic.

Algebra.

Rikitaro Fujisawa.—Revised Text-Book of Elementary Algebra.

Tsuruichi Hayashi.—New Text-Book of Algebra.

Hisashi Terao and Kōkurō Yoshida.—Mathematical Text-Book for Middle Schools: Algebra.

Teiji Takagi.—Text-Book of Algebra for Common Education.

Seito Kaba.—Revised Text-Book of Algebra.

Kamenosuke Nagasawa.—New Text-Book of Algebra.

Masanosuke Iijima and Ichinojō Amano.—Elementry Algebra.

Geometry.

Dairoku Kikuchi.—Elementary Text-Book of Geometry.

Tsuruichi Hayashi.—New Text-Book of Geometry.

Hisashi Terao and Kōkurō Yoshida.—Mathematical Text-Book for Middle Schools: Geometry.

Tota Yasuda and Denzaburo Shirai.—Text-Book of Geometry.

Seito Kaba.—Text-Book of Geometry.

Kamenosuke Nagasawa.—New Text-Book of Geometry.

Kwanichiro Miwa.—Revised Text-Book of Geometry.

Trigonometry.

Dairoku Kikuchi and Goichi Sawada.—Elementary Text-Book of Plane Trigonometry.

Matazō Endō—Text-Book of Plane Trigonometry.

Tsuruichi Hayashi.—New Text-Book of Plane Trigonometry.

Hisashi Terao and Kōkurō Yoshida.—Mathematical Text-Book for Middle Schools: Plane Trigonometry.

Kamenosuke Nagasawa.—New Text-Book of Plane Trigonometry.

Logarithmic Tables.

Meikō Imamura.—Ordinary Logarithmic Tables.

Tōkichi Miyamoto.—Five Places Logarithmic Table by Gauss, revised with Directions for Use.

All the text-books for middle schools have been compiled in accordance with the syllabus of teaching for middle schools, and consequently there is not much difference either in the kind or in the grade of the subject matter. And yet, in recent times, the contents have undergone notable improvements. Compared with what was prevalent some ten years ago, a complete renovation is apparent. To point out the principal improvements introduced, to the great benefit of both instructors and learners:—

1. Explanations and demonstrations have been simplified.
2. Strict and exact terminology has been introduced.
3. Problems have been judiciously selected and properly arranged.
4. There has been an adjusting to pupils' grade.
5. Recitation hours have been well distributed.
6. Stress is put on practical applications.
7. Systematic connection between various branches of mathematics is established.
8. Definitions, rules, formulae, theorems, etc. are clearly described.

But as to the essential character of text-books, there is much room for improvement. In the compilation of text-books, it is a grave problem whether the systematic or the methodic treatment of the subject matter is to be preferred. The adoption of a text-book compiled methodically involves the method of teaching prescribed by it. As in this case the characteristic benefits of free and creative activity in teaching are not available, text-books of this kind are not generally welcomed by first-class teachers. But by adopting these text-books inexperienced teachers may avoid the danger of courting failure. If, on the contrary, a book with systematic treatment is adopted, teachers themselves must plan the method of teaching, and the results will be the reverse

from what is mentioned above. Accordingly the problem of the choice of text-books cannot be settled apodictically. The text-books in use in our country mostly belong to the former class and are generally welcomed by the teachers.

At any rate, text-books are not transcripts of lectures, but are collections of problems in addition to instructions. If teachers are not thereby hindered in their teaching activity and pupils find therein what they have assimilated, developed and constructed during their recitations, it is sufficient. Preparation must be made through the pupils' own exertions. Review is not mere repetition. The methodically compiled text-books with their minute explanations and complete demonstrations, not only hinder the teachers' activity in teaching but also limit the pupils' activity in acquisition. It is often said that pupils prefer private study from text-books to the teacher's direct instruction. If teachers and pupils attach much importance to text-books and rely mainly on them, then the cultivation of the reflective, constructive and expressive faculties, which is the most important point in the teaching of mathematics, can not take place. This is the usual defect in the teaching of mathematics. The fault, however, is not on the part of the text-books, but rather on the part of the teachers. The improvement of teachers is more imperative than that of text-books.

In regard to the collections of problems, they are not much used in our country. One reason for this is that no adequate collections of problems exist. Another is that the text-book contains problems more than enough for the prescribed hours. If, however, the same problems be assigned year after year, pupils in the lower classes may learn their solution directly from the copy books of those in the higher classes. To remedy this defect and, at the same time, to afford matter for home-tasks during the summer vacation, teachers sometimes specially collect problems for their own use.

As the pupils in middle schools have as yet no sure fundamental knowledge and are in a grade which does not require reference books, there is no middle school where reference books are in use.

About the reference books for teachers, there are very few written by Japanese authors. As the teachers' knowledge of foreign languages has been limited generally to English, books published in England and America have been mostly used. In recent years, however, works composed by our own scholars are gradually increasing. Also an increasing number of teachers study German. The limit of of reference books, therefore, is gradually extending.

The following are the chief reference books most extensively used in our middle schools.

RIKITARO FUJISAWA.—Topics of Arithmetic and its Method of Teaching.

RIKITARO FUJISAWA.—Text Book of Arithmetic.

RIKITARO FUJISAWA.—Text Book of Elementary Algebra.

RIKITARO FUJISAWA.—A Sequel to the Text Book of Elementary Algebra.

RIKITARO FUJISAWA.—Lectures on the Teaching of Mathematics.

DAIROKU KIKUCHI.—Exposition of Mathematics.

DAIROKU KIKUCHI.—Text Book of Elementary Geometry.

DAIROKU KIKUCHI.—Lectures on Geometry.

HISASHI TERAO.—Text Book of Arithmetic for Intermediate Education.

TEIJI TAKAGI.—New Select Arithmetic.

TEIJI TAKAGI.—Lectures on New Style Arithmetic.

TSURUICHI HAYASHI.—New Select Geometry.

TSURUICHI HAYASHI and MOTOHARU KUNIEDA.—Equations.

TSURUICHI HAYASHI and MOTOHARU KUNIEDA.—Applications of Equations.

TSURUICHI HAYASHI and TANIJIRO KARIYA.—Inequality.

TSURUICHI HAYASHI.—Problems of Arithmetic: Four Rules.

TSURUICHI HAYASHI and MOTOYA SHIBAYAMA.—Conception of Numbers.

TSURUICHI HAYASHI.—Combinatory Analysis.

TSURUICHI HAYASHI and TANIJIRO KARIYA.—Theory of Probability.

TSURUICHI HAYASHI.—Determinants.

TSURUICHI HAYASHI.—Problems of Geometrical Loci.

TSURUICHI HAYASHI.—Problems of Impossible Geometrical Constructions.

TSURUICHI HAYASHI.—Problems of Maxima and Minima in Elementary Geometry.

HENRI POINCARÉ.—Science and Hypothesis; trans. from French into Jap. by T. HAYASHI.

TSURUICHI HAYASHI.—Rudiments of Differential and Integral Calculus.

TOKUSUKE KŌNO.—Lectures on Differential and Integral Calculus.

TOKUSUKE KŌNO.—Treatise on Algebraic Analysis.

OTOKICHI ARAKAWA and YASUJIRO TAKAGI.—Classifications of Constructions of Triangles.

OTOKICHI ARAKAWA.—Exposition of Classifications of Constructions of Triangles.

CATALAN.—Theorems and Problems in Geometry; trans. from French into Jap. by KAMENOSUKE NAGASAWA.

JITSUO YOSHIKAWA.—Modern Synthetic Geometry.

KIYOSHI IKEDA and RYOHEI TAGUCHI.—Treatise on Algebraic Analysis.

IWATARO TOMINAGA.—Psychology of Numbers and Arithmetic Teaching.

KICHISABURO SASAKI.—Essence of the Teaching of Arithmetic and Principle of Counting.

TODHUNTER.—Algebra.

HALL AND KNIGHT.—Elementary Algebra.

HALL AND KNIGHT.—Higher Algebra.

SMITH.—A Treatise on Algebra.

CHRYTAL.—Introduction to Algebra.

CHRYSTAL.—Text-Book of Algebra

WILSON.—Elementary Geometry.

CASEY.—Elements of Euclid.

CASEY.—A Sequel to Euclid.

HALSTED.—Elements of Geometry.

NIXON.—Euclid Revised.

NIXON.—Geometry in Space.

CHAUVENET.—A Treatise on Elementary Geometry.

TAYLOR.—Euclid.

PHILLIPS AND FISHER.—Elements of Geometry.

GODFREY AND SIDDONS.—Elementary Geometry.

CASEY.—A Treatise on Elementary Trigonometry.

NIXON.—Elementary Plane Trigonometry.

TODHUNTER.—Plane Trigonometry.

HOBSON.—Plane Trigonometry.

LONEY.—Plane Trigonometry.

BURNSIDE AND PANTON.—Theory of Equation.

PUCKLE.—Elementary Treatise on Conic Sections.

SALMON.—A Treatise on Conic Sections.

TODHUNTER.—Differential Calculus and Integral Calculus.

WILLIAMSON.—Differential Calculus and Integral Calculus.

FINK.—A Brief History of Mathematics.

BALL.—A Short History of Mathematics.

CAJORI.—A History of Mathematics.

SMITH.—The Teaching of Elementary Mathematics.

YOUNG.—The Teaching of Mathematics.

BENCHARA BRANFORD.—A Study of Mathematical Education.

BALL.—Mathematical Recreations and Problems.

SECTION 4. Models, Specimens and Instruments.

All acknowledge the need of models, specimens and instruments for teaching arithmetic, geometry and trigonometry, and of making use of them in observation and experiment.

There is no need of limiting the use in arithmetic of models and specimens, of weights and measures, as well as of coins ; in geometry, of rules, measures, compasses, circular instruments ; and in trigonometry, of surveying instruments to measure angles, distances, heights, etc. On the contrary, it should rather be encouraged. In regard to the use of models in teaching geometry, however, it must be limited to the beginning of teaching plane geometry in the lower class and solid geometry in the higher, when there is need of giving the pupils accurate ideas. We must, however, gradually bring pupils to understand geometrical conceptions without the aid of models. In geometry models may simply be used to make pupils acquire accurate ideas by way of actual observation. But, it is not easy for beginners to understand or to express complex figures. Hence for both purposes actual observation is necessary.

In case theorems relating to area and volume are given, they should be given in connection with their practical application, for which purpose models may also be utilized. To construct models enhances the value of their use. With the exception of such as can be constructed of string, thick paper, wire, etc., however, it is generally difficult to assign the construction of models to pupils at present.

It is also necessary in trigonometry to teach pupils actual measurement by means of surveying instruments. But as this requires time and equipment, and as it is difficult to allow many pupils at it, it has not yet extensively been put into practice.

The following are the kinds of models, specimens and instruments, which are generally provided in most of our middle schools :—

Implements of measures and weights, implements of metric measures and weights, implements of English measures and weights, a model of vernier, a tape measure, a Gunter's chain, flag staffs, tally pins, theodolite, a slide

rule, a black board compass, rulers;

Figures of Japanese and foreign coins, figures of standard implements of measures and weights, various kinds of loan bonds, blanks of bills of exchange and cheques, figures of geometrical solids, wall-map for showing logarithmic table, models of geometrical solids.

SECTION 5. Relation between the Course in Mathematics and Other Courses.

In the course of drawing, geometrical figures and patterns are given in sketches, copies and designs in the third year and below, and geometrical drawing is given in the fourth and the fifth years. The matter taught in geometrical drawing is as follows:

Fourth Year

Straight line, angle, circle, polygon, curve, horizontal plane, vertical plane, lateral plane, and development.

Fifth Year

In addition to what is given in the previous year, intersecting planes, isometric drawing, and an outline of perspective.

The drawing lesson is one hour a week for each year. And, as free-hand drawing is also given in addition to geometrical drawing, the hours used for geometrical drawing are exceedingly few.

The apportionment of geometrical drawing in the course of drawing is as above, and geometry, as previously mentioned, is to be taught in the third year only. Hence there must be planned some such mutual connection between the drawing lesson and geometry teaching, that when in the former geometrical figures are taken in the second year or lower they may afford by way of observation an intuitive geometrical knowledge preparatory for the latter, whilst when

in the latter geometrical theorems are treated in the third year and upward, they may in return afford a rational geometrical knowledge helpful to the better understanding of the former. It is also to be desired that the knowledge afforded by the teaching of geometry may be utilized in geometrical drawing, whilst the ability to produce elaborate figures acquired in the drawing lessons may be utilized in geometrical constructions, not only to elucidate the method but also to make it actual, and so forth.

In chemistry, biology, mineralogy and geography, in as far as they are taken in middle schools, mathematics are seldom used except for some simple computations, proportions and graphic representations. To these may, perhaps, be added some knowledge of solid geometry for crystallography, some computations of weights and measures as well as of money for geography, and some knowledge of solid geometry and trigonometry for the teaching of longitude and latitude as well as for map drawing. And yet in the middle school grade these mathematics are seldom applied, and at least there is no need in the teaching of mathematics to pay special attention thereto.

Mathematics are most extensively applied in physics. And what is most extensively applied in physics is the knowledge relating to proportions. Of the physical laws contained in the ordinary text-book, those relating to proportion are more than thirty.

Equations are used in dynamics, properties of matter, theory of heat, acoustics, optics, and theory of electricity and magnetism. There are not a few cases in which the solution of simple, quadratic, and irrational equations is required.

Of the theorems given in elementary geometry those are made use of in physics, which relate to triangles, parallelograms, similar figures, and symmetrical figures. They are most needed in dynamics and optics.

Even though middle school physics are not generally so advanced as to utilize trigonometry, yet some knowledge concerning trigononometrical figures is needed in teaching the reflection of light, the dip of the magnetic needle and the galvanometer.

Certainly it is desirable in the teaching of physics, not only to apply the principles acquired in mathematics so as to help the understanding, but also to endeavour thereby to reveal fully the value of the application of mathematics as such. It is also desirable in the teaching of mathematics, not to be satisfied with merely giving the principles, but also to endeavour fully to train pupils in applying them to actual problems,—especially when in algebra proportion and equation are treated,—taking the subjects extensively from physics as well as from other branches. Teachers of mathematics are blamed for the insufficiency of the pupils' knowledge of mathematics in learning physics. This insufficiency, however, is due not to the inferiority of their mathematical knowledge, but rather to the lack of training in its appilication. This is one of the ordinary defects in the teaching of mathematics. Undoubtedly it is due to the fact that thoughtlessly too much importance is attached to the exercise of problems in pure mathematics to prepare for the impending entrance examinations; that there is a cramming of such matter and a consequent neglect of the practical side of mathematics.

Of mathematics, arithmetic and algebra have the closest relation to daily life. As a matter of course, computations in the four rules, fractions and compound numbers are necessary for every one. As the result of material civilization, however, such conveniences as water-pipes, gas-fittings, electrical appliances, etc. have become increasingly prevalent, and the conditions of daily life have undergone a wonderful change. Consequently the amount of knowledge necessary for every day computations has become greatly extended, so that

for those who are well to do and keeping pace with the progress of national opulence, the knowledge of computations relating to taxes, loan bonds, stocks, interest, annuities, insurance. etc., has become very necessary. Hence in the teaching of mathematics in the middle school, efforts must be made amply to provide the knowledge required for all these problems of daily life.

CHAPTER VIII. To remove the Dislike of Mathematics, and to make that Study More Generally Acceptable.

Generally speaking mathematics are not to the taste of the pupils and frequently are a subject of dislike. This is owing to the following reasons. Abstract and well ordered as it is the study of mathematics is naturally monotonous, and its acquisition appeals to the reasoning power only. In numerical computations errors easily occur. Such an error, be it ever so insignificant at first, finally becomes so magnified that no computer can overlook it, and consequently he finds himself obliged to repeat the process. For these reasons mathematics do not appeal to beginners.

The science of mathematics is also no favourite in the home or in general society. As mathematical knowledge is wanting there, children can come into contact with it only at school. Left without help, they find their home-task in mathematics, reviewing and preparing, not so easy as other subjects. Children thus encourage a feeling of dislike against arithmetic and lose all taste for it already in the elementary schools.

Prior to entering elementary schools, however, children have a liking for measuring and enumerating things, and comparing them in size and number. Taking this aspect of their mental development into careful consideration, therefore, they ought to be so instructed in elementary schools as to deepen their taste for mathematics. In order to attain this

aim the following directions are to be specially attended to:—

Even if the teaching of mathematics should at first proceed, as a matter of course, from the intuitive and experimental side, yet do not idly consider its abstract side difficult, but try to impress mathematical truth gradually upon the children's mind so as to cultivate their taste for it.

Keeping in mind the nature of mathematical teaching, take up one item and teach it till it be fully acquired and becomes thoroughly familiar, and then proceed to the next item. Make children review at times items already taken, so that their clear understanding never be lost.

Paying special attention to the notation of figures and to the method of computation, do not thoughtlessly blame the slowness of children's computation, but rather try to cultivate in them the habit of computing correctly. Even if an error be committed, do not immediately ascribe it to their lack of scholarship and make them dispense with computation. Pay attention to the selection of subject matter so as to make it accord with the state of the children's knowledge.

Devise a plan to make home review easy, so as to lead children to take it up of their own accord as they do with other subjects.

Even in teaching other subjects, apply mathematics to those that relate to numbers thus making it practical.

In regard to backward children, make a thorough investigation and apply some remedy accordingly. Teach mathematical games. Also give interesting lectures relating to mathematics at class meetings and the like.

If attention be paid to some other points as well, so that the taste for mathematics be aroused in children and the dislike against it be removed whilst they are attending elementary schools, the greater part of the present problem may be regarded as already solved.

Also in middle schools similar tactics should be pursued. The more advanced pupils in middle schools acquire a sufficient liking for mathematics, but the backward pupils find it increasingly difficult as they advance, and come to dislike it altogether. Hence, some means should be devised from the very beginning to alleviate the difficulty experienced by backward pupils.

Concerning the plan of making mathematics more prevalent than at present no investigation has been made from the standpoint of popular education. Last year the Department of Education organized a Popular Educational Research Committee and arranged for a general investigation concerning popular education. Therefore the problem just referred to may sooner or later be investigated. The following may afford some help in cultivating a general taste for mathematics and to make clear its necessity.

To encourage popular works on mathematics, and provide them in the popular lecture-meetings, and to give interesting lectures on mathematics.

To publish interesting mathematical topics in the newspapers and magazines extensively circulating in the home circle.

To post up at schools, shrines, temples, parks, stations, etc., where large numbers of people assemble, the survey-maps of the neighbourhood and the results of surveying the distances, heights, areas, etc. of the principal mountains, rivers, high-ways, buildings, etc. in their neighbourhood.

To post up in schools, village-offices, town-offices, principal highways, statistics concerning local industries. To give in apprentices' schools, supplementary schools, young men's associations, etc. some easy problems in applied mathematics.

CHAPTER IX. Training of Teachers.

As the report concerning the training of teachers is to

be specially prepared by some other committee, we shall here limit ourselves to its main features.

Teachers of middle schools ought to have a teachers' licence granted in accordance with the Ordinance relating to the authorization of teachers' licences. These licences are to be granted by the Minister of State for Education to the graduates of government schools and institutes specially established for teachers' training as well as to those who have successfully passed a qualification test. The schools specially established for the training of teachers consist of higher normal schools, of which there are two at present, namely, the Tokyo Higher Normal School and the Hiroshima Higher Normal School. The teachers' training temporary institutes are the institutes specially established as a kind of emergency measure for the training of teachers. There is only one at present, concerned with the training of mathematics teachers. It is the Third Temporary Institute for the Training of Teachers in Sendai.

The qualification tests consist of two kinds, namely, the one without examination and the other with examination.

The test without examination is mainly applied to graduates of the schools established or approved by the Minister of State for Education.

The test by examination for mathematics, takes place according to four grades, the first consisting of arithmetic, algebra, and geometry, the second of trigonometry, the third of analytical geometry, and the fourth of differential and integral calculus,—the standard being in accord with the degree of ability required to teach satisfactorily in middle schools,---and the examination is held separately for each grade and at the same time there is a test as to sufficient knowledge of the outlines of pedagogics and of pedagogical method.

The number of those to whom teachers' licences have been granted in 1909 is as follows :—

Those needing no test	24
Successful non-examinees	7
Successful examinees	21
Total	52

Of those to whom teachers' licences are granted, the graduates of schools and institutes specially established for the training of teachers were given, during their studies, various subjects relating to pedagogy as well as various exercises relating to practical teaching, but those who merely pass the test generally lack pedagogical knowledge and tact as well as skill in teaching. Even after a few years' experience, they cannot free themselves from various defects in their practical teaching. Hitherto, as there was no sufficient number of middle school teachers and as it was difficult to make up the deficiency, efficiency of knowledge and skill could not be expected of these teachers. Neverthless, as the teachers' amount of knowledge and degree of skill have much to do, not only with the teaching itself, but also with the progress and development of school education at large, the Department of Education has been constantly endeavouring to supplement their scholarship and to improve the method of practical teaching. For this purpose, it used to organize in former days a summer lecture class during the long vacation. In order to enhance the value of this course, however, the higher normal schools have in recent years been instructed to organize courses during the ordinary school season, for the purpose of teaching in addition to mathematics, some pedagogics and pedagogical methods and of instituting researches as to practical methods of teaching. Good results have not been lacking.

For all that, the progress and improvement of middle school education depend much upon the teachers' knowledge and tact in education as well as upon their personality. And it seems most appropriate to the present condition of Japan to improve the method of qualification tests by

establishing a new course in practical education in the higher normal schools, and taking all the successful applicants for the test into this course as candidates for teachers in order to train them in education, theoretical and practical, and then granting teachers' licences only to those whose attainments prove satisfactory.

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CONTENTS.

The Teaching of Mathematics

in

Higher Middle Schools [High Schools].

THE COURSE OF MATHEMATICS

IN

THE SECOND DEPARTMENT.

PRELIMINARY REMARKS.

I. Kinds of Students.

A higher middle school is a preparatory institution of the Imperial University. Graduates of middle schools are admitted to higher middle schools after passing through a competitive examination, and they all intend, after their graduation, to proceed to the Imperial University with the object of pursuing their respective special courses of study. In every higher middle school, those students who are going to enter the Faculties of Science, Technology, and Agriculture are put into one group designated by the name of *Second Department*, in the first and second years of which course all the students receive exactly the same mathematical lessons, but in the third year, calculus is taught to all those who are going to enter the College of Engineering and also to some of those who intend to enter the College of Science.

II. The Principle according to which the Subject-matters of Mathematical Instruction are selected.

As above stated, the amount of knowledge of the students at the time of their admission, is that of the middle school graduates, and after three years' study in this

institution, they enter the Imperial University. It is, therefore, necessary that we should keep up, on one hand, a close connection with the middle school education, and, on the other, with the university education. At the same time, we must not lose sight of the affinity of mathematics with other branches of study, physics, dynamics, surveying, and so forth, preparing the mind so as to enable the students understand the lectures on these subjects. These considerations determine the selection of the subject-matters to be taught as well as the method of instruction.

III. The General Considerations regarding Mathematical Teaching.

As by far a great part of our student is going to enter the College of Engineering—such being the present tendency in this country—it cannot be helped that we have to teach the students of the second department as if they were all going to enter that college; but, as mathematics taught in higher middle schools must also have the character of mental culture, we ought not to make its applications alone our sole object, even though we may teach it as a preparatory study for engineering students. In teaching mathematics for its own sake, the utmost importance is to be given to deductive reasoning, keeping the logical principle in view as far as possible. And at the same time we have to make the students skilful in numerical calculations, whereby the subject-matter to be taught are selected from practical point of view. Now if we put too much weight on the logic, we would naturally meet with great difficulties in proving some certain theorems, and it would be almost impossible to teach them rigorously in higher middle schools; therefore, in actual teaching, we believe it sometimes necessary to have recourse to heuristic methods, whereby we appeal to the common sense of the students, or else explaining them by graphs and making use of the knowledge they have already acquired.

IV. Relations with other Branches of Study.

In order to establish connections between mathematics and other subjects, especially physics and dynamics, we shall choose examples and problems in mathematics from among those of physics and dynamics as much as we can, along with geometrical problems. The detailed discussion on this subject will be found in the answer to the question regarding Chapter II in the second Part.

V. Syllabus.

According to what we have stated above, we have prepared the detailed syllabus of teaching, and every particular will be found in it. Remarks have been added in order to make our views clear.

Table of the hours allotted to the mathematical lessons of the 2nd department.

	Hours per week	Total hours in one year
First year	5	about 160
Second year	4	,, 128
Third year	4	,, 128
		416

These hours are distributed among the different branches of mathematics in the following way.

		Trigonometry	Algebra	Analytical geometry	Calculus
First year	1st term	39	26		
	2nd term	24	20	6	
	3rd term		18	27	
Second year	1st term		26	26	
	2nd term		20	20	
	3rd term		18	18	
Third year	1st term				52
	2nd term				40
	3rd term				36
		63	128	97	128

Table of the Text-books now in use.

Trigonometry	Todhunter—Plane trigonometry for the use of colleges and schools.
Algebra	Todhunter—Algebra for the use of colleges and schools. Smith—A treatise on algebra. Fujisawa—*Zoku Syotô Daisûgaku Kyôkwasyo* (Text-book of Continuation of Elementary Algebra).
Analytical geometry	Kikuchi—Analytical geometry. Miwada—Solid analytical geometry. Puckle—An elementary treatise on conic section and algebraic geometry. Aldis—An elementary treatise on solid geometry.
Differential and integral calculus	Todhunter—A treatise on the differential calculus. " A treatise on the integral calculus. Williamson—An elementary treatise on the differential calculus. " An elementary treatise on the integral calculus.

Trigonometry.

GENERAL REMARKS.

I. The Object of teaching Trigonometry to the Students of the Second Department.

We should teach trigonometry, not only because it is an important study in itself from mathematical point of view, but also because it has not a little relation with analytical geometry, calculus, and other subjects, especially with physics, dynamics, and surveying, which will be studied later.

II. The Selection of the Materials to be taught and the Care that should be taken in Teaching.

The students have already learned elementary trigonometry in middle schools, but it will be assuming too much to think that their knowledge of the subject is otherwise than uncertain; so the first half of the trigonometry taught in higher middle schools is the same as that which the students ought to have learned in middle schools, with only this difference that the subject is treated more minutely; and after thus making their knowledge more certain step by step, we proceed to teach the remaining half which is entirely new to them.

As circular measure is not taught in middle schools, it should be explained clearly with special care, so that the students may fully understand it. Again, as circular measure is being constantly used in calculus and various branches of applied mathematics, it should be explained fully in order to avoid the mistakes which the students are liable to make. The students, so far as our experience goes, will be able to state the definition of circular measure clearly and correctly, but when they once come to apply it to numerical calculations, they often forget the fact that the

radian is the unit of the angle used in the calculation and commit blunders. Further, the relation between circular measure and sexagesimal divisions should be well taught.

Regarding trigonometrical ratios, it is futile to confine ourselves to acute angles, as is done in middle schools; but we should further give the definitions and explanations concerning the cases of angles in general, and we should also explain the periodicity of circular functions, and thus introduce a notion of periodic functions.

It has long been customary to devote one chapter to the properties of triangle, but from practical point of view there is no great need of treating the properties of triangle minutely and this chapter should, therefore, be entirely omitted. But the properties of triangle can be best explained and demonstrated by the use of trigonometical functions, and by so doing, not only the relation between geometry and trigonometry is made clear, but it also leads to the applications of trigonometry. Therefore, it will be only proper to retain this chapter as ever. Accordingly, we have preserved this chapter, but the properties of triangle are not so minutely described as has hitherto been done. We believe that the problems regarding the properties of triangle which are given to the students, have the effect of awaking their interest for trigonometry.

As the construction of logarithmic and trigonometrical tables is of no practical use whatever, and as its omission will bring no inconvenience, it has been left out; but there are some who advocate that it should be taught at this stage in view of the students having hereafter no opportunity of learning how these tables were made.

PLANE TRIGONOMETRY.

1. Circular measure of an angle. Definition of circular measure and its relation to the sexagesimal divisions.

2. Trigonometrical ratios. Definition of trigonometrical ratios of any angle from geometrical point of view; periodicity of circular functions.
3. Formulæ expressing sine and cosine of the sum or difference of two angles in terms of sines and cosines of the angles themselves. Addition theorems for tangent and cotangent. Multiplication and division of circular functions. Formulæ expressing the sum or difference of two sines or two cosines in the form of a product. Extension of the formulæ for the sum of three angles. Use of subsidiary angle. Special trigonometrical ratios.
4. Logarithm. Definition of logarithm and its fundamental properties. Transformation of the base. Use of logarithmic and trigonometrical tables.
5. Relations between the sides and the trigonometrical ratios of the angles of a triangle.
6. Solution of triangles.
7. Properties of a triangle.
8. Summation of the trigonometrical series :—

$$\sin \alpha + \sin (\alpha + h) + \sin (\alpha + 2h) + \ldots . + \sin (\alpha + \overline{n-1}\, h),$$

and $\cos \alpha + \cos (\alpha + h) + \cos (\alpha + 2h) + \ldots . + \cos (\alpha + \overline{n-1}\, h)$.

9. Methods of elimination, explained by several examples.
10. Inverse trigonometrical functions.
11. Solution of trigonometrical equations, explained by several examples.

Algebra.

GENERAL REMARKS.

Algebra taught in higher middle schools is to begin with the continuation of the same subject taught in middle schools, and then we go on to integral functions which are followed by calculus. In a portion of its first part, therefore,

it would appear that the same thing is taught over again, but we teach algebra more or less deeper than it is taught in middle schools. As there is no text-book suited to our object, and in that portion of algebra that comes after permutation and combination the subject-matters are, so to speak, independent of one another; so they are taught in different order by different teachers. The theory of rational integral function is, however, taught generally in the order prescribed in the syllabus.

ALGEBRA.

I. Equation of the First Degree.

Solution of the linear equation with one unknown quantity.

Solution of the simultaneous linear equations with two unknown quantities. It should be explained what relations among the known quantities make the equations inconsistent, or the unknown quantities indeterminate.

Equivalencies (in Rouché-Comberoussé sense) of two successive systems of equations which lead to their solution.

[The term "equivalencies" is not absolutely important, but it is convenient to use it in teaching.]

Variation of a linear function and its graph, and geometrical interpretation of the solution of the simultaneous linear equations with two unknown quantities. Solution of the simultaneous linear equations with three unknown quantities, whereby the notion of a determinant is to be introduced and explained.

II. Inequalities.

[Since it is useful for the discussion of the nature of a quadratic function, the theory of inequalities is inserted here. The fundamental principles on inequalities should be treated minutely and fully explained, for the subject is new to the students, and, moreover, it is not difficult to under-

stand.]

The fundamental theorems on inequalities.

Method of solving a linear inequality with single unknown quantity.

The arithmetical mean of any number of positive quantities is greater than or equal to their geometrical mean; and several other important theorems of the like kind.

The application of the preceding theorems:—Some problems of maxima and minima.

III. Equation of the Second Degree.

Solution of the equation of the second degree. [We should explain the necessity of introducing a complex number.]

Solution of a quadratic inequality.

The variation of the value of a quadratic function ax^2+bx+c. [In order to explain the variation of the value of a quadratic function ax^2+bx+c, it is convenient and interesting to interpret it by means of graph, and to show the connection between algebra and geometry.]

IV. Rational Integral Functions.

Remainder theorem and its consequences. Euclid's algorithm.

If the product of two rational integral functions $A(x)$ and $B(x)$ be divisible by a rational integral function $C(x)$, and $A(x)$ be prime to $C(x)$, $B(x)$ must be divisible by $C(x)$.

Decomposition of a rational fraction into partial fractions.

V. Permutation and Combination.

[This chapter is introduced here to give review-lessons to the students who have already acquired some knowledge of permutation and combination in middle schools.]

Number of r-permutation and r-combination of n letters.

Vandermonde's theorem.

Permutation and combination of n things when each of them may be repeated.

[In solving the problems of permutation and combination, the students must be very careful in reasoning, for otherwise they are liable to be led into erroneous results. This chapter will perhaps give a very good exercise of careful reckoning and reasoning for students.]

VI. Binomial Theorem.

The binomial theorem for positive integral exponents.

The properties of binomial coefficients.

Applications of the binomial theorem.

Multinomial theorem.

Summation of series by means of the method of finite differences.

VII. Theory of Determinant.

Definition of a determinant.

Elementary properties of a determinant.

Minor determinant, and expansion of a determinant.

Multiplication of determinants.

Application of the theory of determinant to the solution of the simultaneous linear equations with n unknown quantities.

VIII. Theory of Probability.

Fundamental notion.

Definition of probability.

Addition rule for mutually exclusive events.

Multiplication rule for mutually independent events.

Probability of the concurrence of dependent events.

Chance of event happening r times in n trials.

Mathematical measure of expectation.

Inverse probability.

IX. Theory of Numbers.

[The knowledges of the students concerning integers, which they have acquired in elementary and middle schools, are isolated and have no systematic connection with each

other. Despite the fact that the theory of numbers is rarely used for practical purposes, we introduce it here with the object of systematizing the students' knowledge and making them familiar with deductive reasoning. There is an opinion that it is advisable to teach the scale of notation also in this chapter.]

Elementary and fundamental properties of numbers.

Determination of $\phi(n)$.

Notion of congruence.

Elementary properties of congruence.

Fermat's and Wilson's theorems.

X. Simple Continued Fractions and Indeterminate Equations.

Properties of convergents. [We may teach some of the theorems in such a way that they can be applied with slight modification to general continued fractions.]

Periodic continued fractions and quadratic surd.

Solution of indeterminate equation.

Solution of $x^2+y^2=z^2$.

XI. Convergency and Divergency of Infinite Series.

Notion of a limit.

Definition of convergency and divergency of an infinite series.

Necessary and sufficient condition for convergency and divergency.

Tests of convergency:—comparison test. d'Alembert's test. Cauchy's test.

The series $\Sigma \frac{1}{n^p}$.

Alternating series.

Definition of unconditionally convergent series and conditionally convergent series.

Rearrangement of terms of a series. [Dirichlet's theorem on the rearrangement of terms of the convergent series with positive terms, and its extension to an unconditionally convergent

series. Some examples of conditionally convergent series.]

Sum or difference of unconditionally convergent series.

Multiplication of unconditionally convergent series.

Unconditional convergency of double series.

Binomial series, and binomial theorem for any index.

Exponential and logarithmic series.

Notion of an infinte product.

XII. Complex Number.

Definition of a complex number and the fundamental operations in complex numbers.

Argand's or Gauss' geometrical representation.

Fundamental properties of complex numbers.

De Moivre's theorem and its applications, including the solution of binomial equations. Convergency of infinite series with complex terms. [This theorem is important in view of the connection between exponential and circular functions.]

XIII. Rational Integral Functions.

[The chief object of the following chapters is to explain, on one hand, how a numerical equation can be solved, and on the other, to give the theory of rational integral functions as a preliminary to calculus. Algebraic equations are discussed systematically in such a manner that it may be looked upon as a model of systematic investigations.]

Rational integral functions and their graphs.

Derived functions and the special case of Taylor's theorem in differential calculus.

Outlines of the continuity of rational integral functions.

Geometrical interpretation of the first derived function.

Maxima and minima of rational integral functions.

XIV. Properties of Algebraic Equations.

The statement of the theorem:—every algebraic equation has at least one root.

Recapitulation of the theorem:—If a rational integral function vanishes for $x=a$, then it is divisible by $x-a$.

Every algebraic equation of degree n has n roots and no more.

Imaginary roots of an algebraic equation whose coefficients are real, occur in pairs.

An algebraic equation $f(x)=o$ has no real root, or an even number of real roots between two real numbers a and b, if $f(a)$ and $f(b)$ have the same sign, and the equation has odd number of real roots when they are of opposite signs.

XV. Symmetric Functions of Roots.

Relation between roots and coefficients.

Statement of the theorem :—Every rational integral symmetric function of roots can be expressed as a rational function of the elementary symmetric function.

XVI. Transformation.

Subtracting from the roots a given quantity.

Multiplying the roots by a given constant.

Transformation into reciprocal roots.

Some other transformations.

[It would be desirable to give some account of the general linear transformation.]

XVII. Cubic and Biquadratic Equations.

[Among the professors of higher middle schools, there is one who thinks that it would be advisable to teach the outline of the theory of groups in order to lead the students into algebraic solutions of cubic and biquadratic equations by its aid, while there is another who insists that the latter subject ought to be struck out from the higher middle school curriculum.

We believe that for the benefit of the students the algebraic solutions of the cubic and biquadratic equations should be taught, although it may not be necessary to give numerical examples.]

Algebraic solutions of the cubic and biquadratic equations.

Solution of the simultaneous equations of the second degree with two unknown quantities.

Solution of a cubic equation by the use of trigonometric functions.

XVIII. Properties of Derived Functions.

Special form of Rolle's theorem in differential calculus.

A multiple root of the order m of an algebraic equation is a multiple root of the order $m-1$ of its derived equation.

XIX. Separation of Roots.

Theorem of Fourier and Budan, and Descartes' rule of signs as its particular case.

Sturm's theorem, and its application to finding the conditions of the reality of the roots of cubic and biquadratic equations.

XX. Solution of Numerical Equations.

Determination of rational roots of a numerical equation.

Methods of Newton and Fourier.

Regula Falsi.

Horner's method.

XXI. Elimination.

Elimination by symmetric functions. [This method may be considered as an interesting application of symmetric functions, though it is not well adapted for practical purposes.]

Euler's method.

Sylvester's method.

Bezout's method.

Discriminant of an algebraic equation.

Analytical Geometry.

I. Object of teaching Analytical Geometry.

The object chiefly aimed at in the teaching of analytical

geometry is to make the students fully understand the way of representing curves by means of co-ordinates, teaching at the same time the principal properties of conic sections.

II. Topics of Teaching and our View concerning them.

The projective properties of conic sections are very interesting, but in the elementary part of analytical geometry we are chiefly concerned with their quantitative properties. In higher middle schools we confine ourselves, with a few exceptions, to the most important of the quantitative properties.

The idea of direction cosines is generally introduced for the first time in solid geometry, but it would be better, we believe, to introduce it in that part of plane analytical geometry where straight line is treated, and later to extend it to three dimensions.

To give to the area of a triangle a sense, either positive or negative according to the order in which we take its sides, is convenient in view of applying it later to differential and integral calculus.

A certain professor is of the opinion that it would be better to introduce one more chapter, in order to examine the nature of different surfaces represented by the general quadratic equation, and to find the condition that it may represent any particular kind of surface. However desirable this may be, it was decided not to adopt that opinion, owing to the limitation in available time.

Tangent plane and normal are to be given within the course of differential calculus.

ANALYTICAL GEOMETRY.

I. Representation of a Point by Co-ordinates.

Cartesian co-ordinates (rectangular as well as oblique), polar co-ordinates.

Transformation of a system of co-ordinates into another.

Cartesian co-ordinates of the point which divides the segment between two fixed points in a given ratio.

The distance between two given points expressed by their co-ordinates in rectangular and oblique Cartesian systems.

The area of the triangle formed by the lines joining three given points, and its expression in the form of a determinant.

II. Straight Line.

The locus represented by an equation and the equation of a curve.

Any linear equation in Cartesian co-ordinates represents a straight line, and conversely every straight line is represented by a linear equation in Cartesian co-ordinates.

Various forms of the equation of a straight line.

a) $\frac{x}{a}+\frac{y}{b}=1.$

b) $lx+my=p.$

c) Parametric equation of a straight line:—

$$x=x_1+lr, \quad y=y_1+mr.$$

d) $y=mx+b.$

e) The equation in the form of a determinant.

f) Equation of a straight line referred to polar co-ordinates, $\rho \cos(\theta-\alpha)=p.$

The angle between two straight lines whose equations are given in Cartesian co-ordinates, the axes being rectangular or oblique.

The condition of parallelism and perpendicularity.

Length of the perpendicular let fall from a point upon a straight line.

The equation to a straight line passing through the intersection of two straight lines.

The equation to the bisector of the angle between two given straight lines.

III. Transformation of Co-ordinates.

Parallel transformation.

Transformation of co-ordinates when the axes (orthogonal) are turned round the origin.

Transformation from oblique co-ordinates into rectangular and *vice versâ*, the origin and x axis remaining the same.

IV. Geometrical Applications.

Solution of several geometrical problems for the illustration of the application of analytical methods.

V. The Circle.

The equation to a circle referred to rectangular and oblique axes derived from its definition.

Parametric equation of a circle:

$$x = r\cos\theta, \quad y = r\sin\theta.$$

Equation to a circle referred to polar co-ordinates:

$$\rho^2 - 2\rho a\cos(\theta - a) + a^2 = r^2.$$

Equations to the tangent to a circle:

(1) $x x_1 + y y_1 = r^2$,

(2) $x\cos\theta + y\sin\theta = r$,

(3) $y = m x + r\sqrt{1+m^2}$.

The equation to the circle passing through two points of intersection of two given circles.

Radical axis for two circles.

Radical centre for three circles.

VI. General Equation of the Second Degree.

Determination of the co-ordinates of the centre of the curve. Classification of conics into central and non-central.

Condition that the equation of the second degree represents two straight lines.

[In this chapter, up to the preceding article, it has been so treated that the discussion is applicable to both rectangular and oblique co-ordinates, but, in the

following, it will be confined to the case of rectangular co-ordinates alone.]

Elimination of the term involving xy by turning the axes of co-ordinates by an angle ω.

Normal form of the equation to a central conic referred to its principal axes.

Distinction between the ellipse and the hyperbola.

Transformation of the equation of a non-central conic into its normal form.

VII. Central Conics.

Parametric equation to the ellipse:

$$x = a\cos\theta, \quad y = b\sin\theta.$$

Auxiliary circle and eccentric angle.

Orthogonal projection of a circle upon a plane neither parallel nor perpendicular to the plane of the circle is an ellipse.

[We have introduced this theorem here, for it is very important and useful and can easily be proved.]

Equation to the ellipse referred to polar co-ordinates, centre being the pole.

Eccentricity.

Hyperbola and its conjugate hyperbola.

Parametric equation of the hyperbola, and the eccentric angle.

Equation to the hyperbola referred to polar co-ordinates, the centre being the pole.

Eccentricity.

Tangent and normal to a central conic at a point on the curve.

$$(1) \quad \frac{x x_1}{a^2} \pm \frac{y y_1}{b^2} = 1,$$

$$(2) \quad x\cos\alpha + y\sin\alpha = \sqrt{a^2\cos^2\alpha \pm b^2\sin^2\alpha},$$

$$(3) \quad y = mx \pm \sqrt{m^2 a^2 \pm b^2}.$$

Equation to a pair of tangents from a point to a central conic.

Properties of pole and polar.

Definition of foci and directrices of a central conic. The distance of any point on the curve from the focus bears a constant ratio to its distance from the corresponding directrix.

The sum (or difference) of the distances of any point in an ellipse (or hyperbola) from the foci is constant. In the ellipse the normal bisects the interior angle between the focal distances, and in the hyperbola the exterior angle; and the focal radii make equal angles with the tangent.

Polar equation to a central conic, the focus being the pole.

Confocal conics.

Conjugate diameters; the equation to a central conic referred to its conjugate diameters.

In the ellipse (or hyperbola), the sum (or difference) of the squares of any two semi-conjugate diameters is equal to the sum (or difference) of the square of the semi-axes.

All parallelograms, whose sides are formed by straight lines passing through the extremities of one of conjugate diameters and parallel to the other, are equal in area.

Asymptotes; the equation to the hyperbola referred to its asymptotes.

VIII. Parabola.

Equations of a tangent:

$$y\,y' = 2\,d\,(x + x'), \quad y = m\,x + \frac{d}{m}$$

Pole and polar. The pair of tangents drawn from a point to a parabola.

The locus of the point from which two tangents at right angles to each other can be drawn, is the directrix. Definition of focus.

The tangent at any point makes equal angles with the focal distance of the point and the line drawn through it parallel to the axis.

Polar equation to the parabola, the focus being the pole.

The equation to the parabola referred to a diameter and the tangent at its extremity.

SOLID GEOMETRY.

I. Co-ordinates.

Cartesian and polar co-ordinates of a point.

The distance between two points, expressed in Cartesian co-ordinates.

The co-ordinates of a point which divides the straight line joining two given points in a given ratio.

Direction cosines, and the angle between two straight lines.

Theorems concerning orthogonal projection: the algebraic sum of the orthogonal projections on a given line of a series of lines joining two points is equal to the orthogonal projection of the straight line joining them, due regards being had to giving proper signs. The area of the orthogonal projection of a triangle on a plane, is equal to the area of the triangle multiplied by the cosine of the angle between the plane of the triangle and the given plane.

II. Plane and straight line.

[It is to be explained that a single equation in x, y and z represents in general a surface, and the two equations in x, y and z a curve in space.]

Direction cosines of a plane and the equations of a plane.

$$(a) \quad Ax+By+Cz+D=o,$$

(b) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$,

(c) $l\,x+m\,y+n\,z=p$.

The distance of a point from a plane.

Volume of the tetrahedron formed by four given points, and the equation of a plane in determinant form.

Equations of a straight line:

(a) $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$,

(b) $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$,

(c) $A_1x+B_1y+C_1z+D_1=o$ and

$A_2x+B_2y+C_2z+D_2=o$

The angle between two planes, and the conditions of their parallelism and perpendicularity.

The shortest distance between two non-intersecting lines.

Transformation of co-ordinates.

III. Surface of the Second Degree.

Normal forms of the equations of the ellipsoid, the hyperboloids of one sheet and of two sheets, the elliptic paraboloid and the hyperbolic paraboloid.

Cone and cylinder.

Proof of the theorem that the section of a right circular cone by a plane, is in general a conic, and the section of a right circular cylinder is an ellipse.

[Tangent plane and normal to a surface are to be taught within the course of differential calculus, as they furnish an excellent example of the applications of differential calculus to geometry, and as, moreover, it would be somewhat difficult to discuss them without the aid of differential calculus.]

Differential and Integral Calculus

Although the differential calculus can be most elegantly treated by the sole use of differential coefficients to the exclusion of that of infinitesimals, the notion of the latter is indispensable in such problems as are connected with applied mathematics. No course on differential calculus can ever be considered as perfect, unless infinitesimals are introduced or even extensively made use of. By so doing logical strictness may be impaired more or less, but the advantage to be gained more than compensates this draw-back, by enabling the students to be ready to proceed to solve any problem in such a manner as promises to be most effective and successful.

The theory of calculus should not be given in such a form that recalls a rule of thumb, and that may result in presenting the calculus as a merely mechanical manipulation of symbols. At the same time it is equally undesirable to go to the other extreme of elaborate treatment running into hair-splitting details. In many cases graphical interpretations and explanations are very much more to be preferred than confusing and incomprehensible analytical treatment.

It would be hardly necessary to add that throughout the whole course of calculus every opportunity should be taken advantage of, to bring it in contact with physics and other branches of mathematics.

I. Function of a Single Variable.

1. Variable and function.

Continuous variation of a real variable.

Definition of a function of a real variable.

Definitions of one-valued and many-valued functions; elementary algebraic and transcendental functions; explicit and implicit functions; inverse functions.

Graph of a function. Graph of elementary functions.

2. Limit.

Limiting value of a variable.

Limiting value of a function.

Limits of the sum, difference, product and quotient of functions.

Special limiting values:

$$\lim_{x=a}\frac{x^m-a^m}{x-a}=ma^{m-1},\ \lim_{x=\infty}\left(1+\frac{1}{x}\right)^x=e,\ \lim_{x=0}\frac{\sin x}{x}=1.$$

3. Continuity of functions.

Continuity of a function at a point.

Continuity of a function in an interval.

[This is limited to the case of a closed interval.]

Theorem. If $f(x)$ is continuous in an interval (a, b), and $f(a)$ and $f(b)$ have opposite signs, then $f(x)$ vanishes for some value of x lying between a and b.

[The proof of this theorem may better be omitted, and some graphical explanations given instead.]

Cor. A continuous function takes any value lying between any two of its values.

Continuity of the sum, difference, product and quotient of continuous functions.

Continuity of the continuous function of a continuous function.

Continuity of elementary functions.

Examples of discontinuous functions.

4. Infinitesimals.

The notion of infinitesimals. Order of infinitesimals.

Some mathematical and physical examples.

5. Differentiation.

Definition of derivatives.

Geometrical meaning of the derivative of a function.

Differentials.

Differentiation of the sum, difference, product and quotient of functions.

Differentiation of the function of a function.
Differentiation of inverse functions.
Relation of the change of the value of a function to the sign of its derivative.
Rolle's theorem.
[This theorem is explained by the aid of graph.]
Mean value theorem.

6. Successive differentiation.

Successive derivatives.
Theorem of Leibnitz.
n^{th} derivatives of certain functions.
Determination of $f^{(n)}(o)$ by means of differential equations.
Successive differentiation of the function of a function.
Dependency of the sign of $f''(x)$ with the convexity and concavity of a curve $y=f(x)$. Point of inflexion.

7. Taylor's theorem and expansion of functions.

Taylor's and Maclaurin's theorems. Lagrange's and Cauchy's forms of the remainder.
Expansion of functions e^x, $\sin x$, $\cos x$, $(1+x)^m$, $\log(1+x)$, $\operatorname{arctg}(x)$ into power series of x.
Expansion of a function by means of indeterminate coefficients.

8. Indeterminate forms.

Evaluation of indeterminate forms.

II. Function of Many Variables.

Definition of a function of two or more variables.
Limiting value of a function.
Continuity of a function at a point.
Continuity of a function in a domain.
Statement of theorems concerning the continuity of a function.
First partial derivative.
Geometrical meaning of the first partial derivative of

a function of two variables.
Total derivative and total differential. Geometrical and physical examples on total derivative and total differential.
Successive partial derivatives.
Change of the order of differentiation.
Change of independent variables.
Differentiation of a composite and an implicit function.
Taylor's theorem.

III. Maxima and Minima of Functions.

Maxima and minima of a function of a single variable.
Maxima and minima of an implicit function.
Maxima and minima of the function $f(x, y)$, wherein x and y are connected by a functional relation.
Maxima and minima of a function of two variables.
Method of indeterminate coefficients for finding the maxima and minima of a function of several variables connected by some functional relations.

IV. Application of Differential Calculus to Geometry.

Tangent and normal in rectangular and polar co-ordinates.

Ex. Ellipse, astroid, catenary, inverse curves, limaçon (elliplic and hyperbolic), cardioid, lemniscate of Bernoullil.

Asymptote.

1. Two kinds of definition.
2. Relation between the two definitions.
3. Determination of asymptotes, particularly of those of algebraic curves.
4. Coincidence of the two definitions in the case of algebraic curves.
5. Curvilinear asymptote.

   Ex. Hyperbola, folium of Descartes, strophoid,

$$y^3 = ax^2 + x^3, \quad y^3 - xy^2 = a\,(x^2 + y^2), \quad y = \frac{x^3 + x}{x - 1},$$

$$y = ax + b + \frac{c \sin x}{x}, \text{ hyperbolic spiral.}$$

Convexity and concavity of a curve. Point of inflexion.

Ex. $y=\frac{1-x}{1+x^2}$, cubic parabola.

Relation of two curves at their common point.

Definition and geometrical meaning of the order of contact.

Multiple points.

1. Double point, triple point, etc.
2. Determination of multiple points of algebraic curves.

   Ex. Lemniscate of Bernoulli, $y^2=(x-a)(x-b)(x-c)$.

Envelope. Locus of double points of a system of curves.

Ex. Astroid, strophoid.

Length of arc. Angle of contingence.

Radius of curvature. Osculating circle and centre of curvature.

Ex. Ellipse, parabola, spiral of Archimedes, catenary.

Evolute and involute.

Ex. Ellipse, parabola, cycloid, involute of a circle.

Curve tracing.

Ex. Epicycloid, hypocycloid, epitrochoid, hypotrochoid, conchoid of Nicomedes, $x^4+xy^2+y^4=x(ax^2-by^2)$, $a^3y^2=bx^4+x^5$.

Graphic solution of an equation.

Surface and space curve.

1. Equation to right helicoid, cylinder and cone.
2. Tangent plane and normal to a surface.
3. Space curve.

   Ex. Helix.
4. Tangent line and osculating plane to a space curve.

INTEGRAL CALCULUS.

I. Indefinite Integral.

Definition of indefinite integral.

[The term "integrand" to be used for the sake of convenience.]

Elementary integrals of fundamental importance.

Methods of integration :—

(a) integration of a function by decomposing it into the sum of the functions whose integrals are known,

(b) integration by substitution,

(c) integration by parts.

Integration of various kinds of functions :—

(1) integration of rational functions,

(2) integration of some irrational functions,

(3) integration of the following transcendental functions,

$$\int x^m (\log x)^n \, d\,x, \quad \int x^n \cos x \, d\,x, \quad \int x^n \sin x \, d\,x,$$

$$\int \sin^m x \cos^n x \, d\,x, \quad \int e^{ax} \cos^n x \, d\,x.$$

II. Definite Integral.

Definition of definite integral.

[Definite integration to be defined as summation, the limits being finite, and the existance of definite integral to be explained by means of the area of a curve.]

Calculation of definite integral directly from its definition, for example,

$$\int_a^b x^3 \, d\,x, \quad \int_a^b e^{kx} \, d\,x, \quad \int_a^b \sin x \, d\,x, \text{ etc.}$$

Approximate integration :—trapezoidal rule and simpson's method.

Definite integrals whose indefinite integrals are known.

Theorems on definite integrals :—

(a) interchange of limits,

(b) decomposition of the interval of integration,

(c) change of the variable of integration.

A special case of the first mean value theorem in integral calculus.

Elementary definite integrals :—

$$\int_a^b \log x\,dx, \int_0^{\frac{\pi}{2}} \sin^n x\,dx, \int_0^{\frac{\pi}{2}} \cos^n x\,dx, \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x\,dx,$$

in which m and n are positive integers.

Extension of the meaning of definite integral to the integral whose limits are infinite.

III. Quadrature of Plane Curves.

Quadrature of plane curves in rectangular and oblique co-ordinates.

Ex. Ellipse, circle, hyperbola, parabola, and cycloid.

Qudrature of plane curves in polar co-ordinates.

Application to the folium of Descartes, parabola $\left(r=\dfrac{a}{1+\cos\theta}\right)$, lemniscate of Bernoulli, cardioid.

Quadrature of plane curves whose equations are given in parametric representation.

Application to an ellipse given by the equations

$$x=\frac{1}{\sqrt{a+2h\mu+b\mu^2}}, \quad y=\frac{\mu}{\sqrt{a+2h\mu+b\mu^2}}$$

in which $ab-h^2>o$ and $a>o$.

IV. Rectification of Plane Curves.

Rectification of plane curves in rectangular co-ordinates.

Ex. Circle, parabola, astroid, and catenary.

Rectification of plane curves in polar co-ordinates.

Ex. Parabola $\left(r=\dfrac{a}{1+\cos\theta}\right)$, lemniscate of Bernoulli $(r^2=a^2\cos 2\theta)$.

[The term "the elliptic integral of the first kind" to be here introduced.]

Rectification of plane curves represented by parametric equations.

Ex. Cycloid, astroid, and ellipse.

[The term "the elliptic integral of the second kind" to be introduced on this opportunity].

V. Cubature of Solids.

Cubature of a solid, the area of its section by a plane being given as a function of its distance from a given plane.

Ex. Sphere, cone, elliptic paraboloid, ellipsoid and hyperboloid of one sheet.

Cubature of a solid of revolution.

Ex. Solids of revolution obtained by rotating parabola, astroid round the axis of x, and cycloid round its base.

VI. Complanation of the Surface of Revolution.

Complanation of the surface of revolution.

Ex. Spherical zone, the surfaces of revolution obtained by rotating parabola $y^2 = 2px$, astroid round x-axis, and cycloid round its base.

VII. Integration and Differentiation of Infinite Series.

Definition of the uniform convergence of an infinite series.

Uniform convergency of a power series.

Continuity of uniformly convergent series.

Integration and differentiation of series.

Deduction of the expanded form of arcsin x by integration from the expansion of $\frac{1}{\sqrt{1-x^2}}$

Integration of the elliptic integral

$$\int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1-k^2\sin^2\phi}}$$

by expansion.

VIII. Double and triple Integrals.

Definition of double integral.

Finding of double integral by two successive single integrations.

Division of the domain of integration,

(a) into the series of elementary rectangles, whose sides

are parallel to the axes of co-ordinates,

(b) into the series of elementary areas bounded by concentric circles, and straight lines passing through the origin of co-ordinates.

Applications of the above method to the cubature of a solid and to the complanation of a surface.

Definition of triple integral.

Finding of triple integral by three successive single integrations.

Division of the domain of integration,

(a) into the series of elementary rectangular parallelopipeds, whose faces are parallel to the planes of coordinates.

(b) into the series of elementary volumes formed by concentric spheres, planes passing through z-axis, and right circular cones having z-axis for its axis and the origin of co-ordinates for its vertex.

Application: The cubature of a solid, the centre of mass, the moment and the moment of inertia of a surface and of a solid.

The Course of Mathematics

in

the Third Department.

INTRODUCTORY REMARKS.

In higher middle schools, there are students whose intention is to enter the Faculty of Medicine of the Imperial University. They are collected together in the group designated by the name of the Third Department. The number of hours allotted to the teaching of mathematics in the third department is considerably less than the corresponding number in the second department; namely, 3 hours a week in the first year, 2 hours a week in the second year, and thus, in total, about 160 hours. They are distributed in the following manner as regards the study of trigonometry, algebra, analytical geometry, and calculus:—

for trigonometry and algebra, about 40 hours in the first year,

for analytical geometry, about 56 hours in the first year,

for differential and integral calculus, about 64 hours in the second year.

Text-books at present in use are the following:—

Kambly—Die Elementar-Mathematik III. (Ebene und sphärische trigonometrie.)

Todhundter—Plane Trigonometry for the use of colleges and schools.

Kambly—Die Elementar-Mathematik I (Arithmetik and Algebra.)

Smith—A treatise on Algebra.

Sammlung Göschen: Höhere Analysis I und II. (Dif-

ferential und Integralrechnung.)

PLANE TRIGONOMETRY.

Circular measure of an angle.

Trigonometrical ratios.

Formulæ for the trigonometrical ratios of the sum or difference of two angles, and various formulæ deducible from them. Formulæ for the division of angle.

Relation between the sides and trigonometric ratios of the angles of a triangle.

Logarithm. Definition of logarithm, and its fundamental properties. Use of logarithmic and trigonometric tables.

Solution of triangles.

Solution of trigonometric equation.

Inverse circular functions.

ALGEBRA.

Review of the solution of quadratic equation, permutation and combination, taught at middle schools.

Inequalities. Fundamental theorems on inequalities.

Decomposition of rational fraction into partial fractions.

Binomial theorem for positive integral exponent.

Convergency and divergency of an infinite series.

Notion of a limiting value. Definitions of convergency and divergency of an infinite series. Theorems concerning the test for determining the convergency or divergency of a series:

(*a*) by comparing each term of a series with corresponding term of another series, whose convergency or divergency is known,

(*b*) by examining the ratio of each term to the preceeding.

Theorems for the test of the convergency or divergency

of a series, whose terms are alternately positive and negative.

Convergency of some power series.

ANALYTICAL PLANE GEOMETRY.

I. Representation of a Point by Co-ordinates.

Rectangular Cartesian co-ordinates and polar co-ordinates.

Cartesian co-ordinates of the point which divides the segment between two given points in a given ratio.

Distance between two given points expressed by their co-ordinates, the axes being rectangular.

II. Straight Line.

Any linear equation in Cartesian co-ordinates represents a straight line, and *vice versâ.*

Various forms of the equation to a straight line:

a) $\frac{x}{a}+\frac{y}{b}=1,$

b) $x=x_1+l\,r$ and $y=y_1+m\,r,$

c) $l\,x+m\,y=p,$

d) $y=m\,x+b,$

e) the equation of a straight line referred to polar co-ordinates.

Angle between two straight lines, whose equations are given in rectangular Cartesian co-ordinates.

Condition of parallelism and perpendicularity.

Length of the perpendicular let fall from a point upon a straight line.

Solution of several problems for the illustration of the applications of analytical method.

III. Circle.

Equation to a circle referred to rectangular axes.

Polar equation to a circle.

Equation of a tangent to a circle: $x\,x_1+y\,y_1=r^2.$

VI. Central Conics.

Definition of the ellipse, and its equation. [We define an ellipse as the locus of a point, the sum of whose distance from two given points is constant.]

Centre and diameter of the ellipse.

Polar equation to the ellipse referred to its centre.

Definition of the hyperbola, and its equation. [Hyperbola is defined as the locus of a point, the difference of whose distances from two given points is constant.]

Centre and diameter of the hyperbola.

Polar equation of the hyperbola with its centre for the origin.

Equation of a tangent to a central conic:

$$\frac{x\,x_1}{a^2} \pm \frac{y\,y_1}{b^2} = 1$$

Asymptotes, as the tangents at infinity.

Definitions of the foci and directrices of the ellipse and hyperbola.

The distances to a focus and the corresponding directrix from a point on a central conic, are in a constant ratio.

Focal radii to a point on a central conic make equal angles with the tangent at that point.

V. Parabola.

Definition of the parabola, and its equation. [We define a parabola as the locus of a point whose distances from a given point and a given line are equal to each other.]

Definition of the focus of the parabola.

Equation of a tangent to a parabola: $y\,y_1 = 2\,d\,(x + x_1)$.

The tangent at any point on the curve makes equal angles with the radius vector to the point from the focus and the line drawn through it parallel to the axis.

ANALYTICAL SOLID GEOMETRY.

I. Co-ordinates.

Rectangular Cartesian and polar co-ordinates of a point.

The distance between two given points, expressed in Cartesian co-ordinates.

The co-ordinates of a point which divides the straight line joining two given points in a given ratio.

Direction cosines, and the angle between two straight lines.

II. Plane and straight Line.

Equation to a plane:

(a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$,

(b) $Ax+By+Cz+D=o$.

Equations to a straight line:

(a) $\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}$,

(b) $A_1x+B_1y+C_1z+D_1=o$ and
$A_2x+B_2y+C_2z+D_2=o$.

Angle between two planes.

III. Surfaces of the Second Degree.

Definition of a sphere, and its equation.

Ellipsoid, hyperboloid of one sheet and of two sheets, elliptic and hyperbolic paraboids.

[The order of treatment we follow in this case is exactly the opposite to that adopted in Aldis' elementary treatise on solid geometry. Thus, we define, in the first place, ellipsoid, hyperboloid of one sheet and of two sheets, &c., by their equations, and then proceed to find their sections by a plane parallel to the axes.]

Definitions of right circular cone and cylinder, and their equations.

DIFFERENTIAL AND INTEGRAL CALCULUS.

Differential Calculus.

I. Introduction.

Definition of a function and variable.

Limiting value of a function.

Theorems on limits:

(*a*) limit of the sum or difference of two functions,

(*b*) limit of the product of two functions,

(*c*) limit of the quotient of two functions.

The limit of $\left(1+\frac{1}{x}\right)^x$ for $x=\infty$.

The limit of $\frac{\sin x}{x}$ for $x=o$.

On the continuity of a function and the graphic representation of a function.

Infinitesimals.

II. Derivatives.

Definition of derivatives.

Geometrical interpretation of the derivative of a function by means of the graph of the function.

Differential.

Theorems concerning differentiation:

(*a*) differentiation of the sum or difference of two functions,

(*b*) differentiation of the product and quotient of two functions.

Differentiation of the function of a function.

Differentiation of an inverse function.

Differentiation of elementary functions.

Relation between the variation of a function and the sign of its derivative.

Rolle's theorem and the mean value theorem.

Higher derivatives.
Relation of the sign of $f''(x)$ to the convexity or concavity of a curve $y=f(x)$ and to the point of inflexion.

III. Taylor's Theorem and its Applications.

Taylor's and Maclaurin's theorems.
[We shall treat the theorems by assuming the expansibility of a function, the remainder being not given.]
Expansion of the following functions into power series:—

e^x, $\sin x$, $\cos x$, $\text{arctg}\, x$, $(1+x)^n$, $\log(1+x)$.

Evaluation of the indeterminate forms.

IV. Function of Many Variables.

Definition of a function of two or more variables.
Continuity of a function at a point.
Continuity of a function in a given domain.
First partial derivative.
Geometrical representation of a function of two independent variables, and the geometrical interpretation of the first partial derivatives.
Successive derivatives.
Change of the order of differentiation.
Differentiation of a composite and an implicit function. Total derivative.
Total differential.

V. Maxima and Minima.

Maxima and minima of a function of a single variable.
Maxima and minima of an implicit function.

VI. Application of Differential Calculus to Geometry.

Tangent and normal to plane curves.
Asymptote.
Envelope.
Curvature and evolute.

Integral Calculus.

I. Indefinite integrals.

Definition of an indefinite integral.
Elementary integrals of fundamental importance.

Methods of integration :—

(a) integration of a function by decomposing into the sum of certain number of functions whose integrals are known,

(b) integration by substitution,

(c) integration by parts.

Integration of rational functions.

Integration of $\int \phi (\sin x, \cos x)\, dx$, where ϕ denotes a rational function of the arguments.

II. Definite integrals.

[In the first place, we shall interpret the existence of definite integral by using the area of a plane curve, and give a few examples of definite integrals which may be directly calculated from the definition of definite integral.]

Evaluation of a definite integral, when the indefinite integral is known.

Theorems on definite integrals :

(a) interchange of the limits,

(b) decomposition of the interval of integration,

(c) change of the variable of integration, &c.

Some simple definite integrals.

III. Quadrature of Plane Curves.

Quadrature of plane curves expressed in rectangular co-ordinates.

Application to ellipse, hyperbola, parabola, and cycloid.

Area of plane curves in polar co-ordinates.

Application to circle and parabola.

IV. Rectification of Plane Curves.

Rectification of plane curves in rectangular co-ordinates.

Application to parabola, catenary, and astroid.

V. Cubature of Solids.

Cubature of a solid, the area of its section by a plane being given as a function of its distance from a given plane.

Application to ellipsoid and sphere.

Cubature of solids of revolution.

Ex. Solid of revolution obtained by rotating the parabola $y=px^2$ round the axis of x.

VI. Complanation of the Surfaces of Revolution.

Complanation of the surface of revolution.

Ex. Surface obtained by the revolution of the parabola $y^2=2px$ round the axis of x. Surface obtained by revolving a cycloid round its base.

The Course of Mathematics

in

the First Department.

INTRODUCTORY REMARKS.

Of the various subjects taught to those students in the first department who are going to study Philosophy at the Faculty of Literature of the Imperial University, mathematics has its share. Two hours a week throughout the year in the first year course, *i.e.*, in all about 62 hours, are assigned to teaching the elements of analytical geometry and calculus. In addition, trigonometry taught in middle schools is reviewed and something of probability is also given as it will be of use in studying philosophy. Keeping in view the shortness of time at disposal we have adopted the following syllabus.

Text-book. In one of the higher middle schools, Junker's Höhere Analysis is used as text-book, but in all others, mathematics is taught by lecture, no text-book being used.

I. Trigonometry.

Circular measure of an angle.

Trigonometrical ratios.

Addition theorems for sine, cosine, and tangent; and various formulæ deducible from them.

Relations between the sides and angles of a triangle.

Inverse circular functions.

II. Analytical Geometry.

Rectangular co-ordinates and polar co-ordinates.

Empirical curves.

Notion of a function and variable.

Curve of uniform velocity with respect to time.
Curve of the velocity uniformly accelerated.
Equation of a straight line:

$$\frac{x}{a}+\frac{y}{b}=1, \quad y=mx+b.$$

Solution of simultaneous equations of the first degree with two unknown quantities, and its geometical interpretation.
Solution of the equation of the second degree.
Graphic representation of the functions,

$$y=x^2, \ y=ax^2+bx+c, \ y=\frac{1}{x}, \quad y=k\sqrt{a^2-x^2}$$

Definition of a circle, and its rectangular and polar equations.
Definitions of ellipse, hyperbola and parabola.
Definitions of focus and directrix.
Solution of simultaneous equations.

$$\frac{x^2}{a^2}\pm\frac{y^2}{b^2}=1 \text{ and } y=mx+b,$$

and the graphic interpretation of the solution.
Graphic representations of the functions,

$\sin x$, $\cos x$, $\operatorname{tg} x$, $\operatorname{arc} \sin x$, $\operatorname{arc} \operatorname{tg} x$, a^x, $\log_a x$,

x^3, x^3+ax+b, $x^{\frac{3}{2}}$

Graphic solution of simultaneous equations

$y=ax+b$ and $y=\operatorname{tg} x$,

$y=x^3$ and $y=ax+b$.

Definition of a tangent to a curve, and the equation of a tangent to a circle, ellipse, hyperbola, and parabola.

III. Differential Calculus.

Notion of limit.
Limit of certain functions, among which are to be included

$$\operatorname*{Lim}_{x=\infty}\left(1+\frac{1}{x}\right)^x \text{ and } \operatorname*{Lim}_{x=0}\frac{\sin x}{x}.$$

Continuity and discontinuity of functions.
Infinitesimals.
Differentiation and differentials.
Successive derivatives.
Convergency and divergency of infinite series.
Necessary and sufficient condition for convergency and divergency.
Tests of convergency:
comparison test,
d'Alembert's test,
Cauchy's test,
test for alternating series.
Taylor's and Maclaurin's theorems without the remainder, the expansibility of functions being assumed.
Binomial theorem, expansions of e^x, $\sin x$, $\cos x$, etc.
Maxima and minima of a function.

IV. Integral Calculus.
Notion of indefinite integrals.
Direct integration.
Methods of integration:
by decomposition into a certain number of functions,
by substitution,
by parts.
Notion of definite integrals.
Quadrature of plane curves.
Approximate integration: — trapezoidal rule and Simpson's method.

V. Probability.
Definition of probability.
Addition rule for mutually exclusive events.
Multiplication rule for mutually independent events.

Probability of the concurrence of dependent events.
Repeated trials.
Mathematical measure of expectation.
Inverse probability.

Answers to the Question Proposed.

PART I.

1. The answer to the question given in Chapter I. of the first part of the General Plan of the Work of *Rapport Préliminaire.*

The table of the general mean of the average ages of the students in higher middle schools.

	First department	Second department	Third department
First year	20 Y. 6 M.	19 Y. 8 M.	19 Y. 7 M.
Second year		21 Y. 1 M.	20 Y. 8 M.
Third year		22 Y. 1 M.	

2. The answer to the question given in Chapter II.

How to establish the relations between mathematics and other branches of science, especially physics and dynamics, is an important question but difficult to answer.

The object of teaching physics and dynamics in higher middle schools is chiefly to impart to the students physical concept, keeping in view that the knowledge of these subjects they have already acquired is very meagre and imperfect.

In teaching physics and dynamics, if we make use of mathematics which the students are assumed to have learned, explanations will be made easy, and consequently, less time will be needed in their teaching. But if we use mathematics too much, the imparting of physical concept may be hampered by the very reason of the difficulty of mathematics. This

is an undesirable thing, which we ought to avoid. It would be superfluous to add here that we should endeavour to make the relations between mathematics and both physics and dynamics as close as possible, so long as such endeavour does not endanger the purpose of teaching the latter subjects. Indeed such a method seems to have hitherto been adopted.

In many text-books of mathematics which have been in general use, their authors seem to have paid great attention to the close relations that are necessarily found to exist among the different branches of mathematics. To give an example, what have been used in the explanations and applications of calculus are nearly all the problems of geometry, thus showing that great attention and pains have been taken in making the relations between these two branches intimate; but it will be found at the same time that physical problems are rarely nserted in the text-books of mathematics. If we give the students the physical as well as geometrical applications of calculus, the introduction of the physical problems will most likely excite in them great interest, and will well accord with the object of teaching mathematics, for mathematics is, unlike physics and dynamics, one of the subjects which they have been taught all the time ever since they entered the elementary schools and which have been deeply impressed on their minds.

In a word, we believe that the relations between mathematics and the sciences in which mathematics are extensively used, should be kept intimate, by applying the former to the latter in a suitable manner, and at the same time choosing the examples and problems used in mathematics from among the problems of physics and dynamics.

PART II.

1. The answer to the question given in Chapter I. of the second Part of the General Plan of the Work

of *Rapport Préliminaire*.

The remarks appended to the syllabus would serve as the answer to this question.

2. The answers to the questions given in Chapter III.

(*a*) Examinations should not be done away with.

Examination is not only a sort of stimulus for making students diligent, but also causes them to review what they have been taught during a term or a year. It will make their knowledge certain, and what they have learned piece by piece is transformed into a systematic whole. Though it is needless to say that the students should not study merely for the sake of examination, yet if we dispense with it, they will become, it is feared, not diligent at all. If there be some good means to cause them to be self-diligent, it would be well to abolish examination altogether. We believe, however, there is at present no such means of fulfilling our purpose, that will prove more efficient than examination.

(*b*) The number of examinations.

From what we have just stated, it would appear that oftener the examination is held the better it is, but this is far from being true. The students are not, of course, fond of examination, and if we were to hold it too often, they would come to hate the study of mathematics. Moreover, it would not only diminish the number of the teaching hours of mathematics but also bring about injurious effects on the teaching of other subjects. We believe, therefore, that it is most appropriate to hold it about six times a year, that is, three term examinations, each to be held at the end of a term, and three special examinations to be held once in each term. But at present only term examinations are held in most of the higher middle schools.

(*c*) How to select the problems in examinations.

Many comparatively easy problems are to be preferred to a few difficult ones. As to the nature of the problems,

they should be such as will test how far the students have comprehended what have been taught. In case the time allotted to examination is exceptionally long and many problems could be given, such problems as will test the ability of the students may also be included.

3. The answers to the question given in Chapter IV.

(*a*) The method of teaching.

In every branch of mathematics, there are many fundamental principles which may be recognized intuitively to be truths, but which requires, strictly speaking, logical proofs which are exceedingly difficult. In such cases, it would be better to teach them intuitively, dispensing with the logical mode of reasoning, and explaining them by means of graphs. In higher middle schools, we have to start by frequently having recourse to the intuitive method of teaching, and then to go on step by step to the logical method. As to the harmonious relations between the intuitive and logical methods of teaching, we have already stated them in the remark on the syllabus.

We add the following remarks, to which we wish to call special attention. When there are many data necessary in proving a certain theorem, we ought to point out clearly why and where in the course of proof these data are necessary, or else the students will commit these data into memory without understanding what they are good for. Again it is very important, we believe, to call attention of the students to the following points :—

(1) the nature of θ in the remainder of Taylor's theorem,

(2) the order of going to limits in $\operatorname*{Lim}_{h=o}\operatorname*{Lim}_{k=o} f(h\,k)$,

(3) examining of discontinuous points within the interval of a definite integral,

(4) care to be bestowed in taking the proper sign in extracting square root, for example :

$$\frac{d}{dx}\operatorname{arc\,sec} x = \frac{\pm 1}{x\sqrt{x^2-1}} \qquad x \gtrless o,$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \pm \operatorname{arc\,sin}\frac{x}{a} \qquad a \gtrless o.$$

(*b*) On the conventional boundary between the different branches of mathematics.

It would be better, generally speaking, not to observe strictly the conventional boundary between the different branches of mathematics. However, if we put away entirely the conventional boundary, the students are liable to become confused. Therefore, the question converges to in what measure the boundary may be made to disappear. To give an example or two, as to the relation between algebra and calculus, it would be better to give the general notion of limit as early as possible in algebra; and with regard to differential and integral calculus, we think it will be at least convenient to begin with integral calculus as soon as possible, and thenceforth to teach differential and integral calculus side by side. We have often tried such method of teaching, and have always found it very convenient. Indefinite integration immediately following differentiation as its inverse process is more easily comprehensible, and also this method gives the students more chances of applying their knowledge of integral calculus to physics, dynamics, and surveying. And as far as our experience goes, we have found no incovenience whatever in following the method just mentioned.

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Article IV.—The Teaching of Mathematics in the Faculty of Science (College of Science) **of Imperial Universities.** By T. Yoshiye, *Professor at the College of Science, Tokio Imperial University*, and S. Nakagawa, *Assistant-professor at the College of Science, Tokio Imperial University.*

TOKIO
1912

DIVISIONAL REPORTS
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TOKIO
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Article V.—The Teaching of Mathematics in the Faculty of Technology (College of Engineering) **of the Tokio Imperial University.** By K. Shibata and S. Yokota, *Professors at the College of Engineering, Tokio Imperial University.*

TOKIO
1912

DIVISIONAL REPORTS

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TOKIO
1912

THE TEACHING OF MATHEMATICS
IN THE
FACULTY OF TECHNOLOGY

(College of Engineering)

TOKIO IMPERIAL UNIVERSITY

The object of teaching mathematics in this college is to give students mathematical knowledge necessary for engineering purposes. For this reason, not only the operations in symbols and their method of reasoning should be taught, but also much stress should be laid on their practical applications based on actual examples. In solving a problem, therefore, answers expressed in symbols are not sufficient, and students should become accustomed to the practice of expressing in numbers what they have got in symbols in such units as may be easily understood by a common workman or mechanic.

Students should be instructed so that before they begin calculations of any kind, they may be able to form general idea regarding the order of magnitudes of the results to be obtained in order to avoid such answers as are sometimes very far from being correct.

As regards numerical calculations, students should understand the degree of approximation of the result aimed at, and so avoid unnecessary calculations. For instance, when a student estimates the horse-powers of an engine by means of an indicator diagram, he sometimes gives a result consisting of ten figures, despite the fact that there may be an

error of some 10 % in the data; it is extremely necessary to caution him against such obsurdity.

Therefore, all the branches of mathematics taught in this college must be closely connected with the engineering course.

The time allotted to the teaching of mathematics in this college is three hours per week during the first term (from the middle of September to the middle of December) and the second term (from the beginning of January to the end of March) of the first year course.

An examination is held at the end of each term i.e. twice during the whole course. At each examination, from four to six questions are generally given, and the students are requested to give their answers within three or four hours.

As to the method of teaching, a professor delivers a course of lectures either his own or taken from some text-books, and the students take notes of the essential points in these lectures.

The following are some of the reference books now in use:—

> J. Lüroth—Vorlesungen über numerisches Rechnen. Teubner, Leipzig.
>
> J. Perry—Calculus for engineers. Edward Arnold, London.
>
> A. G. Greenhill—Differential and Integral Calculus. Macmillan & Co., London.
>
> P. Appell—Éléments d'analyse mathématique. Gauthier Villars, Paris.

The standard of teaching. Differential and integral calculus and other branches of mathematics which are being taught in higher middle schools are used as basis, and various ways of applying them for practical purposes are taught in the college. The differential equations, from page 548 to page 674 of Appell's Mathematical Analysis (published in 1905

and given in the foregoing list) are taught, if possible, before the end of the first term, and the applications of them and other branches of mathemetics are taught during the second term.

We annex here as reference Professor Inokuchi's "Outlines of Practical Mathematics," as his views are recognized in the college and agree with what have been stated above.

Appendix——A General Outline of Practical Mathematics. By Professor A. Inokuchi.

The following is a general outline of a course of Summer lectures on practical mathematics delivered by me in 1902, at the request of the Minister of Education, to those engaged in technical education.

The order in which the different branches of mathematics are taught, and the method of teaching which are generally adopted, waste days, months, and even years of the student's life. The subject itself is one suggestive of headache to many children; while the study of it in the opinion of many people is practically as useless as the study of the theory of swimming, of horsemanship, or of bicycle riding. What a pity it is that the study of mathematics, the most useful and sharpest tool in the hands of those engaged in various branches of industry, and the one which can be got for nothing, should be regarded by school children and by some of their parents as mere waste of time!

Professor John Perry of England, an earnest advocate of reform in technical education, says:—

"There are some persons among those who are engaged in various branches of industry, who apply mathematics to the industry they are engaged in, and these persons almost always come down to be teachers and professors and seem to get away farther and farther from their former occupations; and again there are persons who are very skilful in framing the order and method of teaching, and these persons have instinctively been very skilful in abstract reasonings from their childhood, and have come to be what they are as easily as ducks have been very skilful in swimming since the days when they were mere ducklings. There are many persons of this stamp and in their eyes, those children who can not learn elementary Euclid, even though they spend a year in its study, must be said to be dunces; but to tell the truth, the mode of learning things and training mental faculties by abstract reasonings is limited to persons of extraordinary brains, and moreover they will perhaps be persons of abnormal brains.

When they teach children geometry, why do they not entirely lay aside Euclidean reasonings? Why do they not teach theorem 47 in Part I of Euclid without demonstration, saying it is an evident theorem? Why do they not teach two or three pages of elementary algebra instead of Part II and Part V of Euclid, or why do they not say a great many theorems in Part VI are axioms? If they say logical inferences are necessary in any way, let them consider well—there should not be, strictly speaking, anything like an axiom.

Newton applied geometrical conic sections to the study of astronomy, and dynamics has made its development. Seeing this fact, is there any reason for saying that the pupils of technical schools should go from astronomy to dynamics?.........."

This plan of Perry's for the reformation of the method of teaching mathematics has already been recognized by the Science and Art Department of England, and at Battersea Polytechnic of the same country, mathematics is now being taught after the same remodelled method. A certain M. T. Ormsby is a lecturer in the said institution and he is now teaching applied mathematics, civil engineering and surveying. This same person has written a book of practical mathematics, which makes a demy-octavo volume of 410 pages. The contents of the book are arithmetic, algebra, geometry, trigonometry, solid geometry, mensuration and calculus, and, though there are lines of demarkation between the various branches, the explanations, demonstrations, operations and so on, given therein, are quite different from those that have hitherto been adopted, and the whole is designed to be taught in two years.

I believe such a method to be the most appropriate for teaching mathematics, as an essential subject in technical education, and I shall now try to state in what points my views differ from the methods that have been hitherto adopted.

In arithmetic, as soon as the four common rules have been mastered, the addition, subtraction, multiplication, and division of decimals, should be taken up, and in fractions, the work should be confined to such simple ones as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and so on. That the greatest common divisor and least common multiple are not easily understood by beginners is universally recognized. At this point those axioms relating to general quantities should be taught and at the same time the applications of the sign of equality, and it would be better to teach ratio, direct proportion, inverse proportion, compound proportion, percentage, proportional parts and so on, by applying the above stated axioms and the sign of equality. Square root and cube root can be taught in arithmetic, but it would be better to let pupils use tables, as they use almanacs and railway time tables, and it would also be better to teach them the use of the table of logarithms to five figures.

In algebra, the beginning should be made at equations avoiding difficult factoring and other work of the same nature, where pupils are drilled, not

only in common equations, but also in those equations which contain decimals and incommensurables. It is also necessary to teach the theory of exponents, logarithms, and the binomial theorem. The pupils should be required to show on plotting paper the manner of variation of an expression of the third degree and of a transcendental expression, maximum and minimum, and the root of an equation.

Length, breadth, and size are more than sufficiently minutely explained in common text-books of geometry, and a great many applications of them are found in algebra, but time, interval, speed, and acceleration are nowhere clearly explained in any text-books of mathematics, and problems regarding them are only found here and there.

This seems to me a gross mistake, and I believe the loss of balance will be evident to any one who will look through what is taught in mathematics from beginning to end. I shall, therefore, try to make up for this defect.

Regarding arithmetic and algebra as immaterial tools, and instruments, pencil, ruler, compass and protractor as material tools, the properties of lines, angles, triangles, circles, and circular functions may be taught far more concisely than is generally the case.

In mensuration, arithmetical, algebraical, logarithmical, geometrical, trigonometrical, and graphical problems regarding triangles, circles, ellipses, spheres, prisms, cylinders, pyramids and so on, may be taught promiscuously, and also approximate calculations according to the rules of the trapezoid, and Simpson's method.

What I have just stated is a general outline of the method which seems to me the most appropriate. For putting it in practice, actual objects models, drawings, diagrams, plotting paper, slide-rules, table of quantities, and drawing instruments, will be required.

So much by way of introduction; now as to the essential points of my lectures;

(a) An inaccuracy in the known numbers renders an exact result impossible.

(b) The accuracy and inaccuracy of numbers are independent of the position of decimal point.

(c) In a calculation, which is necessary from the standpoint of industry it is usually sufficient, if the result is expressed by two or three figures, and therefore it is sufficient in such operations, if three or four figures are given.

(d) The errors of a number cut off at the fourth figure; and the same of the result got by multiplying or dividing that number, and of the power or root of the same.

(e) It is easier and also more convenient to solve the problems of proportion according to the theory of ratio.

(f) The application of the binomial theorem to approximate calculations, or, $(1+x)^n=1+nx$, $(1+x)^m(1+y)^n=1+mx+ny$, and so on.

(g) The explanations of logarithms and antilogarithms, and the use of the logarithmic table of five figures.

(h) The application of the formula of interpolation, $y_x = y_0 + x\triangle y_0 + \frac{1}{\underline{|2}}x(x-1)\ \triangle^2 y_0 + \frac{1}{\underline{|3}}x(x-1)\ (x-2)\triangle^3 y_0 + \ldots\ldots$

(i) The theory of the slide-rule and its usefulness.

(j) Miscellaneous problems:

1. Find the value of m and c in $y = mx + c$ by finding the two sets of values for x and y.
2. Find the value of x in $1.8^x = 4.2$
3. Find the value of h in $Q = \mu\, b\, h \sqrt{2gh}$

etc. etc.

(k) The explanation of speed, the mode of measuring speed, the explanation of simple harmonic motion, examples, graphs, and so forth.

(l) The circumference of a circle, and the surface and volume of a sphere, prism, cylinder, and pyramid.

(m) The volume of a truncated pyramid, wedge, pseudo-prism and spherical segment.

(n) Similar plane figures and solids and their applications.

(o) Methods of drawing similar ellipses quickly, of describing a part of a large circle, of estimating an approximate area by sight, and of forming a general idea regarding the limit of errors in the estimation.

(p) The rules of the trapezoid, Simpson's and other approximate methods of integration.

(q) The meaning of mean value and how it is found.

(r) The great usefulness of graphical mathematics and the readiness with which it can be applied.

Seeing that nearly all problems can be solved by graphical methods, very big books, nearly as microscopical as the map of Tokio, have been written. The large does not always cover the small, and rough calculations not necessarily inferior to detailed ones. The minutely accurate methods found in such big books may be necessary to those scholars who have a plenty of leisure, but to those who are engaged in practical industry, they are utterly useless.

(s) The artificial variations of quantities determined by rules, regulations, laws, or treaties are generally discontinuous; but their natural variations (such as the rate of cooling of hot water in a kettle, the average temperature of each day in the year for many years, the average weight of tens of thousands of Japanese of a certain age, the relation between temperature, pressure, and specific gravity of a certain gas, and so on) are continuous quantities and can be represented by smooth curves.

(t) The variations that can be represented by straight lines, the equation of a straight line, the determination of the constants in the equation of a straight line by using plotting paper, and many other useful examples.

(*u*) Finding maxima and minima by curves of the second degree, curves of the third degree, curves of circular functions, exponential function and so forth; and finding the rate of variation.

(*v*) Finding the root of any equation, such as the roots of $3x^3-3x^2+\frac{4}{3}=0$, $\theta - sin\theta = \frac{2}{3}\pi$, etc.

(*w*) To find the constants in an equation of a curve got by plotting the result of observations, for example, to find the value of n and c in the equation $pv^n=c$ by the indicator diagrams of steam engines, gas engines, and oil engines, etc.

(*x*) Given a few instances of the prices of some complex commodities and the expenses of some works, how to infer the prices and expenses in similar cases. Some other important items, problems and cautions.

THE END.

DIVISIONAL REPORTS

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Article VI.—The Teaching of Mathematics in Normal Schools (for preparing Male Teachers for Elementary Schools). Prepared by T. Okada, *Teacher at the Aoyama (Tokio) Normal School,* under the direction and supervision of T. Hayashi, till recently *Professor at the Tokio Higher Normal School* and now *Professor at the Tōhoku Imperial University.*

TOKIO

1912

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TOKIO
1912

CONTENTS.

The Teaching of Mathematics in Normal Schools

(for preparing Male Teachers for Elementary Schools)

The Japanese Sub-Commision received reports from no less than thirty-one, out of the total number fifty-two, of prefectural normal schools for preparing male teachers for elementary schools. The present divisional report was compiled, due regard being had to the opinions expressed in the reports just mentioned.

CHAPTER I.

Aim, Constitution, and Curriculum of the Normal School.

(*A*) Aim.—The normal school is an institute for the training of elementary school teachers. From its very nature, it is a school where a kind of special professional education is to be given. And yet, the standard required for entering it being only that of the graduation grade of only eight or nine years' course of elementary education, it is found necessary to give to its pupils not only the said special professional education, but also the higher common education in order to make them fit, when they assume as elementary school teachers an appropriate place in society to fulfil the duty due to that position.

(*B*) Constitution.—In the normal school there are provided a preparatory and a regular course. The latter divides itself into the first and second division.

The preparatory course aims at giving education necessary for those entering into the first division of the regular course, and is open to graduates of two years' course of

higher elementary school or those who are above fourteen years of age and have the scholarship of equivalent grade. It extends over one year.

Those who are entitled to enter into the first division of the regular course are either graduates of the preparatory course or graduates of three years' course of higher elementary school or those who are above fifteen years of age and have the scholarship of equivalent grade. It extends over four years.

Those who are entitled to enter into the second division of the regular course are either graduates of middle schools or those who are above seventeen years of age and have the scholarship of equivalent grade. It extends over one year. This division is specially established for giving to those who have completed the higher common education, the normal education of a short period so as to fit them for elementary school teachers.

(*C*) Curriculum and its distribution.—The curriculum of the preparatory course includes morals, Japanese and Chinese mathematics, writing, drawing, music, and gymnastics.

The curriculum of the first division of the regular course includes morals, pedagogics, Japanese and Chinese, English, history, geography, mathematics, natural history, physics chemistry, political economy, writing, drawing, manual works, music, and gymnastics; but English is optional. Besides it includes one or both of agriculture and commerce. In case when both are included, pupils are required to take either one of the two.

The curriculum of the second division of the regular course includes morals, pedagogics, Japanese and Chinese, mathematics, natural history, physics and chemistry, political economy, drawing, manual works, music, and gymnastics.

The distribution of subjects and recitation hours per week are to be seen from the following table.

Subject \ Years	Preparatory Course	First division of the regular course. First year	Second year	Third year	Fourth year	second division of the regular course
Morals	2	2	1	1	1	
Pedagogics			2	4	3 } *9 } 12	3 } *8 } 15
Japanese and Chinese	10	6	4	3	2	2
English		3	3	3	2	
History		2	2	2		
Geography		2	2	1		
Mathematics	6	4	3	3	2	2
Natural history		3	2	1		} 3
Physics and chemistry			2	3	4	
Political economy..					2	2
Writing	3	2	1	1		
Drawing	2	} 3	3	3	3	3
Manual works						
Music	2	2	2	2	1	2
Gymnastics	6	5	5	5	3	3
Agriculture and commeace			2	2	2	
Total	31	34	34	34	34	34

Notice. The asterisk * in the table shows the number of hours for the practice in pedagogics.

CHAPTER II.

Aim and Subject-matter of Mathematical Instruction.

(I.) Aim and Subject-matter.

The aim of teaching mathematics is to make clear to the pupils the relation between number and quantity, to make them proficient in computation, to make them understand the method of teaching arithmetic in elementary schools, to help them to acquire the knowledge necessary for daily life and to make them sound and accurate in thinking. The subjects taught are arithmetic, algebra, geometry, outline of book-keeping, and method of teaching arithmetic.

Quite recently the Department of Education has compiled the syllabus of teaching for normal schools. The principal of each normal schools is required to frame, in accordance with the same, details of teaching suitable to local circumstances. The syllabus of mathematics runs as follows.

Syllabus of Mathematics.

(Preparatory course).

Arithmetic. .. .. 6 hours a week.

Exercises in mental and written arithmetic and *soroban*-calculation of the same grade as that of the third year of the higher elementary school.

(First division).

First year.. .. ..4 hours a week.

(*A*) Arithmetic and algebra.

1. Integer, decimal, fraction.
2. Negative number.
3. Integral expression.
4. Four rules, linear equation, measure.

(*B*) Geometry and appendant arithmetic.

1. Straight line.

Angle, parallel lines.

2. Linear figure.
   Triangle, parallelogram, mensuration.
3. Circle.
   Arc, chord, segment, tangent.

Second year .. ..3 hours a week.

Arithmetic, algebra, and geometry.

1. Fractional expression.
   Reduction to the lowest terms, reduction to common denominator, four rules, fractional equation.
2. Evolution.
   Square root, cube root, quadratic equation, irrational expression.
3. Proportion.
   Ratio, proportion.
   Proportional lines, similar figures, mensuration.

Third Year .. ..3 hours a week.

(*A*) Arithmetic, algebra and geometry.

1. Trigonometrical functions.
   Trigonometrical functions of an acute angle.
   Solution of a right-angled triangle.
   Logarithm.
2. Progression.
   Arithmetical progression, geometrical progression.
3. Percentage.
   Percent, interest.

(*B*) Book-keeping.

1. Double entry book-keeping.
   Loan, items of account, entry, settlement.
2. Simple entry book-keeping.

(*C*) Method of teaching arithmetic in elementary schools.

1. Aim.
2. Selection and arrangement of subject-matters.
3. Methods of instructions.
4. Instruments and useful advices.
5. Investigation of text-books and diagrams.

Fourth Year .. ..2 hours a week.

Geometry and appendant arithmetic.

1. Planes.

Plane and straight lines, dihedral angle, polyhedral angle.

2. Polyhedrons.

Prism, pyramid, mensuration.

3. Solids with curved surface.

Circular cylinder, circular cone, sphere, mensuration.

(Second division).

2 hours a week.

(*A*) Arithmetic.

Exercise in mental and written arithmetic and *soroban*-calculation.

(*B*) Book-keeping (the same as the first division of the regular course).

(*C*) Method of teaching arithmetic in elementary schools (the same as the first division of the regular course).

Remarks.

1. Lay stress on the mutual relations between arithmetic, algebra and geometry. And especially make some matters relating to arithmetic accurately understood in the teaching of algebra and geometry.
2. Let mental arithmetic and *soroban*-calculation be exercised even in the first division of the regular course.
3. Locus and construction in geometry to be taught on suitable occasions according to convenience.
4. Let matters relating to enactments be given in connection with political economy, commerce, etc. according to convenience.
5. Aim at simplicity in book-keeping under the subject of mathematics, and give its advanced part under the subject of commerce.

(**II.**) Connection among various subjects.

That connections between various subjects should be made

as close as possible is generally approved. In regard to the method, the scope and the grade thereof, however, no definite opinion has yet been agreed upon. The aim of the above syllabus is to plan to attain the mutual connections as much as possible, to teach the subjects of the same name in the three different branches of mathematics, like ratio and proportion, at the same time possible, to let arithmetic be the centre and connecting link between it and other branches of mathematics, so that the items relating to arithmetic must be made accurately understood even when algebra and geometry are given, and to abolish the method of teaching arithmetic as an independent branch, by means of making arithmetic, algebra and geometry into a compact systematic whole. The syllabus, however, having been but recently compiled, how it works is not yet clear. Moreover at present, as there are very few higher elementary schools of three years' course and as there are only twenty-one normal schools having preparatory course, the scholarship of matriculates is rather poor, and consequently there is reason enough to expect not a little difficulty in carrying out the aim of the syllabus all at once. In case the preparatory course be not established, many are of opinion that arithmetic is better to be taught as an independent branch in a systematic way prior to geometry and algebra. To put it minutely, even if such considerate measure be taken as to give over rigorous proofs and general theories in geometry and algebra and to reserve only their arithmetical applications or to eliminate progression, logarithm, irrational number, complicated problems on interest, etc. from arithmetic and to teach them in algebra and geometry, yet it is regarded as better to avoid the confusion between arithmetical, algebraical and geometrical treatments, and anyhow to complete arithmetic as one independent branch, thereby, on one hand, to regulate and systematize the arithmetical knowledge the pupils have got in elementary schools, and, on the other, to give instruction in a compact

form immediately available when they engage themselves in elementary school teaching.

This is the point at which the mathematical education of normal schools differs from that of middle schools. Even in the case in hand, it is a matter of course to pay attention to the mutual connections between various branches of mathematics by endeavouring, in the teaching of algebra and geometry, to make the items relating to arithmetic accurately understood, or by assigning computation exercises as arithmetical application of those theorems and rules given in algebra and geometry, or by comparing arithmetical solution with algebraical solution when problems are to be solved by means of equation. But we are not able at present amalgamate the three branches of mathematics too much.

In case the preparatory course be established, arithmetic can be satisfactorily regulated and systematized in the course, and consequently the above-said amalgamation teaching in the regular course may go forward with ease.

In either case, however, care should be taken to keep the connection among the various branches and at the same time to bear the characteristics peculiar to each in mind. The adherence to the principle regarding arithmetic as centre so to speak, regarding both algebra and geometry as if they were the subordinate subjects, should not only be likely to annihilate the characteristics of each, but also, tending toward the utilitarianistic extremity, threaten to demonlish the very footing of mathematics in the common education. The majority believe that it is not only unwise to teach mixed algebra and geometry, as it promotes confusion and profits nothing special, but that it is rather impossible.

As to which of the two, algebra or geometry, should be taught first, there exist a few differences of opinion. But, as algebra is more closely connected with arithmetic than is geometry, looking it either from the standpoint of its character or from the standpoint of easiness with which it appeals to

beginners, it is better to teach algebra prior to geometry.

The amalgamation teaching of plane and solid geometries has never been experimented in our normal schools. But this is not only possible but also profitable for attaining the aim of the teaching of geometry, and is an interesting problem worth careful investigation in future.

In short, in our normal schools at present, it is not adequate to take off the partition between or to amalgamate the various branches of mathematics, if the present constitution of these schools and pupils' scholarship be taken into consideration. Nevertheless in regard to the need of making the mutual connections as close as possible, all unite in approval of it.

CHAPTER III.

Examination.

(**I.**) Aim of Examination.

As there are very few reports in which the aim of examination is clearly stated, I regret much that I have not sufficient material to make an statement of the point of view from which examinations are carried on in our normal schools. But so far as the committee has inquired, the following three items seem roughly to cover the ground.

1. In order thereby to decide the merits of attainments in various subjects for promotion and graduation.
2. As a means of making the process of teaching and acquisition rational and complete.
3. In order to stimulate and to encourage indirectly for study.

The view relating to the aim of examinations is one of the most important problems in education, that demands serious consideration, because the view depends directly upon the method of examination and the attitudes of teachers and pupils towards examination and indirectly upon the daily

teaching and acquisition. It is to be regretted, however, that this matter has never been seriously studied. Examination seems as if it is carried out and repeated simply as a custom without any special care for its real aim either by teachers or by pupils. Generally speaking, pupils are anxious only for promotion and graduation, and are forced to study unwillingly for examinations' sake, laying weight on nothing but the marks. As a result, constant and earnest study in daily recitations is neglected, and interest in the subjects tends to be decreased. Some reports state the fundamental aim of examinations to lie in judging the merits of attainments to decide promotion and graduation and the others regard examinations even as a convenient means of encouraging pupils to study.

It is a matter of course that a direct aim of examinations is to test the pupils' scholarship. The reason for this is that teachers can judge the result of their teaching by it, pupils can acknowledge the merits of their attainments, and accordingly the rational proceeding of teaching and acquisition can be effective. After all, the fundamental aim of examinations may be said to lie in this point. As promotion and graduation are but the natural results of the rational proceeding of teaching and acquisition, they should never be regarded as the principal object of examinations.

It is an undeniable fact that examinations indirectly stimulate pupils to study. And therefore, it is not absolutely objectionable to make the best use of the ambitions and emulative spirit which is at its highest point in youth. Yet it will lead at times to an irrevocable evil, if this is done to an excess.

(**II.**) Method of Examination.

Methods of examinations now prevalent in normal schools are not all alike, but in general they are within the limits of the following items.

(*a*) An examination is given at the close of each term

on the matters taught during the term. This method is most widely adopted.

(*b*) Besides the term examination, special or periodical examinations are held once or more a term. This is another method adopted widely too.

(*c*) An examination is given at the close of each school year on the matters taught during the year. A few schools adopt this method.

(*d*) An examination is held at the time of graduation on all subjects taught throughout the whole course. Only one school Tochigi Normal School adopts this method.

(*e*) There are some schools where such works as questions and answers in daily recitations, exercises, and home tasks are regarded as parts of examinations. Also there are some schools where daily result is regarded as part of the merits of attainments and is made use of as a means of deciding promotion and graduation.

(*f*) As for the time of examinations there are two kinds, special and periodical.

(*g*) As regards the dates of examinations some are given on the appointed days, and the other on an unexpected day.

(*h*) As to the limits of subjects covered by an examination, it is left to the discretion of teachers.

Term examinations in most normal schools are held at the close of each term; in some schools they are held in the last week of the term; and in some, on suitable occasions in the last few weeks. Special examinations which are held several times each term are customarily given unexpectedly, but some schools give them periodically. During periodical examinations there are generally no regular daily recitations, while in the case of special ones daily lessons go on as usual.

The dates for examination are sometimes previously announced and sometimes not; the question, when it is appropriate to announce previously and when not, has never been publicly studied and remains unresolved. My private opinion with regard this question is as follows:

The examination without any preliminary announcement of the date should be held only when it is necessary to test the understanding and memory of the pupils and the extent of their power of application to the subjects in which special review and practice have been assigned (which is practically the same as announcing the date beforehand) to the matters of importance to which special attention must be directed or which must always be kept in mind, etc.

In the other case, whether periodical or special, announcement should be made of the appointed days, and the scope of set subjects so as to give time enough for satisfactory preparation; sometimes necessity may arise for giving hints for preparation and showing the way of review. It is in this way only that the fundamental aim of examinations can be attained.

To give examinations often in unexpocted ways for the purpose of directing pupils' constant attention to study and of inciting them to untiring effort is an antieducational measure. It confuses the pupils' calm and sincere attitude to study, and perverts their genuine interest in the subjects owing to continual uneasiness and anxiety on account of examinations. When the number and time of examinations are fixed it is inevitable that the subjects be mechanically limited for examinations, but it must be proper to oblige pupils to review the important matters taught for them. This way will greatly help pupils in reviewing the matters connected with one another, to make their mutual relations clear as a whole, to make all subjects understood as united and systematized knowledge.

Judging from this, it would be better to give special examinations on occasions suitable to the proceeding of teaching rather than mechanically decide the number and time of examinations. In this way, though the dates of the examinations are not definite, yet they may be known beforehand from the progress of instruction. This gives them the nature of periodical examinations on the one hand and it avoids the feeling of uneasiness of pupils for special examinations on the other hand.

(**III.**) Methods of Weighing the Merits of Attainments.

Promotion and graduation are the natural results of the rational proceeding of teaching and acquisition, but by no means the fundamental aim of education. It is unavoidable that promotion and graduation exert great influence upon the pupils' mind; therefore the pupils' attitude towards examinations may depend directly or undirectly upon the methods of weighing the merits of attainments for promotion and graduation, and consequently the rational proceeding of teaching and acquisition may depend directly or indirectly upon the methods of weighing the merits of attainments for promotion and graduation.

There are two kinds of methods of weighing the merits of attainments now prevalent in the normal schools at present.

(*a*) To estimate simply by scholarship.

(*b*) To estimate mainly by scholarship, taking into consideration the pupils' attitude in daily recitations, and their degree of interest in the subjects, etc.

The conditions necessary to the weighing of the merits of attainments are such as facility of investigation, accuracy and impartiality of the result attained and so forth. From this standpoint, the method of weighing simply by scholarship may be the safest and most convenient one; but since, as a matter of course, the aim of didactic teaching consists not only in the direct promotion of scholarship, but also in

fostering settled habit of investigation and in developing interest in subjects, it may be proper in inquiring students' attainments to take these points also into consideration.

Examinations as means of testing scholarship should be of the following qualities:—

1. Examinations aim at exclusively testing true scholarship of the pupils.

What I mean by scholarship includes accuracy of memory of the knowledge acquired, thoroughness of understanding, degree of application-ability.

2. Examinations should stand absolutely independent of the means of acquisition.

This means that pupils should be examined on the matters they have acquired, and examinations must not be the means of acquisition, or must not be used as the means thereof.

3. Examinations should stand entirely independent of the means of teaching.

By this it is meant that examinations should be given on the matters taught in due forms, and never be regarded as part of the teaching or used as the means of making up for deficiency in teaching.

There may still remain room for further investigation as to whether it is appropriate to define the meaning of scholarship examinations as above, but the standard to be adopted as a proper and safe means of testing scholarship, I believe, can not be anything but the result of scholarship examinations based upon this view.

Questions and answers in daily recitations, exercise problems, home tasks, etc. all serve as means of teaching and acquisition whereby rational proceeding of teaching may be rightly carried on, by developing pupils' scholarship and inciting them to study. But so called daily merit can not be adopted as the standard of judging pupils' true scholorship as the result of their acquisition. For students' scholarship, from the pedagogical point of view, has no import but

as the result of teaching and acquisition. But it may not be improper to use it as a reference of judgment on the attitude of their study, and interest in the subjects, and so on. It is exceedingly important to pay attention to these matters and to make as thorough an investigation as possible.

CHAPTER IV.

(I.) Method of Teaching.

The aim of the teaching of mathematics in the normal school is neither to prepare pupils for schools of higher grades nor to foster specialists, but it is mainly to give them training in computation for daily uses, to cultivate their mathematical interest and at the same time to make them understand the principle of the teaching of arithmetic in elementary schools, thus giving them sufficient qualification as cultured educators of the nation. This aim must always be kept in view. And again, due attention should be paid to psychological tendencies of youth in general, for the majority of pupils belong to the period of adolescence.

The study of methods of teaching for intermediate education has not yet been sufficiently investigated in Japan Though there are some who are engaged in earnest enquiry thereinto, no remarkable progress has been seen owing to the fact that there is no adequate organization for such an investigation. In spite of comparative uniformity and immutability of teaching materials, methods of teaching in normal schools are quite disconnected and lack unity, there being very few opinions which are widely approved. The report which follows shows only the result of investigation with reference to the general tendency.

(II.) Details of Teaching, Text-Books, Reference-Books, etc.

(*a*) Details of teaching mean giving general direction and comments with regard to items to be taught,

and their arrangement, order of teaching, mutual connections among items, recitation hours for each item, etc. The principal is responsible for its compilation and the proceeding of teaching in general is required to conform to these details.

(*b*) Text-books are used in every normal school. The choice of text-books is the duty of the principal. He chooses them from among the text-books which are examined and approved by the Department of Education. But the decision is usually submitted to the faculty-meeting of the school. In case when, by necessity, any text-books which are not approved by the Department of Education, must be used, the sanction of the Department is required.

(*c*) Sometimes reference-books are used besides text-books, but there exist no regulations concerning their use. In fact there are few schools where reference-books are definitely decided.

(*d*) Not many schools adopt the collection of problems outside text-books for the purpose of exercise. Exercise-problems attached to text-books serve for the purpose, and sometimes the teacher himself makes choice of problems and assigns them to the pupils.

(*e*) The use of collection of solutions of problems and explanatory notes is generally forbidden. Such kinds of books are considered to accompany more demerits than merits for pupils' acquisition.

(**III.**) Form of Teaching.

The form of teaching most widely adopted in normal schools is the dialogue form. The reason is this, that teaching in normal schools is mainly done on the base of the prescribed details, text-books as the centre, and the age of pupils is so advanced as to make self-investigation easy,

and moreover, from the very nature of the subject great emphasis is laid on preparation, exercise and review. The use of lecture form is limited to special occasions. Consequently, though pupils take notes of the most important points in the lessons, yet that bad practice of "dictation teaching," so to speak, is not committed.

As understanding, memory, and application are the three essential conditions of the teaching of mathematics there is no need of mere extensiveness and abundance of teaching material, but it is desirable to repeat important points, omitting matters of secondary importance. Teaching should proceed in such a way as to bring about correct understanding, faithful memory and free application.

Scholarship of teachers, confidence and respect of pupils toward their teachers, all combined, have a very great effect on bringing about good results of teaching. For teachers it is advisable to take heed to these points. In intermediate education such a thing as the study of the forms of teaching is of secondary importance.

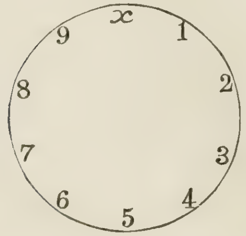
(IV.) Proficiency in Computation.

The aim of the teaching of arithmetic in normal schools is to affirm and regulate the knowledge, which pupils have already acquired in elementary schools, to make up deficiency and unify fragmental knowledge, and at the same time, to improve pupils' computing ability. As computation, above all, has fundamental importance in mathematics, ability in that must be so developed as to form the basis for valid teaching. As a matter of course elementary schools are expected to give such a thorough training in computation that in schools of higher grades, teaching can proceed with full confidence in the pupils' proficiency. But generally speaking in the present state of things the computing ability of matriculates is incomplete and not uniform, it can not be relied upon at all. This fact gives rise to the

necessity of making up the deficiency in the lower classes in normal schools.

At present incompleteness of computation ability is a fault common to all the pupils of schools of every grade. This is the cause of the unsatisfactory result of the teaching of mathematics, and common antipathy against mathematics may, to a great extent, be traced to this lack of computation ability. This is an undeniable fact. No one fails to acknowledge the necessity of taking some measures in the way of investigating the causes of this defect, with the making up of the deficiency in view. Truly how to develop the computing ability of pupils is the fundamental problem to be studied first of all, when the question of the reformation in the teaching of mathematics is touched upon. In normal schools which are institutes where elementary school teachers are trained, this problem assumes greater importance.

My private opinion on this point is that immaturity in computations comes from inexpertness in the fundamental computation. The first and best plan for increasing computation ability is to complete the fundamental computation ability. I draw this conclusion from the fact that all computations are combinations of the fundamental computation, and from the results of my experience and investigation. A simple method devised by myself based on this conclusion is the use of the following diagram.



Replacing x by numbers from 0 to 9, ten series of successive addition problems are made. Replacing x by number from 45 to 54, ten series of successive subtraction problems are made. Each pupil is required to practice computation by himself, the standard of proficiency in computation being considered attained

when he can compute all the series of addition and subtraction with no mistake in less than a minute.

Both theory and experiments prove this method to be most valid and at the same time most valuable for the purpose of promoting and regulating computation ability in a short space of time.

(V.) Promotion of Interest in Mathematics.

In order to promote interest in mathematics, to foster the habit of accurate and logical observation and treatment "the funnel method," as it were, must be carefully avoided and pupils must be led in such a way as to make spontaneous study possible.

The following points may be mentioned in relation to method of teaching concerned with this point.

(*a*) Stress must be laid on preparation work, and pupils should be encouraged to solve problems by their own effort, carefully thinking over (factual and operational) relations.

(*b*) Since mathematical theory develops step by step the knowledge already acquired should be always reviewed so as to serve as the basis for the next lesson.

(*c*) To assign pupils always many hard problems brings no good effect; on the contrary, it often blunts the pupils' contemplative faculty and suppresses their interest in mathematics. To give them problems within their power and adequate to the state of their progress, and to make them enjoy full gratification of success by solving them, to make them conscious of their progress, and to let them have chances of strengthening their confidence in their own ability, these are the sources by which to call out the interest of pupils in mathematics.

(*d*) In solving problems pupils should be accustomed

not to be satisfied until they have got answers complete and adequate to the problems. This is not only proper process of solution of problems, but it is also exceedingly effective for promoting interest in reasoning and for strengthening self-confidence.

(*e*) Accuracy of thought advances hand in hand with accuracy of expression. Language used by both teachers and pupils in explaining problems, in proving theorems or rather in all teaching process, must be concise and accurate. This is most important in order to make thought accurate and to develop rigorous reasoning power.

(*f*) Verification of identity in algebra, examination of roots of equation, exercise problem of geometry, especially, problems concerning locus and construction, are most efficacious for training of reasoning and promotion of interest.

The following items may be counted among the causes of that general antipathy against mathematics.

(1) Too much and too hard materials make pupils' understanding imperfect and memory inaccurate, making acquired knowledge unfit for practical use.

(2) Owing to immature training in computation, pupils' computing ability, in general, is incomplete so that pupils lack ease and quickness in it. Feelings of uneasiness and trouble always tend to accompany their computation.

(3) Lack of clearness of ideas, inaccuracy of fundamental knowledge and incomplete drill all combined hinder the application of acquired knowledge.

(4) Fundmental ideas and knowledge concerning mathematical quantities which have relations with daily life being inaccurate, they become unpractical.

(5) Choice of exercise problems, above all, problems of application is improper.

a. Inadequate to the real fact.
b. Too hard.
c. Too easy.
d. Redundant and tedious.
e. Lacking unity and connection.

(6) Ignorance or misapprehension concerning general purposes or principal aims of branches.
(7) Methods of teaching are unsuitable. Teachers are not zealous enough and moreover they are not quite sufficiently qualified.
(8) Methods of study are not suitable, especially pupils lack investigating spirit.
(9) That evil custom of cramming which results from defective examination-systems and their application, is prevailing.
(10) Youth generally tends to indulge in reverie, and in consequence, is not inclined to exact mathematical observation or to logical reasoning.

(VI.) Cultivation of Mathematical Common Sense.

(*a*) Utilization of objects, models and diagrams, etc., is not only helpful in making ideas accurate and understanding easier, but it is also necessary in cultivating mathematical common sense. In teaching arithmetic or geometry, etc., it is desirable that attention be paid to this respect.

The following may be mentioned as expedient articles available for teaching purpose.

(1) Objects, instruments, models, diagrams, etc. necessary to teaching systems of weights and measures, money, time and so forth.
(2) Objects, models, specimens, etc. concerning percentage, mail, telegraph, etc. which have special formulae and customs.
(3) Instruments for explanation of mensuration and

materials of various forms used for measurement.

(4) Simple surveying-instruments and drawing-instruments.

(5) Models, and diagrams necessary for the teaching of geometry.

(6) Books and tables of statistics.

(*b*) Simple surveying, practice in the use of weights and measures, drawing, making of statistical tables or making of models, etc., are all effective methods.

(*c*) Rough computation, rough estimation of measures are favorable for cultivation of mathematical common sense. Practice in rough computation of length, area, capacity, weight, angles, time, rough estimation of length or distance by means of the finger-breadth or foot-breadth, or by velocity of light; rough estimation of time by the number of pulse-beatings and respiration; computation of capacity of utensils by the application of the law of specific gravity, the relation which exists between the weight of water and its capacity; these are all to be encouraged.

(*d*) As regards problems of daily necessity, imparting of accurate knowledge and promoting of application ability must be aimed at. Adequate ideas concerning social customs and public regulations must be given to pupils.

(*e*) Exercise problems, above all, application problems must be so chosen that they have relation with practical life as closely as possible. Numbers and measures should be selected with actuality in view. Problems of daily life, matters learnt in other branches are all available materials. Imaginary problems which have no direct relation to life rather tend to diminish the common sense of the pupils.

(VII.) Characteristics of Lessons Concerning Methods of Teaching.

(*a*) The principal aim of the teaching of arithmetic is to regulate and complete the knowledge of pupils, already acquired in elementary schools. The practical side of mathematics such as thorough familiarization with computation, and solution of applied problem should receive greater attention than subjects of general theory which should rather be taught in algebra and geometry. In arithmetic thorough familiarization with the application of fundamental rules is most desirable. In teaching arithmetic attention must be paid to letting pupils know well the matters connected with the teaching of arithmetic in elementary schools. For example, use of black-board, presentation of problems, demonstration of principles and rules of calculation, and simple explanation of solutions of applied problems, construction of simple models and the ways of using them, should be taught in such a way that they can be applied to the teaching in elementary school or at any rate with such object in view.

(*b*) As algebra is a lesson which is to be taught for the first time in normal schools, it will be better to connect it with arithmetic, and to use the knowledge of arithmetic as a basis. Theorems and rules taught in algebra must be kept in close touch with arithmetic when they relate to similar problems. In arithmetic, application or practical computation is more important than theory, while in algebra the reverse is the case, i.e. rigour of theory is of greater impostance.

(*c*) Geometry is a lesson entirely new to pupils; no similar subject being taught in elementary schools,

pupils should be, from the very begining, carefully accustomed to rigorous treatment. Since the average age of pupils in normal schools is above fifteen, no special difficulty or trouble is felt in the teaching of systematic geometry from the start, and yet beginners will receive great help by means of models and other expedient teaching articles and real instances. Thus they will give correct ideas of geometrical figures and quicken pupils' understanding. The essential object of the teaching of geometry is to make thinking accurate, to develop reasoning power, to promote interest in mathematics by rigorous proofs and logical treatment. Therefore accurate understanding of propositions, faithful memory of axioms and theorems, rigorous demonstration of theorems, and expression in concise and accurate language, are strictly demanded from the outset. Pupils find the greatest difficulty in the demonstration of problems of locus and in the examination of problems of construction, but it is the best material for the teaching of geometry. In teaching them, rigorous demonstration and careful examination must always be kept in mind. Among the theorems of geometry there are many which are related with the elements of mensuration, elements or application of ratio. Their application to arithmetic must not be forgotten.

(VIII.) Methods of Teaching Arithmetic in Elementary Schools.

Methods of teaching arithmetic in elementary schools should, as a matter of course, be such that it is taught as a part of lessons in arithmetic. But in most cases explanation of separate problems and methods of computation thereof being given, pupils receive only some fragmental hints of

instruction as to methods of teaching. They need systematic instruction in the general elements of the methods of teaching and their applications. Essentials of teaching, choice and arrangement of teaching materials, and their treatment, detailed notices concerning teaching, instruments and books, must be taught, and at the same time pupils should have practice in them. Usually about twenty hours are assigned to the methods of teaching, and its practice is done in the attached elementary schools under supervision of teachers.

(IX.) Connections with Other Lessons.

Not a few matters which are concerned with number, measure, and computation, are found in other lessons, especially, in natural science. Such branches as physics or geography can never be treated independently of mathematical knowledge. In order to have connections and unity among several branches it is very important to get therefrom materials for problems in arithmetic, algebra, etc., Moreover, as imaginary problems make very often the practical value of mathematics questionable, it is most desirable to give pupils problems relating to the real facts, learnt in other branches, if possible. But in the lessons of arithmetic and algebra, etc. in the preparatory course or in the lower classes of the regular course, pupils are unable to understand the meanings of those problems, while in the higher classes more stress is laid on the systematic study of algebra and geometry, arithmetic being taught only by way of supplement. This state of things makes it difficult to introduce these practical problems. Again, since mathematics has its own aim apart from other branches and it is the basis of other branches rather than their application, if too much attention is paid to such connections, it may incur the danger of demolishing its proper aim. As it is the case, there naturally should be a certain limit to

including matters of other branches in the teaching of mathematics.

Judging from the nature of the thing, it is a most reasonable request to make use of the matters taught in mathematics, for the purpose of connection and unification of other branches. For example, such things as to apply principles of mathematics as means of explaining laws of physics; to assign pupils computation problems of physics; to give explanation of calendar in teaching geography; the setting of computation problems concerning longitude-and-time relation; the trial of geometrical proof in teaching mechanical drawing, are to be much encouraged. It may be of no little importance to study what points of mathematics are related to points of other subjects and how and to what extent or degree, but as such a study rather lies within the realm of methods of teaching of other branches, we will not go into details here.

CHAPTER V.

Recent Tendency of the Teaching of Mathematics.

(*A*) Constitution of Branches.

Trigonometry is not taught in normal schools at present but the necessity of teaching it is being gradually felt. It is not as yet taught systematically as an independent subject, though something of it is given as part of gemetry. Some prefer to include trigonometry at the expense of omitting some part of algebra and geometry.

Book-keeping is taught within mathematics, but as it has but little connection with other branches of mathematics some are of the opinion that it would be better to teach it as a part of commerce. No necessity is felt of adding the elements of analytic geometry and differential and integral calculus to the normal school curriculum. To keep the connections between several branches and modifying the system

of teaching with this end in view may be looked upon as the new tendency. But the partitions between different branchs are not entirely removed and again there is no sufficient reason for doing so. The points on which all are agreed upon are these :—To pay attention to the arrangement of teaching materials, to teach related matters at the same time if possible, to omit duplicated matters from one branch, only teaching them in the other branch, to allow freedom of referring to one another, concerning the applicable principles and laws, and thus to try to have unification of branches and at the same time spare time and energy. The main purpose of the syllabus issued most recently is to teach arithmetic in connection with algebra and geometry, not to treat it systematically as an independent branch, while in reality, to teach arithmetic essentially and algebra and geometry only for the purpose of making arithmetical knowledge accurate. But such a method of teaching seems to have many disadvantages, and to cause confusion, and is liable to produce a result contrary to the original purpose.

In normal schools (especially in the case where there is no preparatory course) arithmetic should be taught systematically, and in the teaching of algebra and geometry measures must be taken to keep connections with arithmetic. This is not only more advantageous in fulfilling the original aim of arithmetic itself, but it is also more convenient for the utilization and development of different branches. All acknowledge this truth.

(*B*) Aim and Subject-matter.

The recent tendency concerning the aim of the teaching of mathematics attaches more importance to the practical side and to the knowledge which is only necessary for elementary school teachers. Such a tendency is nothing but the result of transient influence of utilitarianism and short-sightedness now prevalent in the "Zeitgeist."

Normal schools are not professional schools, but institutes to train would-be-educators of the nation. So there must naturally be some difference between pupils of normal schools and those who have made up their minds to go into some special business or profession. The knowledge which is indispensable as the higher common education must receive due attention as well as knowledge and arts directly necessary for their future profession. The tendency to regard algebra and geomotry as sub-branches of arithmetic may have originated in that shallow utilitarian view. However, in the second regular course, pupils are those who have finished their higher common education, and the aim of their training consists in giving professional education necessary as elementary school teachers; so it is right to teach only arithmetic, omitting algebra and geometry.

Utilitarianism tends to reign also in the choice of teaching materials. If too much stress is laid only on the matters of immediate use in elementary schools, pupils' knowledge becomes gradually lower and shallower, and the interest inherent to different branches of study will entirely be lost. Teaching will lose its nobler qualities and will be regarded only as a means of bread winning. Therefore, in the choice of subject-matters care must be given to the practical side on one hand and to the purely theoretical side on the other. The view that the progress of education must go in advance of the tendency of the times may be very appropriately cited also in this case.

Under similar circumstances the importance of development of intellectual powers and promotion of interest is forgotten, and accumulation of knowledge is emphasized too much. As the result of this tendency both teachers and pupils crave for abundance of matters taught, and try to teach and learn by the simplest method possible.

But since the evils of such teaching have been found out, attention has begun to be paid to the choice of

materials, so that materials may be well assimilated and become vivid knowledge. And at the same time development of the intellect and promotion of interest also have begun to be emphasized. Emphasis on computation problems, endeavour to form clear and accurate ideas of function, teaching of methods of utilization of diagrams are other phenomena of the recent trend.

Some topics given below may be omitted.

1. From arithmetic.

    Short methods of calculation.

    Four rules of recurring decimals.

    Multiples of 7, 9, 11, 13, 19, etc.,

    Theory of the greatest common measure and the least common multiple (given in algebra).

    Square roots, cube roots (given in algebra).

    Complex problems of compound interest (given in algebra).

    Progression (given in algebra).

    Foreign systems of weights and measures not in common use.

2. From algebra.

    Complicated study of inequality.

    Theory of maxima and minima.

    Theory of symmetrical expressions and alternating expressions.

    Harmonical progression and other complicated progressions.

    Binomial theorem.

    Complex problems of permutation and combination.

    Complex problems of simultaneous equations of higher than the second degree.

    Complex computation of irrational expressions and general exponential expressions.

3. From geometry.

    Ratio and Proportion based on the Euclidean de-

finition.

Properties of spherical figures.

(*C.*) Examination.

There is a mistaken idea now prevalent that promotion and graduation are the fundamental aim. This is a transient phenomenon that had its origin in utilitarianism. Some have become aware of this deplorable tendency, and are making earnest investigation into the merits and demerits of examinations. Some discuss the reform of methods of examination while others want to abolish them entirely. But most of their opinions are either disputes of minor importance or contentions of trivial moment which were produced from inaccuracy of the definition and vagueness of the aim of examinations.

There are some who go even so far as to say that to abolish examinations entirely is to consider examination to be everything. If we go into details, neglecting the fundamentals, and discuss means not caring for the aim, no convincing conclusion can be reached.

The fundamental problem is to make teachers and pupils truly understand that the true aim of education consists in developing intellectual faculties, refinement of moral feelings, and that promotion is the natural result of right acquisition, and is nothing but impartial and proper means to make the rational proceeding of teaching possible, and that graduation is simply a recognition of the whole acquisition. If this fundamental question is settled to our satisfaction, the examination question will be of minor importance and be solved of itself. Such dispute as the one between those who wish to abolish examination entirely and those who consider examinations as everything, is unworthy of much attention. It surely must be most undesirable to irritate youthful feeling, and to force study by means of examinations, but in cases where teachers be not competent for proper teaching and pupils lack good practice of

acquisition, the measure of utilizing examinations in order to reach the true aim of education may be excusable.

(*D.*) Methods of Teaching.

Methods of teaching in intermediate schools have been neglected and have remained undeveloped, while those in elementary education have been much studied and have made remarkable progress. But lately there have appeared many persons who acknowledge the necessity of studying methods of teaching and who pay much attention to the study of adolescent psychology. Caution, of course, must be taken not to stick too much to troublesome methodology. There remains, however, much room for further progress and development for the study of methods of teaching.

In the teaching of mathematics, as a result of utilitarianic education, only acquisition of knowledge is aimed at, and value of learning itself remains unrecognized (development of observation faculty, promotion of reasoning power, increase of interest in mathematics, etc.).

Consequently teaching tends generally to become gluttonous and the pupils' attitude towards study passive. Resulting from the same cause they bother themselves with applications only neglecting the study of the fundamentals, and devote themselves to the exercise of application problems, putting aside the logical study of theorems and rules. Either from the point of view of the fundamental aim of education or that of mathematics teaching, these tendencies must surely be reformed. It is generally recognised that the deficiency of mathematical common sense and immaturity in computation are primary causes of the general poor result of the teaching of mathematics, and there is some tendency to make effort to solve absolutely the problems relating to methods of training in computation and cultivation of mathematical common sense. As the primary causes of the deficiency of mathematical common sense, that is to say, inaccuracy of practical knowledge of mathematics, the

following may be mentioned.

a. Negligence of intuitive expedient articles.

b. Negligence of experimental methods of teaching.

c. Inadequacy to the real fact concerning the choice of application problems and their treatment.

The time is gone when mathematics was considered to be taught with text-books and chalk only, to-day the value of intuitive teaching is widely recognized and no one doubts the necessity of experimental teaching. Especially for the treatment of common weights and measures for the teaching of practical computation, instruments, specimens, diagrams, etc. are recognized to be indispensable. Experimental teaching is encouraged and the demand for the establishment of the mathematical laboratory and the depository for mathematical reference articles is considered reasonable.

As the result of this tendency the following preparations have come to be generally approved.

(*a*) In teaching systems of weights and measures, money or computation of compound numbers, objects, models, and diagrams, etc. should be used and lessons should be given in actual handling of instruments and in actual surveying.

(*b*) In teaching mensuration accurate ideas of figures should be given intuitively, its computation being explained by means of actual measurement, experiments, and illustrations. Its application should de practiced. In teaching geometrical theorems which have direct bearing on mensuration, computation problems should be assigned, care being taken at the same time to verify by experiments.

(*c*) Mensuration should be taught concerning not only regular forms, but also irregular ones, computation of which may be performed indirectly by means of applications of the law of specific gravity. Its application to common utensils in daily life should

be taken into account.

(*d*) As regards the matters of percentage, mail, telegraph that have definite customs and formulae, objects, specimens should be shown and practical instruction given.

(*e*) In teaching plane geometry when proving theorems, not to speak of construction problems, figures should be as accurately drawn as possible. Straight lines, circle, arcs, etc., should be drawn by the use of rulers, compasses, etc., and thus agreement between thought and expression kept always in view.

(*f*) In solid geometry ideas of space should be given directly, utilizing models of figures or whips, pencils, desks or surface of the black-board in order to help pupils' imagination. Even in cases when explained by plane figures indirectly, pupils should be so trained as to imagine real figures.

(*g*) As regards the matters of surveying in geometry, and trigonometry, methods of practical surveying should be taught and practised.

(*h*) It should be seen to that application problems in arithmetic and algebra be adequate to real life.

(*i*) Let pupils construct themselves simple diagrams, models, instruments of mathematics in their manual-work and drawing lessons.

As regards tendencies of the study of adolescent psychology, the following are some of the problems which are earnestly investigated and studied.

1. Problem of interest.
2. Problem of fatigue.
3. Problem of association of ideas.
4. Problem of concentration of attention.
5. Problem of economy of mental energy.
6. Problem of relations of mind and body.
7. Problem of association between teachers and

pupils.

8. Problem of modes, forms and orders of teaching.

CHAPTER VI.

Candidates for Teachers.

Teachers of normal schools consist mainly of graduates of the higher normal schools and graduates of the teachers training institutes which are attached permanently or temporarily to the higher technical schools, the higher commercial schools, the higher middle schools or the universities. Some are taken from among graduates of the universities or those who passed the teachers' license examinations conducted by the Department of Education. About the training of these teachers of normal schools, Art. VII of these reports is to be consulted.

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CONTENTS.

TRAINING OF MALE TEACHERS

FOR

INTERMEDIATE SCHOOLS.

INTRODUCTORY REMAKS.

The term "intermediate schools" in this report comprises normal schools, normal schools for women, middle schools, and girl's high schools. Separate reports are to be made on the teaching of mathematics in each kind of schools. In this I shall report merely how teachers of mathematics for these schools are trained and supplied in Japan. Of these schools, in girl's high school, normal schools for women, and women departments of some normal schools, both male and female teachers are employed. The present report is to deal with the training of male teachers only for these schools.

Now, these teachers are trained in the higher normal schools and in the teachers' training temporary institute, all of which were established by the Government especially for this purpose. The graduates of all these institutes are given teacher's licenses for intermediate schools throughout the Empire. Besides these government institutes, there is a private school in Tokyo, called the Tokyo Butsuri Gakko (*lit.* Physics School at Tokyo), most of the graduates of which go out to teach in middle schools. The graduates of this school, however, are required, in order to obtain teacher's licenses, to pass the examination conducted by the Educational Department every year. Recently the Waseda University, another private institute in Tokyo, has established a higher normal course, which contains a branch for the

training of teachers of mathematics. Graduates of this course ——as yet there have been none——, has also to pass the examination before they can receive teacher's licenses. However, the directors of these private institutes may ask the Government for the privilege of their graduates of high scholarship receiving teacher's licenses without examination. I am told, that the director of the Tokyo Butsuri Gakko have not, and will not, make such an application.

Teachers of mathematics in Japan, like those of other subject, must have teacher's licenses in order to become regular teachers with the title "*Kyoyu*"; otherwise, however learned and proficient they may be, they are known as "Kyoyu Kokoroe" (temporary or substitute "*Kyoyu*"), assistant "*Kyoyu*," or assistant "*Kyoyu Kokoroe*." These licenses are granted either to those who pass the examination or to those who are considered able to pass. Those to whom teacher's licenses in mathematics are given without examination should come under some one of the following heads, besides the graduates of the higher normal schools and of the teachers' training temporary institute:—

1. Graduates of the below-named schools, or their special courses, having the special recognition of the Minister of Education.

Mathematical course, astronomical course, theoretical physics course, experimental physics course, of the Science College, Tokyo Imperial University.

Mathematical course, physics course of the Science and Engineering College, Kyoto Imperial University.

(Mathematical course, physics course of the Science College of the newly founded Tohoku Imperial University will come under this class, most probably in the near future.)

2. Graduates of a normal, a middle, or a girls' high school, who have studied three or more years at any of the schools recognized by the Minister of Education in con-

nection with teacher's qualifications. (Graduates of the girl's high school having a five years' course are required to take in these schools only a course of two years instead of three.)

3. Graduates of a normal, a middle, or a girls' high school, who have studied abroad at a university or at a school of similar grade, and have degrees or diplomas from such schools.

4. Graduates of schools abroad similar in grade to a normal, a middle, or a girls' high school in Japan, who have studied at a university or at a school of similar grade abroad, and have degrees or diplomas from such schools.

5. Persons who have teachers' licenses for schools equal in grade to, or higher than, the one in which they wish to teach.

(The graduates of the higher normal schools and of the teachers' training temporary institute of course receive teachers' licenses without examination.)

As the higher normal schools and the teachers' training temporary institute were established by the Government specially for the purpose of training teachers for intermediate schools, I shall herein give a description of these institutes. As for the training which other licensed teachers have received, it will be covered by other descriptions specially prepared. For example, the kind of education which the graduates from the science department of the imperial universities have received, will be explained under the special heading of the Universities. As for the examination for intermediate school teachers conducted by the Department of Education, I shall give a description of it at the end of this report.

THE HIGHER NORMAL SCHOOLS

The higher normal schools, which aim at preparing male teachers for intermediate schools, are situated in Tokyo and Hiroshima. (Those for training female teachers are in Tokyo and Nara. As previously stated, however, the subject of female teachers does not fall within the scope of this report.) There is not much difference between these two higher normal schools in their systems and curricula. The former was first established in 1872 and has arrived at its present condition after many changes; while the latter was founded in 1902. Both institutes exist in accordance with the Imperial Ordinance relating to normal education issued in 1897. They are the institutes, existing for the training of teachers for normal schools, middle schools, and girls' high schools.

Each higher normal school has a four years' course, one year preparatory and three years' regular course. The preparatory course is provided equally for students preparing to be teachers of mathematics and for those desiring to be teachers of other subjects. The regular course, in which only special subjects are taught, is divided into five departments: (I) department of Japanese and Chinese, (II) department of English, (III) department of geography and history, (IV) department of mathemstics, physics and chemistry, (V) department of natural history. Mathematics is taught both in the preparatory course, which is common to all the departments, and in the department of mathematics, physics and chemistry, which, in its turn, is divided into two minor divisions, one for mathematics and physics, and the other for physics and chemistry. Teachers of mathematics for intermediate schools are trained in the first of these divisions.

Students entering the preparatory course of the higher

normal schools are mostly graduates of middle schools, and of normal schools, or students recognized to have scholarship equal to that of the graduates of either of these schools. In the Tokyo Higher Normal School, out of the applicants from all parts of the Empire only those are admitted who have the highest grade in the entrance examination. At present, in the department of mathematics, physics and chemistry, about 50 are admitted as preparatory students, out of 300 or 400 applicants. But in the Hiroshima Higher Normal School, two students are admitted from each prefecture who are selected by the prefectural governments from among the applicants in the respective provinces. In this school, therefore there is no special examination for admission. The number of students admitted yearly to this school is less than that of those admitted to the Tokyo Higher Normal School.

In both of these schools, preparatory students are admitted early in April every year, and the average age of the students in the following month (May) is about 21 years. Consequently the average age in the first year of regular course is 22, that in the second year 23, and in the third year 24. The average age of the graduates, therefore, is about 25, which is the lower limit in age of intermediate school teachers supplied by the higher normal schools.

The students in the preparatory course of the Tokyo Higher Normal School are required to take thirty hours' recitations a week—of which four hours a week are devoted to mathematics,—making a total of 160 hours a year, the prescribed vacations not being included. Twenty to twenty-five hours a year are given to arithmetic, forty to fifty-five hours to algebra, some fifty hours to geometry, and about thirty hours to plane trigonometry. The materials of the subjects are arranged with reference to the knowledge of the students already acquired in middle schools or in

normal schools prior to entering this school where fuller and more advanced information on the subject is imparted. Although the requirements in mathematics are somewhat more strict in the case of students preparing for the department of mathematics, physics and chemistry, than for those of any of the other departments, yet in general this subject is taught with equal emphasis to all the students in the preparatory course. Nearly the same instruction is given in the preparatory course of the Hiroshima Higher Normal School.

The list of subjects in geometry and trigonomentry given to the preparatory students two hours a week throughout the whole year is in some cases as follows:—

Rapid review of plane geometry.

Geometry in space: Straight lines and planes in space. Parallel straight lines and planes. Perpendicular straight lines and planes. Dihedral angles and polyhedral angles.

Polyhedrons: parallelepipedons, prisms, pyramids.

Regular polyhedrons: Euler's theorem. Five species of regular polyhedrons.

Comparison of the volumes of polyhedrons.

Circular cylinders, circular cones and spheres: surfaces and volumes of these solids, frusta of cones, spherical sectors and spherical segments.

Plane trigonometry: Definition of trigonometric functions of an acute angle. Relations of trigonometric functions of acute angles. Values of trigonometric functions of particular angles. Tables of natural trigonometric functions.

Relations of sides and angles of a right angled triangle. Applications.

Extension of the definitions of trigonometric functions for any angle.

Sines and cosines of the sum and the difference of the

angles. Sines, cosines and tangents of a double angle.

Definition and properties of the logarithm. Usage of logarithmic tables.

Properties of any scalene triangle. Solution of triangles. Application for some topographic questions.

Of the subjects in arithmetic and algebra given to the same students, the following are some examples:

Definition of number and definition of mathematics. The aim of arithmetic.

Numeration and notation. Four operations: their meaning, and respective methods. Discussion of various methods.

Method of verification. Solution of arithmetical problems.

Compound numbers, especially a full explanation of the origin and history of the metric system of weights and measures.

Properties of integers. Fractions and decimals. Ratio and proportion—meaning of ratio Relation between solutions by proportions and other solutions, and their distinction.

Alligation. (Full explanation of indeterminate problems by means of algebraic knowledge.)

Percentage and interest.

Extraction of square roots and cube roots.

Method of contracted calculation.

Miscellaneous problems.

Introduction to algebra.

Positive and negative numbers.

Algebraic expressions. Mathematical induction. Necessary conditions, sufficient conditions. Properties of a rational integral expression. On $ax+b$, $\frac{ax+b}{cx+d}$ and ax^2+bx+c.

Remainder—theorem, and the theorems deducible therefrom.

Symmetrical expressions and alternating expressions.

Resolution into factors.

Greatest common factor. Least common cmultiple.

Rational fractional expressions.

Equations and identities; kinds, various definitions.

Equivalence of equations, and theorems concerning them.

Solution of fractional equations.

Linear equations with one unknown quantity.

Simultaneous equations; equivalence of simultaneous equations and theorems concerning them.

Fundamental theorems concerning the substitution.

Solution and examination of simultaneous linear equations with two unknown quantities. Simultaneous linear equations with three unknown quantities. Method of indeterminate coefficients. Bezout's method of elimination.

Linear homogeneous equations.

Cases in which the number of the unknown quantities and that of the equations are not the same.

In some cases, the following subjects are given :—

(These are limited to those students who take mathematics only in the preparatory course, and will in future engage in teaching Japanese and Chinese, English, geography and history in intermediate sehools.)

Conception of numbers and magnitudes ; units. Decimal system of numeration ; sexagesimal system ; metric system.

Development of arithmetic in Chaldea, India, Egypt, Arabia, China and in medieval and modern Europe.

Different systems of numerals compared ; their introduction into Japan.

Arithmetic in common life, in commerce and industry.

Arithmetic in schools, its relation to liberal education ; the reason why we study it in the Higher Normal School.

Use of letters and symbols in calculation ; algebra in classical times ; algebra in China and Japan before 1867.

Discovery of plus and minus signs in mediaeval Europe; negative numbers explained. Rule of signs; calculation with negative numbers. Use of letters to represent indeterminate quantities; some indeterminate equations of ancient India and of Diophantus.

Simple equations; simultaneous equations; exercises.

Infinity explained in relation to simultaneous equations.

Quadratic equations with various examples; imaginary roots.

Equations solved like quadratics; exercises. Cubic equations solved like quadratics. Biquadratics solved like quadratics. Binomial equations; cube roots of unity.

Maxima and minima deduced from quadratic equations; various problems. Maxima and minima deduced from symmetry; examples.

Permutations permutations with repetitions, cyclical permutations, transpositions.

Choices and arrangements; problems in connection with words, signals, chess-board.

Combinations, their structure. Homogeneous products.

Some topological problems.

Chance and probability; subjectivity and objectivity.

Probabilities of compound events, of exclusive events, etc.: exercises.

Inverse probabilities and probabilities of testimony: use of the theory for other various purposes.

Binomial theorem with positive integral indices: multinomial theorem.

Proof by mathematical induction, examples of reduction to absurdity.

Arithmetical progressions; arithmetical means. Use of means other than arithmetical; the median and the most frequent; practical examples.

Geometrical progressions; introduction of infinite series; convergency of geometric series; various explanations;

circulating decimals.

Interest, simple and compound. Malthus' law of population explained.

Laws relating to compound interest. Various examples.

The number e introduced, its use and properties; exponential series.

Logarithms, an historical account of: logarithmic computations; logarithmic series; use of tables; exerises.

Compound interest annuities with logarithmic table.

Percentage and interest; explanation of discount, notes, drafts, credits, Wert papier, stocks, insurance, etc.

Prime numbers; Fermat's and Wilson's theorems; factor tables.

Surds and irrational numbers; irrationality of some special numbers.

In the Tokyo Higher Normal School, the first year students of the department of mathematics, physics, and chemistry are required to take twenty-six hours' of lectures or recitations a week for all subjects throughout the whole year. Physical and chemical experiments, however, are not included in these hours. Of these hours, two are devoted to algebra, two to trigonometry, and two to geometry.

The list of topics in algebra is as follows:—

Inequalities.

Theory of quadratic equations.

Maxima and minima.

Equations of higher degrees.

Simultaneous quadratic equations.

Ratio and proportion, and variation.

Series; three kinds of progressions and some other elementary series.

Permutation and combination.

Binomial theorem, multinomial theorem.

Limiting values.

Infinite simple series.

Infinite double series.
General binomial theorem.
Exponential theorem.
Logarithmic series.
Determinants.

The topics in geometry are :—

Foundations of geometry. Definitions and Axioms.
Non-Euclidean geometry.
Demonstrations of theorems.
Maxima and minima.
Geometrical loci.
Problems of geometrical constructions.
Geometry of the triangle.
Modern geometry.
History of elementary geometry.

The subjects in trigonometry are :—

Demonstrations of identities.
Relations between the sides and angles of a triangle and a quadrilateral.
Measurements of heights and distances.
Inverse trigonometric functions.
Solutions of trigonometric equations and discussions.
Trigonometric inequalities.
Maxima and minima.
Subsidiary angles. Trigonometric solution of algebraic equations.
Use of tables of trigonometric functions, tables of logarithms of numbers and tables of logarithms of trigonometric functions.
Construction of these tables.
Rules of proportional parts.

To the students, who from the beginning of the second year are to take physics and chemistry as their specialities, Cartesian geometry is sometimes given instead of the above-mentioned course in geometry which is not necessary to

those students who will not teach mathematics after finishing the school course. Even in such cases, however, the courses in algebra and trigonometry are not changed.

The second and third year students are divided into two sections; the one consisting of those who take mathematics and physics as their specialities, and the other, of those who take physics and chemistry. Teachers' licences for mathematics in intermediate schools are usually given only to the graduates of the former section, but such liceuses may be given to graduates of the latter section whose scholarship-marks give evidence of peculiar fitness. Up to the present time, however, no such cases have occurred. The students of both sections have twenty-three hours' recitations a week for all subjects. To students of the former section are assigned physical experiment, and mathematical exercises, outside of these hours. Of the twenty-three hours, six are devoted to mathematics in which higher algebra, modern geometry, and analytical geometry are assigned. There are also four hours extra a week for mathematical exercises, or lectures on mathematical subjects.

Higher algebra takes up two hours a week, the list of subjects in it being as follows:—

Infinite products.
Continued fractions. Recurring continued fractions.
Indeterminate equations.
Compound continued fractions.
Properties of rational integral functions.
Properties of algebraic equations.
Symmetric functions of roots of equations and their applications.
Algebraic solutions of equations.
Separation of roots of numerical equations.
Solutions of numerical equations.
Elementary theory of numbers.
Divisibility, congruences. Congruences containing un-

known remainders of powers.

Analytical geometry takes up four hours, the subjects in it being as follows:—

Co-ordinates in plane. Cartesian Co-ordinates, polar co-ordinates.

Straight lines, circles. Geometrical loci. Ellipses, hyperbolas, and parabolas, curves of the second degree. Systems of conics.

Homothetic conics, similar conics.

Co-ordinates in space. Planes.

Straight lines. Generalisation of surfaces.

Surfices of the second order.

Of the four hours for mathematical exercises, two are devoted to supplement the course ef algebra above mentioned, with assigned exercises in algebra. The remaining two hours are devoted to lectures, the subjects of which are as follows:—

Complex numbers. Operations in complex numbers.

Different theorems on complex numbers. Demoivre's theorem. Roots of a number. Binomial equations.

Relations of surds. Trigonometric factorisations. Powers of a complex number, the exponent of which is a complex number. Logarithm of a complex number.

Expansions in trigonometric series.

Demonstrations of theorems and solutions of problems in Euclidean geometry in space.

Impossibility of geometrical constructions. Trisection of an angle. Duplication of a cube. Quadrature of a circle.

Transcendency of e and π.

Other problems of impossible construction.

Constructible regular polygons.

To the students of the latter section, i.e. to those who are specialising in physics and chemistry are assigned

physical and chemical experiments, besides the twenty-three hours' recitations. Of these twenty-three hours, three hours are for mathematics, in which analytical geometry and elementary calculus are taught.

To the third year students of the physics-chemistry section, there is no further requirement in mathematics, while the third year students of the mathematics-physics section have twenty-three hours' recitations, besides physical experiments and mathematical exercises. Of the twenty-three hours, six are devoted to lectures, and four to exercises. But in practice even these four hours for exercises are used for lectures. In the last term of the third year the students are engaged in practical teaching, and therefore there are no recitations in this term. The six hours for lectures are devoted to differential and integral calculus. The topics are as follows :—

> Variables and functions. Continuity. Differentials and differential coefficients. Their geometrical and physical meanings. Several formulae in differentials. Differential maxima and minima.
>
> Higher differential co-efficients and differentials.
>
> Curvature of plane curves. Some special curves.
>
> Evolute and involute.
>
> Functions of two variables. Partial differentials. Total differentials.
>
> Surfaces and osculating paraboloids. Indicatrix. Principal curvatures.
>
> Curves in space; tangent, normal, binormal, principal normals, rectifying line and cone, curvatures and spherical curvatures.
>
> Indeterminate forms. Taylor's theorem in Schlömilch's form. Expansion of several functions.
>
> Gregory's series. Evaluation of π.
>
> Inverse operation of differentiation.
>
> Indefinite integration.

Existence theorem of integral calculus.
Definite integrals. Reduction formulae.
Wallis's theorem. Taylor's theorem deduced by integration by parts.
Differentiation and integration under the sign of integration.
Evaluation of some definite integrals.
Quadrature of plane curves. Rectification.
Volumes of solids of revolution.
Quadrature of surface areas.
Calculation of moments. Position of centroids.
Moment of inertia. Pappus-Guldin's theorem.
Lagrange's interpolation formulae.
Mechanical quadrature.
Non-equidistant ordinates of Gauss.
Calculation of π by Simpson's rule, and by other methods of quadrature.
Double and multiple integration.
Change of variables.
Eulerian integrals. Stirling's formulae.
Linear differential equations with constant coefficients.
Characteristic equations.
Particular and general solutions.
Variation of parameter.
Differential equations of the first order; singular solutions.
Existence proof after Lipschitz.
Uniqueness of solutions.
Linear equations of the second order.
Complementary solutions.
Integration by series.
Fourier's functions. Trigonometrical series.
Partial differential equations of the first order.
Several methods of solution.
Geometrical illustrations. Equations of applied ma-

thematics.

Equations of potential functions, of small vibrations, and of heat diffusion.

Two hours for mathematical exercises are devoted to exercises in analytical geometry as already given in the previous year, with additional new lessons. The subjects of the new lessons are as follows:—

General theory of curves.

Construction of curves in rectangular co-ordinates. Tangents, asymptotes. Multiple points and other singular points.

Construction of curves in polar co-ordinates. Tangents and asymptotes. Singular points.

Several special curves: Limaçon de Pascal, epcycloid, spiral of Archimides, logarithmic spiral, strophoid, etc.

The remaining two hours are allotted to exercises in differential and integral calculus, but for the most part, problems in dynamics are treated while exercises are being given. The topics are as follows:—

Velocities and accelerations. Straight motions. Circular motions. Parabolic motions. Elliptic motions. Constrained motions. Elements of rigid dynamics and hydro-dynamics.

It is earnestly hoped that in this school an adequate numder of hours will be devoted to teaching that part of dynamics, to which, mathematics, especially differential and integral calculus, are applied. The elememts of dynamics may with some advantage be taught to students as an introduction to physics even before they have studied differential and integral calculus, or before they have acquired a full knowledge of trigonometry. But I firmly believe that those who are to be teachers of mathematics and physics should pursue that treatment of dynamics, in which differential and integral calculus is freely applied. Again

special hours should be provided for land-surveying,—of course even now a few hours are devoted to this subject. The same opinion holds with reference to descriptive geometry.

For about ten weeks toward the close of the third year the students are engaged in teaching mathematics in the middle school, and in the elementary school attached to the Higher Normal School, under the supervision of the professors of the school and the teachers of the attached schools.

In the Hiroshima Higher Normal School, students belonging to the department of mathematics, physics, and chemistry are required to pursue a regular course of three years more or less like that in the Tokyo Higher Normal School. To point out a few differences, analytic geometry extends from the first into the second year and differential and integral calculus from the second into the third, etc. The recitation hours are six or seven per week for each year.

The aim in teaching mathematics in both of these higher normal schools is to develop in the student a well disciplined logical faculty, the acquisition of practical knowledge, and a heightening of their aesthetic sense for scientific beauty. In the course of time, when they have finished the prescribed course and they themselves take part in the teaching of mathematics in intermediate schools, they should find themselves not only competent through the training they have received in accordance with the said aim in the course pursued by them, but also they should be ready to teach their own students with the same aim in view. Accordingly the professors not only teach their students with this aim in view, but also utilise every opportunity to admonish them to keep the same constantly

in view when they themselves have to teach in the intermediate schools. In order to make the course efficient, the professors not only teach the theory as well as the applications of mathematics, but also explain by what methods the result aimed at is to be attained. The subjects of mathematics assigned to the students in the higher normal schools are certainly far too advanced to be in themselves assignable to students in the intermediate schools. And yet when these very advanced subjects are taught, the professors constantly strive rather to show the connection between them and allied subjects in the intermediate schools or to explain the grounds on which the latter stand, and thus to make it clear to their students with what care they in time must seek to giude students of their own in the intermediate schools. In case the solution of mathematical problems be under discussion, the professors endeavour first of all to correct any bad habits the students may have formed in their intermediate school days, or to enable them to express distinctly what they wish to say, or if need be, even to get them to write out clearly what they wish to express. Or, again, in case one of the students be called to the blackboard to solve for others some mathematical problem, the professors judiciously seek rather to show him the department proper for a teacher, and to point out to him the part of his explanation which he should emphasize and also where the class were mostly misled. The teacher should explain to him how their knowledge should be developed and how their interest be stimulated, and sometimes even such a small matter as the handling of the chalk should be shown. In short, all that teachers of mathematics need to know is practically taught from time to time. Of course there are also some general subjects such as pedagogics and pedagogical methods specially taught in the department. But the practical pedagogical knowledge, which the professors of mathematics impart to their students in

the course of their instruction, naturally proves to be efficacious.

The intellectual progress of students is gauged by means of examinations. The Tokyo Higher Normal School has adopted term and annual examinations, and the Hiroshima Higher Normal School special and term examinations. Yet the professors attach much importance to the daily work of the students and take this also into consideration together with the results of examinations before they fix the relative rank of their students. That examinations are necessary to determine the result of teaching is evident. As they decide the relative rank of the students, they serve as stimulus in making them more diligent. Furthermore preparation for examinations gives them a comprehensive grasp of what they have been over during the term or the year. Certainly the examination system has disadvantages. This no one will deny. But, at the same time, it also has its merits. It can never be entirely dispensed with without great loss to the intellectual development of the students.

Among the graduates of the higher normal schools, there are a very limited number, who, instead of engaging in teaching, take the post graduate course. This course still aiming at the teaching of mathematics in intermediate schools, prescribes for them very advanced theories of mathematics, namely, such as are much higher and more profounder than those prescribed for the students in the regular course. The aim is to strengthen their grasp on the fundamentals of mathematics prescribed for students in intermediate schools. The condition of mathematical instruction throughout the world is generally given. The theory of numbers, higher algebra, theory of curves and surfaces, hydrodynamics, etc. are sometimes given. The recitation hours are from six to twelve per week.

There are also a very few of the graduates of the

higher normal schools, who enter the imperial universities for further study.

In Japan, as the number of licensed teachers of mathematics falls short of the demand, there has been added, in each of the higher normal schools, a special course in mathematics, extending over sometimes two years and a half and sometimes three years, in order to meet this deficiency. On account of its aiming at a quick mastery of the subject, little instruction is given on subjects other than mathematics, on which, as a matter of course, is put the maximum stress. The total number of recitation hours for mathematics in this course during its entire length does not differ much from that in the regular course. Consequently the weekly number of the recitation hours for mathematics in that exceeds the same in this. As a whole, the former is more or less similar to the latter, and includes dynamics, land-surveying, and book-keeping as well. The following is the list of subjects once used in the course of dynamics, covering three hours a week for some two-thirds of one year:—

Transmissibility of forces, parallelogram law of forces, Poisson's proof.

Forces and couples, Poinsot's central axes, Ball's wrench.

Equilibrum of solids. Lagrange's principle of virtual velocities.

Graphical solution of some statical problems.

Simple statical machines, frictions.

Kinematical introduction, velocity and acceleration as differential coefficients.

Rotation, angular velocity, spin, general screw displacement.

Body centrodes and space centrodes, roulettes.

Rectilinear motion of a particle under gravity, under central force, under air resistance.

Parabolic motion, its range, time of flight, envelopes, etc.

Parabolic motion under air resistance, simple cases, some points on external ballistics.

Central motion, the trajectories given to find the law of forces; conics, Cote's spiral, lemniscate etc.

Central acceleration producing simple harmonic motion, also its composition.

Newtonian attraction and motion, motion in conics under other laws of forces; an example of non-conservative force producing motions in conics.

Motion under the inverse cube law of forces; special interest in its relation to the kinetic theory of gases.

Explanation of some problems, methods, and results in the other more difficult parts of the dynamics of particles.

Moment of inertia and radius of gyration.

Poinsot's momental ellipsoid.

Compound pendulum and simple equivalent pendulum, centre of percussion and centre of oscillation identified.

Motion of a top; precession and nutation explained; gyroscopic motion.

Work, energy, potential, conservative force, total differential, etc.

General equation of motion; generalized coordinates; kinetic potential, Lagrangian function, Hamiltonian function.

Integrals of energy and of momentum.

Reduction of Lagrangian equations to the canonical form. Examples of the use of canonical equations.

Equilibrium of strings; common catenary, parabola, catenary of equal strength, etc.

Hydrostatics; floating bodies, surface of buoyancy and its evolute, metacentre.

THE THIRD TEMPORARY INSTITUTE FOR THE TRAINING OF TEACHERS

The Third Temporary Institute, being situated in the city of Sendai, is the only institute other than the higher normal schools which specially aims at the training of teachers of mathematics for intermediate schools and which has government authoritly to grant teachers' licenses to its graduates without examination. Like the special course in mathematics in the higher normal schools, it has been established as an emergency measure to meet the demands of the intermediate schools. The course covers three years. The average age of the matriculates varies between twenty-one and twenty-two.

The curriculum prescribes, in the first year, three hours a week for arithmetic, five for algebra, four for geometry and four for trigonometry throughout the school year; in the second year six for algebra, three for geometry and five for analytic geometry for the first and second terms, and four for algebra, one for geometry, four for analytic geometry and five for differential and integral calculus for the third term; and in the third year, three for algebra, two for geometry, four for analytic geometry and six for differential and integral calculus for the first and second terms, and three for algebra, two for geometry and six for differential and integral calculus for the third term. Besides, two hours a week throughout the second year, and three throughout the third are allotted to land surveying. As the graduates of this institute are to teach mathematics, it is necessary that they be conversant with its pedagogic methods. This, however, is a difficult accomplishment,—and one requiring time for its acquirement. In order to meet the need, therefore, from the very beginning

of the course, a seminary is held on fixed dates. In the exercises of this seminary the students are called in turn to go to the black-board and in the midst of comments to give a demonstration of teaching the subject assigned. In this way they gradually form correct habits of teaching. On account of this need the curriculum prescribes for the method and practice of teaching mathematics one hour a week throughout the first and second years, and in the third year two for the first and second terms, and six for the third. In short, the Third Temporary Institute closely resembles the special course in mathematics in the Tokyo Higher Normal School, and devotes much of its recitation hours to analytic geometry and differential and integral calculus. Even though these two subjects are in themselves of no direct use in an intermediate school, yet they enrich the teacher's stock of mathematical knowledge and makes him more confident in his teaching. This will explain why this special institute and the Hiroshima Higher Normal School devote so many of their recitation hours to analytic geometry and differential and integral calculus. The Tokyo Higher Normal School also sets aside some of its recitation hours for the same purpose, but not to such an extent as the other two do. How far analytic geometry and differential and integral calculus can with propriety be taught to candidates for positions requiring only arithmetic, algebra, geometry, and trigonometry, is certainly a problem not yet definitely solved. General pedagogics also are taught in the Third Temporary Institute, for its graduates of course are expected to know how to teach.

THE DEPARTMENT OF EDUCATION'S EXAMINATION FOR LICENCES TO TEACH IN INTERMEDIATE SCHOOLS.

In the Department of Education, a standing committee has been appointed to take charge of the granting of liceneses to intermediate school teachers. In cases in which the said grant is made without examination, the committee decides. When, however, the fitness of the candidate is tested by examination, a special sxamination committee is appointed from among the professors of the imperial universities, higher normal schools, high schools, higher technical schools, etc., and the said examination is carried out by this examination committee, at least once in each year.

The examinations for teachers' licenses in mathematics are of the following four grades :—

I. Arithmetic, algebra, and geometry;
II. Trigonometry;
III. Analytic geometry;
IV. Differential and integral calculus.

Here, however, it should be stated that a candidate is not entitled to take examination in any of the last three grades unless he has been successful in the preceding one; but he is entitled to be examined successively in all four grades in one year. Only those are regarded as perfectly equipped teachers of mathematics in intermediate schools, who have been successful in passing the last examination, viz. the one in differential and integral calculus. Nevertheless there only a small number have thus far passed all four examinations and received licenses in all four grades. The majority simply have licenses in the first grade, i.e., in arithmetic, algebra, and geometry. The graduates of th

regular, and the special course in mathematics in the higher normal schools as well as those of the Third Temporary Institute are regarded as having successfully passed the examinations in all four grades, and, are as a matter of course, entitled to licenses in them all.

The examination in the first grade is carried out in two divisions, namely, a preliminary and a final examination. Both consist in answering questions in arithmetic, algebra, and geometry. The preliminary examination is held in the provinces where the applicants live. As a matter of course, the special examination committee select the questions, and send them to each of these provinces before the examination takes place. The successful candidates in this preliminary examination are entitled to come to Tokyo in order to take the final examination. This latter includes, in addition to a written examination, an oral examination, whereby the candidates are tested as to their practical ability in teaching. They have also to take a brief examination in pedagogics. The majority of the applicants are non-licensed intermediate school teachers, elementary school teachers, and graduates of the Tokyo Butsuri Gakko.

The following were the questions in mathematics at the Twenty-fifth Examination for Teachers' Licenses, held in 1911.

Preliminary Examination Questions. Arithmetic (3 hours).

(1) Find three fractions, A, B, and C equal to $\frac{3}{5}$, $\frac{4}{9}$, and $\frac{6}{11}$, respectively, such that A's denominator is equal to B's numerator, and B's denominator to C's numerator. Find the simplest forms of such three fractions.

(2) A certain company, dividing its capital in the ratio of $3:5:7$, carried on its business in three divisions. At a semi-annual settlement, it was found that the first division had made 2600 yen, and the second had earned 8 per cent

a year on its capital, but that the third had suffered a loss of 5 per cent a year of its capital. However, the net result was found to be a gain of 6 per cent a year on the total capital. What was the amount of this capital?

(3) A steamer, bound for a certain port, had its engine damaged when one-fifth of its voyage had been completed. As it had to reduce its speed by ten knots for the rest of its course, the average speed was found to be less than the first by four knots. What was the initial speed?

(4) By evaporating 600 grams of water containing 3% of salt, one containing 5% of salt was to be obtained. It was found, however, that 70% of the water had already evaporated. How much water containing 3% of salt must be added in order to obtain the solution of required strength.

(5) Of a cylindrical vessel holding one *shō*, the height and diameter of which are equal; find the height to the hundredth place.

Algebra (3 hours)

(1) When $a+b+c=0$, prove

$$\left(\frac{b-c}{a}+\frac{c-a}{b}+\frac{a-b}{c}\right)\left(\frac{a}{b-c}+\frac{b}{c-a}+\frac{c}{a-b}\right)=9.$$

(2) Solve and discuss the following simultaneous equations:—

$$a\,x-b\,y-z+1=0,$$
$$x+y-a\,z-b=0,$$
$$x+y-z+1=0.$$

(3) Cut a triangle and a rectangle, having equal bases on a straight line, by another straight line so that the sum of the areas cut out between the parallel lines shall be equal to the area of the triangle. Find the distance between the parallel lines.

(4) In how many different ways can ten balls be arranged in a straight line? In arranging them, however, two special balls must in all cases be placed so as to

occupy alternate positions.

*(5) Let $a_1, a_2, a_3, \ldots\ldots$ be an arithmetical progression, and $b_1, b_2, b_3, \ldots\ldots$ be a geometrical having all its terms positive. Prove that a_n is not greater than b_n, if $a_1 = b_1$ and $a_2 = b_2$.

Geometry (3 hours).

(1) Let two circles touch internally at A. Draw a tangent PM to the internal from any point P in the circumfernce of the external, and prove that $PA : PM$ is constant.

(2) Given a vertical angle, the radius of inscribed circle, and the area: construct the triangle.

*(3) The vertex A of the rectangle $ABCD$ is a fixed point, and B and D are on the circumference of a fixed circle. Find the locus of the point C.

(4) Find the limit of the position of a point, the ratio of whose distances from two fixed points is less than a given ratio.

(5) Of a quadrilateral whose four vertices are not all in one plane, three are fixed and one moves along a straight line. Find the locus of the intersection of the lines joining the middle points of its opposite sides.

Remarks. Applicants for teachers in second class schools, (i.e., girls high schools and normal schools for women), need not answer the question marked with the asterisk.

Final examination questions for teachers of mathematics, Arithmetic, algebra and geometry, (written).

PART I. (3 hours).

(1) The sum of a certain irreducible fraction and its reciprocal is equal to $\frac{138794}{400115}$. Find the irreducible fraction.

(2) Eliminate x, y and z from

$$\frac{x}{a}+\frac{a}{x}=\frac{y}{b}+\frac{b}{y}=\frac{z}{c}+\frac{c}{z},$$

$$xyz = abc,$$
$$x^2 + y^2 + 2(ab + bc + ca) = 0.$$

(3) If a, b, p, and q be real, prove that the following equation has real roots,

$$\frac{p^2}{a^2 + x} + \frac{q^2}{b^2 + x} - 1 = 0.$$

(4) Solve the following inequality,

$$x - b > \sqrt{a\,(a - 2x)},$$

where a and b are positive, and $\sqrt{}$ represents the positive square root.

(5) Prove that the following three equalities are consistent with one another,

$$x = 10^{\frac{1}{1 - \log z}}, \qquad y = 10^{\frac{1}{1 - \log x}}, \qquad z = 10^{\frac{1}{1 - \log y}},$$

log representing the common logarithm.

Ditto, (Written).

Part II. (3 hours)

(1) If rectangles $ABDE$, $ACFG$ be externally constructed on the two sides AB and AC of the right angle A of a right angled triangle ABC, prove that the straight lines BF and CD intersect with one another on the perpendicular from A to the hypotenuse BC.

(2) Draw a circle with its centre on a straight line passing through the centre of a given circle, intersecting with this circle at right angles and passing through a given point.

(3) Of a triangle ABC, the vertex A is a fixed point on an edge of a trihedral angle and the other two vertices B and C move respectively along two other edges. Find the locus of the centre of gravity of the triangle.

Ditto, (Oral).

Arithmetic.

In the case of a boat, a certain distance is rowed over,

when there is no tide, in twenty-four hours. With the tide, however, the same distance can be rowed over in fifteen hours. And if against the tides, the boat can be made to go over thirty-two knots in two hours. Find the speed, accordingly, when it is rowed with the tide.

Algebra.

Solve the following simultaneous equations,

$$(1+2k)\,x-(1+k)\,y=1-k,$$
$$3\,(1+k)\,x-(3+k)\,y=3+k.$$

Geometry.

Draw a straight line meeting two straight lines not in the same plane and normal to a given plane.

Trigonometry, (Written).

Theory, (3 hours).

(1) Solve and discuss the following equation,

$$\sin 3\,x=m\sin x.$$

(2) Eliminate θ and φ from

$$c\sin\theta=a\sin(\theta+\varphi),$$
$$a\sin\varphi=b\sin\theta,$$
$$\cos\theta-\cos\varphi=2\,m.$$

(3) Find the maximum value of

$$\frac{\operatorname{cosec}^2\theta-\tan^2\theta}{\cot^2\theta+\tan^2\theta-1}\,.$$

(4) If the length of three bisectors of the three angles A, B, C of a triangle ABC be respectively equal to p, q, r, prove that.

$$\frac{\cos\dfrac{A}{2}}{p}+\frac{\cos\dfrac{B}{2}}{q}+\frac{\cos\dfrac{C}{2}}{r}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\,.$$

(5) Having given one angle, the perimeter and the radius of the circumscribed circle of a triangle, solve the triangle.

Application, (3 hours).

When the three sides of a triangle are known to be

respectively.

$$a=750.74\ m.,$$
$$b=596.42\ m.,$$
$$c=204.68\ \text{m.},$$

compute the three angles and the area.

Analytical geometry, (3 hours).

(1) Given a point (1,1) and a straight line $3x+4y-6=0$, the axes being rectangular. Form an equation to the curve of the second degree, having the point and the straight line for its corresponding focus and directrix and 5 for its eccentricity, and reduce it to the standard form.

(2) Let N be the point of intersection of the normal at any point M on an ellipse and its major axis. Prove that the orthogonal projection of MN on the line passing through M and one of the foci is constant.

(3) Prove that the four vertices of a parallelogram circumscribing an ellipse and its two foci are on the same equilateral hyperbola.

(4) Given an ellipse and a circle concentric with each other, the radius of the circle being equal to the sum of half the major axis and half the minor axis of the ellipse. Prove that the locus of the point of intersection of the two normals to the ellipse at the points at which two tangents are drawn to the ellipse from any point on the circle is a circle.

(5) Let N be the point of contact at which a tangent is drawn from the centre of a fixed circle to the circumscribed circle of a self-conjugate triangle with respect to the fixed circle. Prove that MN is constant.

Differential and Integral calculus, (4 hours).

(1) If $f'(x)\varphi(x)-f(x)\varphi'(x)\neq 0$ within the interval $a\leqq x\leqq b$, and $f(a)=0, f(b)=0$, then prove that $\varphi(x)$ will become zero within the given interval at least once. Here $f'(x)$ and $\varphi'(x)$ are continuous within the given limits.

(2) Let Y be the point at which the line passing through any point X on the diagonal AC of a parallelogram $ABCD$ and the vertex B intersect with the side AD or its extension. Find the minimum of the sum of the sum of the areas of two triangles AXY and BXC.

(3) Take z as the function of two independent variables x and y; substitute.

$$x=r\sin\theta\cos\varphi, \quad y=r\sin\varphi, \quad z=r\cos\theta$$

in $\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}$; taking θ and φ as independent variables eliminate x, y, z.

(4) $B\ (l,\ m)$ represents $\int_0^1 x^{-1}(1-x)^{m-1}dx$, l and m being positive. Prove that

$$B\,(l,\ m)=\frac{l+m}{l}\,B\,(l+1,\ m).$$

(5) Find the whole length of the space curve represented by the equations

$$a\,x=z\,(\mathrm{b}+z), \quad a^2\,(x^2+y^2)=b^2z^2,$$

a and b being positive.

(6) Take $x=\psi\,(u,v)$ and $y=\varphi\,(u,v)$, and change the integration's variables in

$$\iint dx\,dy \quad \text{from } x, y \quad \text{into } u, v.$$

Of those who have taken positions in intermediate schools, a majority, it seems, thus far at least, lack general education and I venture to call it a serious defect in them as teachers of mathematics that they have no qualifications in any subject of general education outside of mathematics, —above all in such a subject as physics. Education in Japan has been rapidly extended, and in spite of the great pains that have been taken to meet the demand for teachers of mathematics the supply is as yet quite inadequate. At such a time, it may be an unavoidable necessity to grant

teachers' licenses in mathematics to those having no qualifications in general education. This lack, however, must somehow be supplied in future. Accordingly, in recent years, the Department of Education has, very wisely, limited the applicants for licenses to the graduates of intermediate schools, and has certainly improved the requirements by adding pedagogics and pedagogical method to the subjects of the same examination in mathematics.

An intermediate school teacher ought not to be only a specialist in one subject. Rather he ought to know at least something of many different subjects,—subjects that are prescribed in the curriculum of intermediate schools—and to qualify himself more or less well in at least one of these. It is indeed to be regretted that there are now so few teachers in Japan who can teach even two subjects,—above all even two such closely related subjects as mathematics and physics. The license examination in mathematics ought not to be limited to testing the teacher's knowledge of mathematics as such. Rather ought it to aim at testing his qualifications as an intermediate school teacher more or less proficient in mathematics. In order to carry out this idea, the examination ought to prove whether or not he is really qualified in subjects of general edutation,—at least in such subjects as physics, chemistry, etc., which are so closely related to mathematics.

Also, the successful examinates have thus far been immediately granted teachers' licenses. Certainly such a course may hitherto have been inevitable. Nevertheless hereafter such persons should be considered simply as candidates for intermediate schools. They should then be taken, for a short time, into an educational institution suitably organized for their further training, and be allowed, on their graduation, to aid in practical teaching in intermediate schools to see whether or not they are really qualified to teach before they are granted teachers' licenses. Certainly the present

custom of granting teachers' licenses to all successful examinates as soon as they have passed the examination is far too hasty.

THE FURTHER EDUCATION OF THOSE ACTUALLY ENGAGED IN TEACHING.

No matter whether they are graduates from government institutes or successful examinates or non-examinates, all those who have once secured licenses as intermediate school teachers ought to devote themselves earnestly to their work, and lest they retrograde in scholarship, they should endeavor by wide reading to become acquainted with the relative superiority of the condition and progress of mathematical education in all advanced countries. This reading should include reference and text-books in English, French and German, not to speak of those in Japanese, mostly to be found in the reports of intermediate schools. They should also pay constant attention to such necessary changes as should be introduced into the mathematical course. Such demands, however, are certainly far too heavy to be met by intermediate school teachers in Japan at present. It is true that among the best teachers there are many who have been very diligent and are really accomplishing good results; but as yet the majority of intermediate school teachers are lamentably ignorant of the world's progress.

Accordingly the Department of Education, aiming constantly to improve the scholarship of its teachers and their methods of teaching, has made use of the long vacation as an occasion for opening summer schools. In order to carry out this object still more effictively, however, it has recently entrusted this work to the Higher Normal Schools which convene for this purpose some forty or fifty teachers of mathematics from all parts of the Empire. The results seem to be really encouraging.

And yet it would defeat its very object, if the teachers

should simply be satisfied with taking this work in a passive way as being compulsory instead of taking in it an active and spontaneous interest of their own. If it is to be of any real benefit to them, they must take it up of their own accord, and must devote themselves to it absolutely.

CONCLUSION.

All that has thus far been said is for the most part simply a report concerning the present condition of the training of intermediate school teachers in Japan. And yet at the same time I have tried here and there to insert my own opinion concerning some future policy that ought to be adopted. The present course of mathematics in intermediate schools generally divides itself into arithmetic, algebra, geometry, and trigonometry, each being taught apart from the rest. In the Syllabus of Teaching recently issued for intermediate schools, there is a certain intermixture of arithmetic and algebra. I can not, however, help wishing to see a course of mathematics so organized as to embrace all four divisions in one unique combination, and to assign the result according to each school year. From a practical point of view, there is no need for a student to have two such different ways of solving a problem as the arithmetical and the algebraical. He simply needs to solve it, no matter how he needs nothing more. And yet I do not mean to imply that only one way need be taught. I would have the one problem solved, once for all; and not have it solved again by algebra, when it had already been solved by arithmetic. Such a unique combination, I wish might be made not only in the case of arithmetic and algebra, but also in the case of geometry and trigonometry as well.

Plane geometry and solid geometry were in former days, as a matter of course, and at present still are taught separatly. And yet I am firmly convinced that the cambination here is not very difficult to make. In the near future, a text book aiming at such a combination will very probably appear in Japan.

I have expressed above my wish to see a course of mathematics so organized out of arithmetic, algebra, geometry and trigonometry as to form a unique whole, and to have a text book thereof so compiled as to accord with class distinctions. It is not my desire to bring up the discussion of any such question as ought to be easier, for it is understood that the text book would be compiled in accordance with the subject distinctions. Certainly there is much in algebra that ought to be easier than geometry. At the same time, there is also much in geometry that ought to be easier than algebra. Should, however, a unique course of mathematics be organized, no such inconvenience would occur anymore,—even if a part of arithmetic came after algebra. Moreover the mathematical knowledge given to the students in intermediate schools, must not be limited to such narrower bounds. Such, for instance, as in analytic geometry, the application of the co-ordinate axes which is really necessary for their daily life. And it is not so very difficult to make them understand it. It would certainly be most unwise to try to teach them any such organized knowledge of analytic geometry as is contained in the text book. But at the same time they should at least have an idea of the co-ordinate axes. It is certainly a regretable fact that here in our country the use even of section-paper is not much thought of.

Differential and integral calculus was formerly regarded as far too advanced a department of mathematics to be attained. On account of a sort of secondary effect of such erroneous ideas still remaining, the majority of teachers are rather inclined to dispense with it entirely, being terrified by the mention of its name or the sight of its symbols. And yet, with proper care and method, much of it can be successfully taught even to a beginner. In fact, we are even at present actually teaching much,—*e.g.* such as the mensuration of volume attached at the end of solid geometry,

—that really depends upon calculus, however, not making use of its name or symbols. And in fact, such teaching is absolutely necessary in intermediate schools. What I would wish to see is the cutting down of so-called higher mathematics, the doing away with too much of the unavailable properties of circle, triangle, etc., which proves useless later. We must unsparingly cut out much of elementary mathematics that ought to be cut out, and must substitute in its place the necessary and indispensable part of higher mathematics. In this, however, it is still uncertain just what to eliminate or just what to substitute.

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CONTENTS.

PART I.

Organization of the Course of Mathematics and the Present Aspect of its Instruction.

PART II.

Modern Tendencies in the Instruction of Mathematics.

The Teaching of Mathematics
in
Girls' High Schools.

The present report was prepared based upon the reports sent in from twenty-nine out of the total number two hundred seven of public and private girls' high schools. The twenty-nine schools just mentioned include the Girls' High School attached to the Tokio Higher Normal School for Women, the Peeress' School or the Girls' Section of the Peers' School, and twenty-seven prefectural girls' high schools.

PART I.

ORGANIZATION OF THE COURSE OF MATHEMATICS AND THE PRESENT ASPECT OF ITS INSTRUCTION.

CHAPTER I. AIM AND ORGANIZATION OF THE GIRLS' HIGH SCHOOL.

The following are extracts from the Ordinance regulations relating to girls' high schools.

(1) Aim.

The girls' high school has for its aim giving higher general education to women.

(2) Length of Course.

The regular course of the girls' high school extends over four years which, according to the local circumstances, may be prolonged by one year.

(3) Qualification for Entrance.

Candidates for admission into girls' high school should

be over twelve years of age, and have completed the elementary school course of six years, or have equivalent attainments.

The following table shows the average ages, in 1911, of the pupils of the Girls' High School attached to the Tokio Higher Normal School for Women.

Class	1st year	2nd year	3rd year	4th year	5th year
Age	12 Y. 7 M.	13 Y. 4 M.	14 Y. 9 M.	15 Y. 9 M.	15 Y. 6 M.

(4) Subjects of Study and Number of Hours of Instruction per week.

Schedule for the Four Years' Course

Class / subject	1st year	2nd year	3rd year	4th year
Morals	2	2	2	2
Japanese language	6	6	5	5
Foreign language	3	3	3	3
History, Geography	3	3	2	3
Mathematics	2	2	2	2
Science	2	2	2	2
Drawing	1	1	1	1
Domestic economy			2	2
Sewing	4	4	4	4
Music	2	2	2	2
Gymnastics	3	3	3	3
Total	28	28	28	28

Schedule for the Five Years' Course

Class / subject	1st year	2nd year	3rd year	4th year	5th year
Morals	2	2	2	2	2
Japanese language	6	6	6	5	5
Foreign language	3	3	3	3	3
History / Geography	3	3	3	2	2
Mathematics	2	2	2	2	2
Science	2	2	2	2	2
Drawing	1	1	1	1	1
Domestic economy				2	4
Sewing	4	4	4	4	4
Music	2	2	2	2	2
Gymnastics	3	3	3	3	3
Total	28	28	28	28	28

(5) Supplementary Course.

The girls' high school may have a supplementary course of less than two years.

(6) Special Course.

The girls' high school may have a special course for the graduates who desire to continue their study of some select subjects.

Note:—In many cases a supplementary course of one year or two is added to the regular course. The subjects of study in supplementary course vary according to local conditions. In some cases only sewing and cookery are taught as necessary preparation for house keeping; in others are

given preparatory study for entrance into higher normal schools for women. Again, in some schools some of their graduates are trained for becoming elementary school teachers, in view of the scarcity of such teachers nowadays. For those pupils desiring to enter a higher normal school for women or to become elementary school teachers, mathematics is taught; and arithmetic, algebra, and geometry are carefully reviewed and supplemented.

At present there are only two schools which have the special course. One is the First Girls' High School in Kioto and the other the Girls' High School attached to Tokio Higher Normal School for Women.

Only in the latter mathematics is being taught. Here the period of study is three years, and the branches of mathematics taught are arithmetic, algebra, and geometry. But as the time given is only two or three hours per week, the work accomplished in these subjects is hardly equivalent to the corresponding work in middle schools.

CHAPTER II.

Aim and subject-matter of Instruction in Mathematics.

(1) Subject-Matter.

Arithmetic, elementary algebra, elementary geometry.

(2) Plan and Number of Hours of Instruction.

First year ....Two hours per week.

ARITHMETIC.

Integers and Decimals.

Numeration and notation, four rules (sometimes Soroban-calculation is taught in addition), mental arithmetic.

Compound Numbers.

Time, common weights and measures, metric system, foreign weights and measures (chiefly yard and pound), money and currencies, foreign money and currencies commonly used in our country, descending and ascending reduction, four rules in compound numbers.

Fractions.

Notations, kinds, reduction, four rules in simple fractions.

Second year . . . . .Two hours per week.

Review of the preceding year's course.

Properties of Numbers.

Multiple, divisor, divisibility by 2, 5, 4, 8, 9, and 3, prime numbers, decomposition into prime factors, greatest common divisor and least common multiple.

Fractions.

Reduction to the lowest terms, reduction to the common denominator, four rules in fractions, decimals as a special case of fraction, conversion of fractions into decimals.

Ratio and Proportion.

Ratio, simple proportion, compound proportion, chain rule.

Percentage.

Percent, percentage, base.

Interest.

Simple interest, compound interest.

Third year . . . . .Two hours per week.

Review of the preceding two years' course.

Ratio and Proportion.

Distributive proportion or proportional parts, alligation.

Percentage.

Taxes, notes, bonds, shares, insurance.

Square root.

Mensuration.

Fourth year . . . . .Two hours per week.

Review of the whole arithmetic.

RUDIMENTS OF ALGEBRA AND ELEMENTARY GEOMETORY.

Algebra.

Simple algebraic expressions, negative numbers, equation of the first degree.

Geometry.

Use of ruler and compasses, kinds and construction of principal geometrical figures, angles, parallel lines, theorems and constructions pertaining to triangle, proportion, simple quadrilateral, circle and regular polygon, area, ratio and solids.

In girls' high schools having five years' course, the rudiments of algebra and elementary geometry are taught in the fourth and the fifth years.

(3) Aim of Instruction.

The direction issued by the Department of Education concerning the teaching of mathematics in girls' high school runs as follows:—

"The mathematics is to be taught in order to make clear the nature of number and quantity, to train girls in calculation, to give them various knowledge necessary in daily life, and to make them accurate in thinking."

In accordance with the above, in arithmetic, practical training is to be more emphasized than the theoretical. In algebra and geometry attention is directed more to the logical side, and to the development of the power of accurate and rigorous thinking, at the same time without losing sight of practical applications.

(4) Relations between Different Branches of Mathematics and Connections with Other Subjects.

(*a*) Relation between arithmetic and algebra.

The teaching of algebra should be closely connected with that of arithmetic. It is important that what has been taught in arithmetic should be generalized and applied to algebraic calculations in letters. In giving rules and solving problems, care should be bestowed upon the inter-relations of these two branches of mathematics. And thus is to be

shown why algebra is more general and yet simpler than arithmetic, and that all this comes from representing numbers by letters.

(*b*) Relation between geometry, arithmetic, and algebra.

In girls' high schools, as regards geometry, our object is to give a general idea. Here simplicity and ease are the desideratum. So sometimes explanations are given by means of concrete examples. Algebraic rules and formulae are often applied in order to make the explanations easy. Mensuration taught in arithmetic is to be explained in geometry.

(*c*) Relation between mathematics and other subjects.

Mashematics and physics are closed allied subjects; but since physics taught in girls' high schools does not make much use of mathematics, we need only consider the relation between the two subjects in connection with weights and measures, composition of forces, specific gravity, thermometry, optics and so forth.

Between geometry and geometrical drawing, there should be close relation.

Between mathematics and geography, there should be relations so far as longitude, calendar, measurement of distance, statistics of productions and the like, are concerned.

Concerning mathematics and domestic economy, there should be connections as regards various items of family income and expenditure, savings, insurance, etc.

Relations should be established between mathematics and book-keeping included within domestic science.

As regards mathematics and sewing, since there are many things requiring calculation in sewing, such are to be included in arithmetical problems and exercises. Attention is also to be drawn to making estimates in sewing lessons.

Formulae of cutting may be simplified by making use of letters as in algebra.

All the problems and exercises given in arithmetic should be such that they may be useful to girls in view of their afterwards becoming home-makers.

CHAPTER III.

Examination.

(1) Object.

(a) Examination is given in order to test and gauge the progress and attainments of individual pupils.

(b) Separate parts are frequently reviewed, and besides, examinations covering the whole of what has been taught during a certain period, are given. Thus pupils gain the chance of renewing their memory and of discovering unity and connection among the parts taught separately.

(c) Examination has the benefit of letting pupils recognize their own ability, and spurs them on to further study with zeal and interest.

(d) Through examination the teacher becomes acquainted with the following particulars :—

(i) Whether pupils have clear comprehension of the subject.

(ii) Whether pupils are skilled in calculation.

(iii) Whether pupils have the habit of writing figures creditably.

(iv) Whether pupils are able to express their thoughts clearly and concisely.

Through the Knowledge of the above particulars, the teacher is able to direct the future course of instruction.

(2) Method.

Some are of the opinion that it is harmful to girls at this stage of mental and physical development, to pile up at one time examinations of various subjects. They wish to see pupils' standing judged by the results of frequent

reviews, given at convenient intervals with or without previous warning.

Again, according to the opinion of some others, examination is to be conducted exactly as in boys' schools *i.e.* once or twice in each of three terms into which one school-year is divided, with previous warning, covering all the parts taught during the term.

Again, there are still some others who wish to see the above two methods combined. In this case the former is called a special examination and the latter the regular.

(3) Kinds of Questions.

(*a*) Questions especially given for judging proficiency in calculation.

(*b*) Questions aimed at testing the power of reasoning.

(*c*) Questions particularly relating to matters of daily life.

(*d*) Questions involving mental calculation and making approximate estimates.

(*e*) Questions given to test the quickness in understanding and calculation.

(4) System of Marking.

According to the kinds of questions, some answers are marked from the point of view of reasoning, and some from that of practical calculation.

The highest mark given is sometimes one hundred, and sometimes ten. In reporting pupils' scholarship to their homes, their standing is classified and expressed in one of the signs of the following sets:—

"high," "middle," "low."

"1st," "2nd," "3rd" "4th."

"Excellent," "good," fair."

By thus reporting pupils' standing to their homes, endeavour is made to co-operate with the homes in furthering the pupils' progress.

CHAPTER IV.

Method of Teaching.

That the method of instruction should vary according to the subject-matter is almost self-evident. But, generally speaking, our teaching of mathematics proceeds in the following order :—

(1) Preparation.

(*a*) The facts already taught and connected with the days' work, are reviewed.

(*b*) The facts already given in other subjects of study and having relations with the day's work, are reviewed and discussed.

(*c*) When weights and measures, money, taxes, insurance, and the like arithmetical matters are to be taught, we appeal to be knowledge which pupils might have got outside the school.

(*d*) In solving problems, sometimes easier preliminary exercises are given.

(2) Actual instruction.

(*a*) New matters are rarely presented in the form of an exposition, but usually in that of a discussion.

(*b*) Real objects, moulds or diagrams are sometimes shown to the pupils to make matters clear.

(*c*) In teaching the solution of a problem, the proof of a theorem, or the method of construction, simple examples are shown first, then pupils are led on to find out the solution or proof by themselves.

(*d*) While pupils are working their exercise, the teacher goes up to each of them in the class, and directs her or gives suggestions as the case may require.

(*e*) When a solution or an answer is shown on the black-board, the passages of particular importance are especially pointed out.

(*f*) Any relations existing between already learned matters and the new are to be thought out by pupils.

(*g*) In case there are many solutions of a problem, pupils are encouraged to think out as many solutions as they can, and to compare their relative merit.

(*h*) Chief points of the day's lesson are grouped and repeated, and especially fundamental axioms and theorems learned during the lesson are clearly written out by pupils.

(3) Applications.

(*a*) Exercises and applied problems which can be solved by the use of newly learned rules, are given. Practical applications of newly acquired rules are always emphasized.

(*b*) When a chain of related matters is completely taught to the end, a certain suitable length of time is devoted to the promiscuous examples relating to such matters.

POINT REQUIRING SPECIAL ATTENTION IN INSTRUCTION.

Arithmetic.

(1) Pupils are always required to write the figures clearly so as not to confuse 1 with 7, 3 with 7, 6 with 0, 4 with 9.

(2) In calculation, pupils are trained to write down figures in right columns in right order; if not, much miscalculation is apt to arise, and much of the teacher's time and energy would be called for in correcting. It is necessary to train girls in this matter at the very start, as it becomes harder afterwards to correct the disorderly habit.

(3) In teaching solution of a problem, it is necessary to consider the problem from all points of view, and find the best solution.

(4) Model problems are carefully treated and are to be made sufficiently understood by pupils that they may be able to solve analogous problems afterwards.

(5) Problems of similar nature are given repeatedly as long the time is allowed.

(6) Calculations of frequent occurrence in daily life are to be practised as much as possible. For this purpose it is advisable to use a part of a lesson hour——either at the beginning or at the end——for mental calculation and approximate calculations in four rules. It is impossible, however, to obtain a satisfactory result in a short time. Only by a long and steady practice any progress could be expected.

(7) Pupils are apt to be careless and err in putting the decimal point in its proper place. Teachers should try to impress on them, how grave a mistake such carelessness may lead to.

(8) Problems of daily life are not so simple as the problems given in a text-book, and if pupils are trained only in easy imaginary ones at school, they would be at a loss afterward when face to face with the complicated matters of life. Hence it is necessary that teachers should make up exercises from matters of every day life, put in some complex numbers, and let the pupil think out the solution, giving her, at the same time, much practice in calculation.

(9) Applied problems become most efficient when chosen from matters relating to local conditions. These help to increase the pupils' knowledge of economics and of household affairs.

(10) From time to time, problems relating to household economics are given to train girls in the complexities of income, expenditure, etc.

(11) Teachers should be always on the look-out for the

market-price of things, and convey to the pupil an appropriate idea of current price of things.

(12) In giving problems of percentage and others relating in any way to percentage, the fundamental points are emphatically explained and it may be well to acquaint the pupil with some reference-books which might be of use to them in future.

(13) In solving an arithmetical problem, an algebraical method or a method deduced therefrom may be used. But for those who have not learned equation, it would be hard to understand. Moreover, there is a certain fundamental difference between arithmetical and algebraical solutions. Therefore in solving an arithmetical problem, preference is to be given to a really arithmetical solution.

(14) Sometimes pupils are seen to use fingers in doing some problems in addition and subtraction: *e. g.* such problems as, "The third day of a certain month is a Sunday. What day of the month would be the succeeding Sunday?" or "How many hours are there between 5 A.M. and 7 P.M." Exercises should be given for the express purpose of abating such habit.

(15) In all cases, verification should be encouraged.

(16) Not only partial reviews, but also reviews including a wider range are being given from time to time, so that pupils may constantly have the clear idea of the fundamentals.

Algebra.

(1) The relation between algebra and arithmetic is to be made clear and the connection between the two is to be kept in view as far as possible.

(2) Pupils studying algebra for the first time often experience more difficulty than teachers anticipate. Therefore it is best to let them first solve an

arithmetical problem arithmetically, then let them write down something like a formula embodying the various steps of the solution, and then replacing numbers therein by letters. This will show them the relationship between a number and a letter.

(3) Let the pupils become gradually convinced in that algebra is simpler and more general in its working.

(4) In spite of the fact that a letter represents an abstract number, pupils are apt to think as if it denotes a concrete number. This common error is to be pointed out and suppressed.

(5) Important formulae are to be committed to memory, so that, whenever necessary these may readily be used.

(6) Verification is necessary in every case, and in solving applied problems by means of equation, the results are to be discussed.

(7) Besides these, all the points mentioned under arithmetic apply to this case also.

Geometry.

(1) The teaching of geometry in girls' high schools aims at giving an accurate conception of space, and also developing the power of thinking. However, in teaching the beginners, it is found advisable not to adhere too strictly to purely logical method, but at first to make use of intuitive method or of inductive method, sometimes simplifying the explanation by making use of numbers. Then after that it may be well to proceed gradually to more logical method.

(2) The words used in explanation must of course be clear and explicit. But, very often in the teaching of geometry, the language used is apt to become too stiff and bookish, far removed from our everyday talk. But it is plain that a clear and concise explanation need not necessarily be in bookish

language. So, except when repeating theorems or definitions, proofs are to be given in ordinary conversational language. This gives also a becoming air of gentleness to girls, and at the same time avoid the misunderstanding that a laudable process of induction can only be expressed in unwonted language.

(3) When pupils are working on geometrical problems, sugsestions and helps should be given rather sparingly, and only at the points of great difficulty and importance. For the rest pupils should be trained to work on by themselves, feeling the way, so to speak. If a teacher is too ready to give a helping hand, pupils —— and particulary incompetent ones—— are apt to avoid the trouble of thinking, and try only to memorize the process of a proof.

(4) In proving a theorem or in solving a problem, there are usually several ways of arriving at the result; among them there is often one way which is recognized by many as the most direct and efficient one. It is important to teach the pupils such ways and methods. In this connection is to be observed the following :—

(*a*) The fundamental theorems should be memorized, and solutions of model problems be reviewed frequently.

(*b*) A theorem or a problem should not be thought out with refrence to a special form or diagram. For instance, if a question is asked of a triangle, some pupils often think of a right angled triangle or an isosceles triangle, and by so doing, upset the whole.

(*c*) Diagrams should be drawn carefully. An accurately drawn diagram is often the means of finding a right solution.

(*d*) The premise and the conclusion of a theorem or a problem should be made clear by refering to the

diagrams. Through neglecting this, pupils often fail in their work.

(*e*) The process of finding a proof may sometimes begin from the conclusion backward in the order somewhat like the following: firstly the recognition of the meaning embodied in the premise; secondly, keeping in mind the result which can be easily deduced from the premise; thirdly, finding out of the relation (A) which must hold good in order that the conclusion be true, fourthly finding out the relation (B) which must hold good in order that the relation A be true; thus proceeding backwards and finally arriving at the result obtained at the second step from the premise. In this way a proof may sometimes be found.

(*f*) Sometimes it is convenient to consider the contraposition of a given theorem and sometimes its obverse.

(*g*) Efforts should be made to reduce any new problem to one of those already solved.

(*h*) A theorem is to be enunciated in the simplest possible words as befits committing to memory.

(*i*) The theorems resembling one another are to be compared and contrasted, in order to facilitate pupils committing them to memory without fear of confusion. Such are, for example theorems concerning equality and similarity of triangles.

General remarks concerning the teaching of mathematics.

(1) Mathematics does not only aim to train the pupil to comprehend clearly, but also to give her the facility of calculation and the ability for application.

(2) Let the pupils be accustomed in expressing exactly and concisely.

(3) In giving problems to the class, the following points are to be particularly noticed:

(*a*) Presentation of a problem should be done orally, or by writing down principal points on the black-board, or by the use of a text-book.

(*b*) Sometimes let pupils themselves make problems and present them to the class.

(*c*) As a model problem or theorem must draw the attention of the whole class, it should be written down on the black-board. Then its meaning must be explained and made clear. And then and only then we have to proceed to the proof or solution. In case there is a sufficient number of black-boards, a theorem or problem may be written on a black-board, before the class begins.

(4) In taking a round of a class the following points are to be observed.

(*a*) Teachers must take a round of a class quickly and thoroughly.

(*b*) If most of the students in the class are found struggling with the solution of a problem, the teacher should open a discussion on the point of difficulty, and so lead them on to the right solution. It is never wise to let the pupil waste their time and energy in vain.

(*c*) It will be necessary to make some special efforts for inferior pupils in the class; but the class as a whole must not be lost sight of for the sake of a few individuals.

(5) Inspection of answers are to be done in the following way:

(*a*) Let the pupil explain the process of the solution of a problem clearly and concisely and avoid falling into the bad habit of being unable to explain the solution thought out by herself.

(*b*) Sometimes let a pupil write down on the black-board the entire processes of the solution of a

problem, and let others criticize. Thus the pupils are to be trained in expressing their own thought clearly and concisely.

(c) In case a pupil was unable to find a right solution, the teacher should find out promptly all the essential points which lead to failure, and teach her in such a way that she shall not fall into the same error.

(d) If the majority of the class failed in solving a problem, an analogous extra problem is to be given.

Text-books.

The text-books generally used are the Girls' High School Text-books compiled according to the directions of the Department of Education. In actual teaching, teachers make suitable alterations and modifications in using these text-books, keeping in view local circumstances and the state of intellectual development of pupils.

Reference Books.

As regards the teaching of mathematics in girls' high schools, it is not desirable merely to enlarge the sphere of the pupils' knowledge and as a consequence lose the grasp of essential points. The aim is to make pupils thoroughly acquainted with the contents of text-books and enable them to apply the knowledge thus gained to matters concerning daily life. For this reason there is not much need of reference-books. Perhaps a collection of appropriate problems for home-work would be a suitable substitute. But there is no such book published as yet; the only way open for a teacher is to give pupils a certain number of select problems for home-work.

Models and Instruments.

Shaku-measure, *Kujira-jaku*-measure, Metre-measure,

Measure giving comparisons of the three kinks of measures, Tape, Yard-measure, Foreign-measure, Grain-measure, Liquid-measure, Metric volume-measure, Various kinds of balances, Clock, Thermometer, Compass, Japanese currencies or their diagrams, Diagrams of foreign currencies. Large compasses for black-board use, Various kinds of rulers, Protractor, Specimens of notes, cheque, share, bond, insurance bill, etc., Models of geometrical figures.

All these models and instruments are to be put and arranged in a certain fixed place, so as to attract the attention of pupils. And so far as is possible, let the pupils use them and make experiment with them.

Practical Work.

(1) In some convenient place in the vicinity of the school, pupils are trained in measuring distance, height, area, volume, and so forth, by counting steps, by estimation and by actual measuring. Pupils are further required to compare the results of measurement by different processes.

(2) By using books, instruments, pebbles, etc., pupils are trained in finding weights of substances by estimating as well as by actual weighing, and in comparing the results obtained.

(3) As for content or volume, practice is done by estimating as well as by actual measuring. A fixed quantity of some thing is put in different kinds of containing vessels, for example one *sho* capacity of rice in a dry measure, in a bag, in a bottle, and the like; and pupils are required to observe and compare them.

PART II.

Modern Tendencies concerning the Instruction of Mathematics.

CHAPTER I.

Current Opinions on the Organization of Schools. Establishment of a new kind of girl's high Schools to be called *girls' high real-school* (or girls' practical high school).

In October 1910, the government regulations for girls' high schools were revised, and a rule was added granting the annexation of a practical course to the girls' high school or the establishment of a new kind of girls' school to be called *girls' high real-school*, for those pupils who desire instructions in domestic line only. This plan was put into execution since April 1911. The aim of the plan may be seen from the following directions issued by the Department of Education.

"With the progress of education for women in recent times, the need of some sorts of practical or utilitarian schools for girls has become urgent. Inconveniences in this connection are being experienced by provincial authorities on account of the lack of directions and regulations concerning such schools. It is true that the girl's high school is allowed to modify its curriculum so as to suit local conditions. And yet there remains much to be desired, so far as the education in domestic line is concerned. On this account, the new regulations allow to annex a practical course to a girls' high school. In this new course, more importance is to be put on sewing, practical subjects are to be added, and more freedom to be given as regards the selection of subjects as well as the number of hours of instruction per week, so as to fit local conditions. Moreover, girls high schools may

admit elective pupils *i. e.* those who choose the subjects of study at their option. And thus is given the chance of easy access to studies. In putting the revised regulations into execution, no efforts should be spared in selecting suitable subjects of study and in keeping within proper bounds as to the grade, so as to fit local conditions as far as possible."

"The education of girls, the moral education, in particular, can only be satisfactorily carried out, when the school and the home work together. That a girl lives far away from her parents for the sake of attending a school, is open to serious anxiety. Bearing this in mind, facilities are given to local authorities for establishing schools for girls. In addition to girls' high schools of old, the local authorities are allowed to establish girls' high real-school either as a separate school or as an annex to a higher elementary school. And thus more chances and convenience are given to girls to be able to receive necessary education without leaving their homes. At the same time, indiscriminate and hap-hazard establishment of such schools does not accord with the spirit and the aim of the revised regulations. Due attention is to be paid to the local financial conditions and supply of teachers."

"The aim of the aforesaid practical course is to rouse interest of girls in industrial work, and bring them up as work-loving women. It is indeed grievious to see, sometimes, young women of middle class not at all so trained as to become competent housewives. And the object of the practical course is to diminish such current grievances. That the course is made optional is due to the consideration of local economic conditions and of the difficulty of securing competent teachers."

The above instructions were immediately put into practice, and no less than forty-four such public schools and seven such private schools have since been established. And thus improvement in women's education is being carried on.

(1) Requirement for Entrance.

The qualification for admission is that the candidate should be above twelve years of age and have completed the elementary school course of six years or have equivalent standing of scholarship.

(2) Duration of the Course.

The duration of the practical course is as follows :—

(*a*) Four years, in case the qualification for admittance is the completion of the elementary school course of six years.

(*b*) Three years, in case the qualification for admittance is the finishing of the 1st year of the higher elementary course.

(*c*) Two years, in case the qualification for admittance is the completion of the 2nd year of the higher elementary course.

(3) Subjects of Study and Number of Hours of Instruction per week.

(*a*) Four years' Course.

Classes / Subjects	1st year	2nd year	3rd year	4th year
Morals	2	2	1	1
Japanese language	6	6	6	6
History	2	2		
Mathematics	2	2	2	2
Science and house-keeping	2	2	3	3
Sewing	14	14	18	18
Drawing	1	1		
Singing	2	2		
Manual work			3	3
Gymnastics	3	3	3	3
Total	34	34	36	36

(b) Three years' course.

Subjects \ Classes	1st year	2nd year	3rd year
Morals	2	1	1
Japanese language	6	6	6
History	2		
Mathematics	2	2	2
Science and house-keeping	2	3	3
Sewing	14	18	18
Drawing	1		
Singing	2		
Manual work		3	3
Gymnastics	3	3	3
Total	34	36	36

(c) Two years' course.

Subjects \ Classes	1st year	2nd year
Morals	1	1
Japanese language	6	6
Mathematics	2	2
House-keeping	3	3
Sewing	18	18
Manual work	3	3
Gymnastics	3	3
Total	36	36

Singing and manual work may be omitted, and manual work may be made optional.

CHAPTER II.

Revision of Syllabuses of Girls' Schools.

A revision of the syllabuses of girls' schools was issued in March 1911. The revised syllabus of mathematics is as follows :—

Girls' High School.

First year : Arithmetic. Two hours per week.
Integer and decimal.
Compound number.

Second year : Arithmetic. Two hours per week.
Measure and multiple.
Fraction.
Proportion (ratio, proportion, compound proportion).

Third year : Arithmetic. Two hours per week.
Proportion (proportional parts, alligation).
Percentage (percentage, interest).

Fourth year : Two hours per week.
Square root.
Mensuration.
Reviews.

In case rudiments of alegebra and elementary geometry are given, they are to be taught in and after the third year according to the following syllabus.

Algebra.

Easy algebraic expressions.
Equations.

Geometry.

Easy plane figures and solids.

In the fifth year of the girls' high school of five years' course, the matters taught in the first four years are to be reviewed and supplemented.

Girls' High Real-School.

Four years' course.

The same as in the girls' high school.

Three years' course.

First year: Two hours per week.
The same as in the first and second year of girls' high school, with suitable omissions and modifications.

Second year: Two hours per week.
The same as in the third year of girls' high school.

Third year: Two hours per week.
The same as in the fourth year of girls' high school.

Two years' course.

First year: Two hours per week.
The same as in the first three years of girls' high school with suitable omissions and alterations.

Second year: Two hours per week.
The same as in the fourth year of girls' high school.

Warning.

(i) The instruction in mathematics should aim not only at the correct and sure understanding of subject-matter, but also at the skill in calculation and the aptitude for application.

(ii) In teaching arithmetic, due attention is to be paid to what the pupils have already learned in the elementary school.

(iii) In arithmetic, besides mental and written calculation, the four operations in *soroban*-calculation are to be taught, as much as circumstances allow.

(iv) In teaching algebra and goemetry, their relation to arithmetic is to be kept always in view. In particular, in case algebra is taught, the extraction of square root is to be taught simultaneously; and in case geometry is taught, mensuration is to be taught at the same time.

The revised syllabuses do not much differ from those of old (chapter II of Part I). The main feature to be noticed is that they do not go into details. The consequent flexibility is to be taken advantage of, so as to suit local circumstances and the intellectual development of pupils. Investigations in this direction are being carried on in every girls' high school.

CHAPTER III.

Various Opinions on the Instruction of Mathematics.

(1) On subject-matter.

Up to the present date, the branch of mathematics chiefly taught in girls' high school was arithmetic; and rudiments of elementary geometry or algebra was added only in the 4th year.

On this account, rigorous proofs were tried to be given and complicated problem were solved, and all these in arithmetic. By so doing much efforts have been wasted and the result was not satisfactory.

Now, some of the instructors of mathematics have come to the conclusion that arithmetic should only aim at enabling the pupils to solve ordinary problems and to do calculation of daily need quickly and accurately and that more difficult

matters should be all left to alegebra and geometry.

Such seems to be more in accord with the peculiar nature of mathematics; and at the same time, this may cause a fresh aspiration of the pupil for further study.

Moreover, a girls' high school is a place where higher general education for women is given, and generally speaking, it means the last stage of women's educations. Therefore, unless there be an inevitable reason against it, due to local circumstances, the teaching of the rudiments of elementary algebra and geometry in conjunction with arithmetic in the 3rd year, and upwards, is considered highly desirable in view of the progress of the world. Indeed, there are many who are in favour of this opinion.

The hours of instruction thus to be used for geometry and algebra can be taken from those of arithmetic in the following manner.

(*a*) Those parts of arithmetic, which the pupils have learned in the elementary school, such as numeration and notation, addition and subtraction of integers and decimals, may be passed over. Consequently, the time formarly spent in teaching such subjects, may now be used only for reviews of the same and could naturally be much reduced. Very often, the girls of the first year of the girls' high school seem to lose interest in arithmetic through the repetition of subject-matters which are so common and with which they are already so well acquainted.

(*b*) As the problems of chain-rule could all be solved by repeating simple proportion, there is no need for giving chain rule as a distinct method.

(*c*) The theory of greatest common measure and that of least common multiple may be referred to algebra, and here in arithmetic, only the processes of finding them may be given.

(*d*) As stated in connection with the revised syllabuses, extraction of square root should better be taught in algebra,

and mensuration in geometry.

(*e*) All the complicated problems of hypothetical character, or those unnecessarily obscure, should be altogether omitted.

(*f*) Everything of purely theoretical nature should be omitted in arithmetic.

(2) On the Revised Syllabus of Algebra.

Up to the date of revision, syllabus for algebra contained, by way of introduction, various definitions, four rules in monomials, negative quantity, equation of the first degree, and lastly simultaneous equations of the first degree; and more than half of the time was spent in calculation. But though calculation is all-important in arithmetic, it is not so important in algebra.

If the work is, as suggested above, limitted to the grade of being able to solve equation of the first degree, exercises in calculation can be made far simpler, and the hours thus freed may be efficaciously used for bringing out more general aspects of algebra.

(3) On the Aim of Instruction.

As mathematics is a subject in close connection with daily life, modern tendency is to emphasize its practical aspect; consequently all theoretical considerations are to be avoided; mental calculation, rough calculation, approximation, etc. should be encouraged; moreover, all the problems should be chosen from matter of daily life, and pupils required to solve them by themselves, so that they may be prepared for practical purposes of everyday need.

(4) On Examination.

Examination was being given heretofore, as explained in Part I, for very good purpose. But pupils were often found to study only for the sake of good marks, and have no interest in the subject-matter itself. Even their parents often took the same attitude. What they take for the gravest

consequence was the result of examination. So when the time for examination drew near, girls had—and were allowed by parents—to make up their daily neglect by abnormal strain of brain. The result of such exertion was, naturally, nothing but faint and casual knowledge of subject-matter and mental and physical weakness. This is a grave question calling forth immediate investigation and remedy.

What seems to be the prevailing opinions on the matter may be summed up as follows:

In schools like the girls' high school, there is no need of fixing the seniority of pupils in a class according to their scholarship. If only discrimination is made between "pass" and "failure," and "pass" classified into "high," "middle," and "low," that would be sufficient for the purpose of home-report.

Regular examinations should be abolished. By carefully observing the pupils' daily progress, by frequently reviewing a part or the whole of the subject taught, and by examining pupils' own exercise-book once in a week or two, the whole object of examination can be satisfactorily gained. Moreover, such a procedure will have the advantages of balanced and steady work on the part of the pupils, such as surer memory, deeper interest, and far steadier progress.

The complete abolition of examination may be unpracticable in some branches of study; but in such subject as mathematics, this can be easily realized, if instructors were willing to make slight extra efforts in their work of teaching.

(5) On the Connection to be made with Elementary School Arithmetic.

In spite of the fact that much of arithmetic had already been taught in the elementary school, this fact was not seldom sufficiently taken into account in teaching mathematics in girls' high schools. Not only was time thus wasted, but it also led to pupils' slighting the subject.

(6) On Calculation.

Calculation is apt to be slighted, and not enough exercise in it given. Very often pupils think that they can easily add, subtract, multiply, or divide, and there is nothing further to be learned in this connection. But the field of calculation is vast, and there are varieties of methods of calculation. Even within the limits of four rules, there are methods like approximation, discussion of errors, etc. which require further study. Therefore, it is best, whenever a chance occurs in practical calculation, to lead pupils on to the first step in advanced calculation. That calculation is usually disliked by pupils as something tedious and dry, is due to the lack of such considerations.

(7) On the Necessity of Preliminary Instruction in Geometry.

The beginnings of a study, no matter what the subject is, are always hard to grasp. But in difficulty of access, no other subject exceeds geometry. In surveying the courses of study already pursued by the pupils of the girls' high school, one finds no trace whatever of geometric figures ; consequently pupils are not at all prepared to learn such things as solid, plane, line, and point in purely geometrical sense. To implant in the brain in such conditions the crystallised issue of hundreds of years of scientific research, is by no means the best pedagogical method. It is no wonder that the first hours of geometry bring to average girls not interest but dislike for the study. The best way to bring about the girls to like the subject is not to push into their brain the crystal, but a more digestible substance by way of preparation.

As a means of such preliminaries, it may be helpful to let pupils draw various geometrical figures, using rules, compasses, protractors, and measures.

Note : If geometrical figures are given in the course of

drawing, such a lesson should be arranged to come before that of geometry.

(8) On the Treatment of Hard and Easy Matters in Mathematics.

It often occurs, unfortunately, that matters taught in school with much pains are completely forgotten and lost sight of in after years, and not even fundamental points remain in memory.

The only remedy for this is to weigh the importance of matters beforehand, and then spend both time and effort in enforcing vital points.

Individually the degrees of difficulty in learning a new thing may vary. But generally speaking, the points which are hard to grasp or are apt to be misunderstood are in common. Notice should, therefore, be taken of such parts, and special studies be made as to the methods of presentation and reviews of such difficult points.

(9) On the Treatment of Pupils differing in Scholarship.

The greatest difficulty experienced in teaching mathematics is the difference in aptitude of pupils. This difference in scholarship is nowhere so marked as in mathematics. If superior ones are to exercise their brain to the full extent, inferior ones are entirely left behind, and are often unable to solve even a single problem, dropping all the while further and further behind the class. On the contrary, if these latter are to be made the central mark of teachers' attention, and time freely used for their sole benefit, the sacrifice of bright pupils and the delay of the work are inevitable.

What is now much in practice in schools is to take medium scholars as the standard, that is, to conduct a class with their comprehension in view. But it necessarily causes

considerable draw-back both for brighter pupils and the dull. How to remedy this is a problem which awaits careful study.

So far there has not been much suggested as to the remedy. Dividing a class in two sections with different work for each section may prove to be helpful; but this requires a great caution, for a girl of high school age is liable to become conceited of their scholarship if they excell, and if not, to despair of her inability.

Also, in keeping incompetent pupils after school hours, though it would be very helpful, a teacher finds an unexpected trouble caused by girls' feeling ashamed of such special treatment.

What is thought to be a comparatively practicable way is to make a collection of suitable problems and examples, and distribute the copies among pupils. Those who have finished ordinary amount of class exercises and must wait for others to finish, can work out the problems given in this book, and for the rest of the class this can be made homework. These supplementary problems and examples are to be solved by the pupils themselves, and to be compared afterwards with correct answers given at the end of the book. By this means, a considerable time can be saved for better pupils, while the duller ones strive for further progress; and the class as a whole will gain practice through coming across with larger number of problems and exercises.

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CONTENTS.

PART I.

ORGANIZATION OF THE INSTRUCTION IN MATHEMATICS AND THE PRESENT ASPECT OF THE METHOD OF TEACHING.

PART II.

MODERN TENDENCIES CONCERNING THE INSTRUCTION IN MATHEMATICS.

PART I.

Organization of the Instruction in Mathematics
and the
Present Aspect of the Method of Teaching.

CHAPTER I.

Kinds of Schools.

I. Aim of the School.

The aim of a normal school is expressed to this effect in the 1st article of the Imperial ordinance issued in 1897, that the normal school is an institution for the training of elementary school teachers. In the 2nd and the 3rd article of the same, it is stated that one or more normal schools may be established in every prefecture, and that they should be under the auspices of the governor of the prefecture.

In the early days of normal schools, as the number of women students was far smaller than that of men, there were no women's normal schools apart from those of men. But owing to the progress in women's education, both the number of those desiring to enter normal schools and the demand for women teachers increased rapidly, and, in consequence, women's normal schools came to be established as independent institutions. The number of these separate women's normal schools at present is thirty againt sixteen of those attached to men's. But as regards the course of study, these two kinds of schools follow exactly the same plan; so they are here treated together under the name of Women's Normal School.

II. Organization of the Women's Normal School.

In the women's normal school, there are two courses, viz.,

the main course and the preparatory course. The main course is again subdivided into the first and the second section. The second section may be left out according to local circumstances.

The course of the first section covers four years, and that of the second section extends over either one or two years.

The preparatory course of one year aims at preparing candidates for the main course.

III. Qualification of the Candidates.

For the first section of the main course.

(*a*) Those who have completed the preparatory course. The conditions for admission into the preparatory are the completion of the higher elementary course of two years, or having scholarship of equivalent standing and being above fourteen years of age.

(*b*) Those who have completed the higher elementary course of three years, or those who are above fifteen years of age and whose scholarship is of equivalent standing.

For the second section of the main course.

Two years' course.

Those who have graduated from the girls' high school of four years' course, or those who are above sixteen years of age and whose scholarship is of equivalent standing.

One year's course.

Those who have graduated from the girls' high school of five years' course, or those who are above seventeen years of age and whose scholarship is of equivalent standing. [Temporarily those who are qualified for entering the above two years' course may also be admitted into this course.]

IV. The Number of Students.

The Imperial ordinance issued in 1897 fixes the number

of normal school students as follows:

"The normal school or schools in a prefecture should admit every year such a number of students as is sufficient to provide the graduates equal in number to one-twentieth of the number of the elementary school classes. Each class is to contain seventy children and there is to be a sufficient number of classes as to contain two-thirds of the number of children of school age in the prefecture. The ratio of the number of male students and that of female students is to be decided by the governor of the prefecture and reported to the Minister of Education. The number of students in one class of the normal school is to be under forty."

CHAPTER II.

Aim and Subject-matter of Instruction in Mathematics.

I. Subjects of Study and Number of Hours of Instruction per Week.

Before going into the aim and subject-matter of instruction in mathematics, it will be necessary to say a few words concerning the relation of mathematics with other subjects of study. The following is the syllabus of subjects and the number of hours of instruction per week according to the regulations issued by the Department of Education and in force since 1908.

(*a*) Subjects given in the preparatory course:
Morals, Japanese language and Chinese classics, mathematics, penmanship, drawing, music, gymnastics, sewing.

(*b*) Subjects given in the first section of the main course:
Morals, pedagogics, Japanese language and Chinese classics, history, geography, mathematics, natural history, physics and chemistry, domestic economy,

sewing, penmanship, drawing, manual work, music, gymnastics. [As an optional subject English language may be added.]

Subjects \ Classes	Preparatory Course	1st Year	2nd Year	3rd Year	4th Year
Morals	2	2	1	1	2
Pedagogics			2	4	Theory 2 Practice 9
Japanese language and Chinese classics	9	6	4	3	2
History		2	2	2	
Geography		2	2	1	
Mathematics	5	3	3	2	2
Natural history		2	2	1	
Physics and chemistry			2	2	4
Domestic economy				2	2
Sewing	4	4	4	4	3
Penmanship	3	2	1	1	
Drawing	2	3	3	3	2
Manual work					
Music	2	2	2	2	1
Gymnastics	4	3	3	3	2
English		(3)	(3)	(3)	(2)
Total	31	31(34)	31(34)	31(34)	32(34)

(*c*) Subjects given in the second section of the main course:

Morals, pedagogics, Japanese language and Chinese classics, mathematics, natural history, physics and chemistry, sewing, drawing, manual work, gymnastics. [In the case of two years' course, history and

geography are to be added, and English language may also be added as an optional subject.]

One Year's Course.

Class / Subjects	1st Year
Morals	2
Pedagogics	Theory 7 Practice 8
Japanese language & Chinese classics ..	3
Mathematics........	3
Natural history......	3
Physics & chemistry	
Sewing	2
Drawing	3
Manual work	
Music	2
Gymnastics	3
Total	34

Two Years' Course.

Classes / Subjects	1st Year	2nd Year
Morals	1	2
Pedagogics	4	Theory 3 Practice 8
Japanese language & Chinese classics ..	5	3
History	2	
Geography	2	
Mathematics........	4	3
Natural history	2	3
Physics & chemistry .		
Sewing	3	3
Drawing	3	2
Manual work		
Music..............	2	1
Gymnastics	3	3
English............	(3)	(3)
Total............	31(34)	31(34)

II. Aim of Instruction in Mathematics.

The fourteenth article of the regulations for normal schools issued in 1907 runs as follows :—

"The object of the teaching of mathematics in normal schools is to make clear to the students the nature of number and quantity, to impart them proficiency in calculation, to acquaint them with the methods of teaching arithmetic in the

elementary school, and, at the same time, to give them general knowledge necessary in daily life and to make them clear and exact in thinking."

Thus both the theoretical aspect and the practical side are equally emphasized. As to the object of the separate branches of mathematics, the following may be mentioned:

(*a*) Arithmetic.

The teaching of arithmetic aims at making clear to the students the synthetic and analytic relations existing between integers, decimals, and fractions, training them in calculation, and acquainting them with the methods of teaching arithmetic in the elementary school. Special emphasis is laid on the skill in calculation, as it is not only useful in everyday life, but also is a necessary qualification of an efficient teacher. Social and economic matters within arithmetic are of special importance for young women who would have very little chance of getting such knowledge later.

(*b*) Algebra.

The object of teaching algebra is to enlarge the students' conception of number, to acquaint them with simple but general solutions of various problems.

(*c*) Geometry.

The aim of teaching geometry is to make clear the real nature of capacity, geometrical figures, and positions, and, at the same time, to develop the power of reasoning and accurate thinking.

III. Syllabuses and Instruction Hours.

Preparatory Course.

Arithmetic .. .. 5 hours per week.

Mental calculation, Written calculation, *Soroban*-calculation.

First Section of the Main Course.

First Year.

Arithmetic and Algebra .. .. .. 3 hours per week.

Integers, Decimals, Fractions.

Negative quantity.

Integral expressions (four operations, equation of the first degree, factor, multiple).

Fractional expressions (reductions, four operations, fractional equation).

Second Year.

Arithmetic, Algebra, and Geometry . . 3 hours per week.

Evolution (square root, equation of the second degree).

Straight line (angle, parallel lines).

Rectilinear figure (triangle, parallelogram, mensuration).

Circle (arc, chord, segment, tangent).

Third Year.

Arithmetic, Algebra, and Geometry . . 2 hours per week.

Proportion (ratio, proportion).

Proportion (proportional lines, similar figures, mensuration).

Progression (arithmetical progression, geometrical progression).

Percentage (percentage, interest).

Methods of teaching arithmetic in the elementary school.

Fourth Year.

Geometry with Arithmetic appended . . 2 hours per week.

Plane (plane and straight line, dihedral angle, solid angle).

Polyhedron (prism, pyramid, mensuration).

Curved surface (circular cylinder, circular cone, sphere, mensuration).

Second Section of the Main Course (two years).

First Year.

Arithmetic, Algebra, and Geometry . . 4 hours per week.

The same as the first and second years of the first section of the main course, with suitable alterations as to the subject-matter and grade of work.

Second Year.

Arithmetic, Algebra, and Geometry . . 3 hours per week.

The same as the third and fourth years of the first section of the main course, with suitable alterations as to the subject-matter and grade of work.

Methods of teaching arithmetic in the elementary school.

Second Section of the Main Course (one year).

First Year.

Arithmetic, Algebra, and Geometry.. 3 hours per week.

The same as the two years' course of the second section of the main course, with suitable alterations as to the subject-matter and grade of work.

Methods of teaching arithmetic in the elementary school.

The above syllabuses which were dictated by the Department of Education in 1910, were accompanied by the instruction that each normal school for women has to prepare a more detailed syllabuses of its own in accordance with the above and so as to suit local circumstances. In fact, arithmetic, algebra, and geometry, all these three branches of mathematics, are being taught in every normal school for women.

In the following are given the essentials agreed upon by most of the normal schools as to the selection of teaching materials in arithmetic, algebra, and geometry respectively.

Arithmetic (Written calculation and *Soroban*-calculation).

(*a*) Arithmetic should be taught for practical use. All theoretical parts are to be left to algebra and geometry, *e.g.*, theory of numbers, theory of ratio, calculation of annuity, progression, square root, cube root to algebra, and mensuration to geometry.

(*b*) In compound numbers, common weights and measures and metric system should be carefully taught. In foreign weights and measures, yard and pound measures, and in foreign money and currencies, those of the countries which

are in close connection with our own, should be taught.

(*c*) In discount, notes and bills, taxes, and insurance, only so much is to be taught as will be necessary in everyday life.

(*d*) *Soroban*-calculation is the method of calculation peculiar to our country, and is very convenient. Therefore students should have practice in this, not only for their own sake, but also for the sake of requirement for elementary school teachers.

(*e*) The origin of number and other psychological study of number are to be given in conjunction with the method of teaching, and even then, only so far as will be desirable in enlightening the student in the principles of teaching.

Algebra.

(*a*) The main object of teaching algebra in women's normal schools should be confined to matters concerning daily life, theoretical explanation of some parts of arithmetic, and its applications. Theory of numbers is to be given only so much as to be of help in comprehending the essentials of arithmetic.

(*b*) Equations are to be taught only so far as the simultaneous equations of the second degree. Equations of higher degrees, except those which can be reduced to equations of the first or the second degree, are not to be taught.

(*c*) The discussion of the roots of an equation is to be confined to simple cases.

(*d*) Logarithm —— Only common logarithm is to be taught for practical use.

(*e*) Progression is to be given without going into details.

(*f*) Probability —— Just a general idea of probability may be given.

Geometry.

(*a*) The object of teaching geometry in women's normal schools is to train the students in theoretical as well as

practical work; so the purely theoretical geometrical methods may very often be replaced by algebraic methods.

(*b*) Problems in loci and construction problems are to be limited to simple cases.

(*c*) Of ratio in geometry, there has been much discussion. The general opinion seems to be not in favour of purely Euclidean definition.

(*d*) Solid geometry is taught only so much as are necessary in connection with mensuration.

IV. Relation Between the Three Branches of Mathematics.

Distribution of Instruction Hours to Each Branch.

The first section of the main course with preparatory course.

Arithmetic	280 hours.
Soroban-calculation	20 ,,
Algebra	200 ,,
Geometry	100 ,,

The first section of the main course without preparatory course.

Arithmetic with *soroban*-calculation	150 hours.
Algebra	150 ,,
Geometry	100 ,,

The second section of the main course (two years).

Arithmetic	100 hours.
Algebra	100 ,,
Geometry	80 ,,

The second section of the main course (one year).

Arithmetic	50 hours.
Algebra	50 ,,
Geometry	20 ,,

Generally speaking, the distribution of recitation hours to each subject is as given above. In actual practice, it is not seldom possible to draw lines of demarkation between different subjects. Sometimes arithmetic is made the principal subject of teaching, and algebra and geometry are taught in conjunction with it. Sometimes algebra is taught as the principal subject and arithmetic as an accessory. But, on the whole, more predominance is given to arithmetic than to any other branch. Below are given a few illustrative examples.

(*a*) Rules and theoretical explanation of the four operations in integers are given parallel to the corresponding operations in algebra.

(*b*) Interest and recurring decimals follow immediately after progression is taught in algebra.

(*c*) After explaining the theory of extraction of square root in algebra, the actual arithmetical operation is given.

(*d*) The theory of the calculation of area and volume is to be given in geometry, algebraic formulae are to be deduced, and thus is to be given the training in applications of algebra and in arithmetical calculation.

CHAPTER III.

EXAMINATION.

Concerning examination, there has been much discussion among teachers of mathematics in women's normal schools. The points requiring careful consideration are when and how often examinations should be carried out, selection of questions, method of giving marks, and, finally, how to make known the results of examinations. The last point and the time when examinations should take place, seem to be of much

concern, particularly in women's normal schools, and special attention is being paid to these considerations. The following give the general aspect of examinations conducted nowadays in women's normal schools.

(1) Object of examination.

(*a*) To test the scholarship of individual student.

(*b*) To test the efficiency of instruction and eventually to improve the method of teaching.

(*c*) To encourage the student for further study.

(*d*) To recapitulate in a connected form matters learned during a certain period of time.

(2) Method of examination.

(*a*) Time and number.

Generally speaking, examinations are being conducted in two ways, occasional and regular.

Occasional examinations are given with or without previous notice when a certain class-work is completed, or when a certain test is needed: *e.g.*, such as test of the proficiency in calculation may be given frequently with or without notice. And generally, examinations in mathematics are given more frequently than in any other subjects.

Regular examinations are given at the end of terms, of which there are three in one school year, and just before graduation. But in some exceptional cases, a schedule of examinations is made out at the beginning of a school year. But even in such cases, students are usually forewarned of the coming examination a week before, and the time for the examination is taken from that of the regular work. The following is one of such schedules adopted in a certain women's normal school during a term in 1910.

Class / Week	3rd Year	2nd Year	1st Year	1st Year (2nd Section)
6th	Pedagogics	Pedagogics	English	Pedagogics
7th	English Mathematics	English	Mathematics	Mathematics
8th	Chinese classics	Chinese classics	Grammar	Chinese classics
9th	Morals	Geography	Chinese classics	Physics Japanese language
10th	Pedagogics Domestic economy	History Pedagogics	History Geography	Pedagogics Geography
11th	History Physics	Physics, Chemistry Natural history	Language Natural history	History
12th	Natural history Geography	Morals	Morals Music	English Natural history
13th	English Mathematics	English Japanese language	Grammar Mathematics	Mathematics Morals
14th	Pedagogics Language	Mathematics	Chinese classics	English Pedagogics

(*b*) Selection of questions.

Questions for term examination are chosen from what have been completed during the term, and those for annual examination from what have been completed during the year. The examination just before graduation covers the whole of the course, but emphasis is laid on the part given during the last term of the last year.

The method usually employed is to present questions to the class and to require written answers within a given limit of time. Oral examination is rarely given even in the case of occasional examination.

In examination of algebra and geometry, the use of reference books is often allowed; and in arithmetic, various tables of numbers are sometimes used.

Examinations in mathematics are usually conducted by the teacher who had to do with the teaching of the subject, and are scarcely ever conducted by the director of the school. Only when government inspectors visit the school, they sometimes wish to test the scholorship of the students. In such a case, the director of the school sometimes himself conducts the examination. But for this, there is no definite time nor any particular method.

As to the selection of questions, the opinions of instructors may vary according to the special needs of the time. Broadly speaking, questions are selected so as to test individual skill in calculation, or to test the power of thinking and memory, or to test the general capacity of a class as a whole.

(*c*) System of marking and treatment of examination papers.

In giving marks to examination papers, some take 100 for full mark, and some simply distinguish four or five grades and mark the papers accordingly.

As for the standard of marking, the following points are taken into account:—

Comprehension of fundamental problems.

Completion of answers.

The power of thinking and its rapidity.

Method of explanation.

Logical expression.

Correctness and rapidity of calculation.

Rough calculation and mental calculation.

Correctness and speed in writing figures, letters, signs, etc.

The treatment of examination papers also varies. In some cases, papers are not returned to the students, but after having been inspected by the director of the school, they are preserved in the school together with the questions given. In other cases, papers are returned to the students witn marks, comments, or advice written on them. This latter method is usually employed in occasional examination. The best papers are shown to the whole class, and errors common to the majority of a class are indicated and criticized in the class.

(*d*) How to make known the results of examinations.

In most of the schools, the results of examinations are not made known lest young girls should become too much given to competition. But individually, each student is informed of a rough idea of her standing and of the points, to which she must thereafter pay special attention.

Under the present regulations, any one, man or woman, who has graduated from a normal school, can, at once, without any further training in teaching work, get the qualification of becoming a regular teacher of an elementary school. The last examination of a normal school serves both for graduation and for qualification as teacher. Hereby it is to be observed, that the result of the practice in teaching work at the attached elementary school is taken account of in determining the result of the last examination.

CHAPTER IV.

Method of Teaching.

I. Arrangement of Teaching Materials.

Teachers of mathematics, as well as those of other subjects, make a detailed plan of work to be used during the year and, this plan having been approved of by the director of the school, follow it in conducting the classes.

Such plans are made by following in the main the official schedule issued by the Department of Education, by keeping in view the contents and arrangement of the text-books adopted, and by following the opinion of the teacher himself, and by suitably distributing the instruction hours.

After having instructed in accordance with such plans, the teacher is required to record in a book every month (in some cases every week), the progress of the work attained during the month. This record serves to be the suggestion for any improvement to be made on the instruction, and is also used as a guide for the next year's plan of work.

As explained before, the revision of the Government regulations for the normal school took place only recently, and there has not been sufficient time for much experience. Such as the teaching of algebra in women's normal schools is quite new. Consequently, the plans of work of these schools are far from being perfect, and need much improvement. In the following is given an example of the plans in use at present. It is that of a women's normal school to which a preparatory course and the second section of one year's course are attached.

The preparatory course, the first, the second, and the third year of the main course have each three terms in one school year.

The fourth year of the main course is divided into four terms instead of three, as the students in this year have to

practice in teaching in the attached elementary school. These four terms are called "short" terms, and during each of them, one third of the fourth year students or the students of the second section are trained in the practice of teaching.

Preparatory Course: Arithmetic.

First term: 5 hours per week (In all, about 65 hours).

Integer and Decimal.

Notation and numeration	1 hour.
Addition, subtraction	6 hours.
Multiplication, division	6 ,,
Practice in calculation	4 ,,
Exercises in four rules	10 ,,

Compound Number.

Meaning of compound number	1 hour.
Metric system	4 hours.
Common weights and measures	3 ,,
Time	1 hour.
Money and currencies	1 ,,
Reduction of compound number	3 hours.
Addition, subtraction	2 ,,
Multiplication, division	2 ,,
Principal foreign weights and measures, and money	2 ,,
Miscellaneous examples	4 ,,

Properties of Number.

Multiple, divisor	3 hours.
Prime number, prime factors	1 hour.
Greatest common divisor	4 hours.
Least common multiple	4 ,,
Miscellaneous examples	2 ,,

Second term: 2 hours per week (In all, about 70 hours).

Fraction.

Notion and nature of fraction	2 hours.

Reductions	3	hours.
Addition, subtraction	4	,,
Multiplication, division	5	,,
Exercises	5	,,
Complex fraction	2	,,
Recurring decimal	3	,,
Miscellaneous examples in fractions	10	,,

Ratio and Proportion.

Ratio	2	hours.
Solution by proportion	2	,,
Problems in simple proportion	5	,,
Problems in compound proportion	5	,,
Chain rule	4	,,
Proportional parts	4	,,
Alligation	4	,,
Miscellaneous problems in proportion	10	,,

Third term: 5 hours per week (In all, about 40 hours).

Percentage.

Percent, percentage, bace amount, difference ..	5	hours.
Discount	4	,,
Taxes	3	,,
Insurance	3	,,

Interest.

Simple interest	6	hours.
Bonds and shares	4	,,
Discount, bank-note	4	,,
Equation of payments	2	,,
Compound interest	4	,,

Supplementary Problems.

Supplementary problems	5	hours.

[*Soroban*-calculation is to be taught in connection with four rules in integers and compound numbers, and practised whenever there is an opportunity.]

First Year of the Main Course: Arithmetic and Algebra.

First term: 3 hours per week (In all, about 39 hours).

Introduction to Algebra.

Meaning of algebra	1 hour.
Signs	1 ,,
Algebraic expressions	1 ,,
Numerical values	2 hours.

Four Rules in Integers and Decimals.

Addition, subtraction	2 hours.
Multiplication, division	2 ,,
Use of bracket	1 hour.
Calculation exercises in four rules	2 hours.

Positive and Negative Quantities.

Meaning of negative quantity	2 hours.
Absolute value	1 hour.
Four rules in positive and negative quantities.	3 hours.

Integral Expressions.

Definition of an integral expression	1 hour.
Arranging integral expression	3 hours.
Addition and subtraction	4 ,,
Use of bracket	2 ,,
Multiplication and division	5 ,,

Linear Equation with One Unknown Quantity.

Meaning of equation	1 hour.
Solution of equation	1 ,,
Exercises	4 hours.

Second term: 3 hours per week (In all, about 42 hours).

Linear Equation with One Unknown Quantity.

(continued)

Applied problems together with arithmetical solution	6 hours.

Simultaneous Equations of the First Degree.

Meaning of simultaneous equations	1 hour.
Solution of simultaneous equations	1 ,,

Exercises	3 hours.
Applied problems	4 ,,
Solution of linear equations with more than one unknown quantity	1 hour.
Exercises	2 hours.
Applied problems	5 ,,
Integral Expressions. (*continued*)	
Formulae of multiplication	3 hours.
Decomposition into factors	2 ,,
Exercises	3 ,,
Properties of Number.	
Multiple, divisor, prime number	1 hour.
Decomposition into prime factors	2 hours.
Greatest common divisor and least common multiple	1 hour.
Applied problems	2 hours.
Integral Expressions. (*continued*)	
Factor, common factor, greatest common factor	1 hour.
How to find greatest common factor	2 hours.

Third term: 3 hours per week (In all, about 24 hours).

Integral Expressions. (*continued*)	
Greatest common factor —— exercises	2 hours.
Least common multiple —— exercises	2 ,,
Fractional Expressions.	
Meaning of fractional expression	1 hour.
Reductions	3 hours.
Addition, subtraction	3 ,,
Multiplication, division	3 ,,
Fraction.	
Exercise in four rules	2 hours.
Applied problems	2 ,,
Fractional Equation.	
Solution of fractional equation	1 hour.

Exercises	2	hours.
Applied problems	3	,,

Second Year of the Main Course: Algebra and Arithmetic.

First term: 2 hours per week (In all, about 26 hours).

Evolution.

Extraction of square root	6	hours.
Extraction of cube root	6	,,

Quadratic Equation.

Solution of quadratic equation	2	hours.
Exercises	3	,,
Resolution of quadratic expression	2	,,
Exercises	3	,,
Miscellaneous exercises	4	,,

Second term: 2 hours per week (In all, about 26 hours).

Relation between the roots and coefficients of quadratic equation	3	hours.
Fractional equation reducible to quadratic equation	5	,,
Simple equations of high degrees..	3	,,
Simple irrational equation	4	,,
Simple simultaneous quadratic equation ..	5	,,
Applied problems in quadratic equation ..	8	,,

Third term: 1 hour per week (In all, 8 hours).

Irrational Expression.

Treatment of simple irrational expression ..	2	hours.
Extension of index law	3	,,
Exercises	3	,,

Second Year of the Main Course: Geometry with Arithmetic.

First term: 1 hour per week (In all, about 13 hours).

Fundamental conceptions	4 hours.
Angle and straight line	6 ,,
Triangle	3 ,,

Second term: 1 hour per week (In all, about 14 hours).

Triangle (*continued*)	6 hours.
Parallelogram	4 ,,
Circle	1 hour.
Chord and arc	3 hours.

Third term: 2 hours per week (In all, about 16 hours).

Segment (of a circle)	3 hours.
Tangent	3 ,,
Inscribed figure, circumscribed figure	2 ,,
Construction problems	4 ,,
Area	4 ,,

Third Year of the Main Couse: Algebra and Arithmetic.

First term: 1 hour per week (In all, about 13 hours).

Ratio and proportion	5 hours.
Equality of three or more ratios	3 ,,
Exercises	5 ,,

Second term: 1 hour per week (In all, about 14 hours).

Progression.

Arithmetical progression	4 hours.
Geometrical progression	4 ,,
Exercises in percentage and interest	6 ,,

Third term: 3 hours per week (In all, about 8 hours).

Method of the teaching of arithmetic	8 hours.

Third Year of the Main Course: Geometry and Arithmetic.

First term: 1 hour per week (In all, about 13 hours).

Introduction to geometrical ratio and proportion	3 hours.
Theorems	4 „
Fundamental theorems	6 „

Second term: 1 hour per week (In all, about 14 hours).

Similar figures	8 hours.
Area	6 „

Third term: 1 hour per week (In all, about 8 hours).

Construction problems	4 hours.
Exercises	4 „

Fourth Year of the Main Course: Geometry and Arithmetic.

Second "short" term for A class / First "short" term for B and C classes — 2 hours per week (In all, about 20 hours).

Plane and straight line	5 hours.
Perpendicular	3 „
Dihedral angle and solid angle	3 „
Polyhedron	5 „
Volume of polyhedron	4 „

Third "short" term for A and B classes / Second "short" term for C class — 1 hour per week (In all, about 20 hours).

Polyhedron, exercises in mensuration	7 hours.
Sphere	3 „
Exercises in mensuration	4 „
Circular cylinder and circular cone	2 „
Exercises in mensuration	4 „

Fourth "short" term for A, B, and C classes: 2 hours per week (In all, about 12 hours).

Exercises in supplementary problems	12 hours.

[Within the hours assigned to exercises in mensuration, is given practical work such as making models and the like.]

Second Section: Arithmetic.

First "short" term: 2 hours per week (In all, about 20 hours).

Integers, decimals and compound numbers ..	15 hours.
Fraction	5 ,,

Second "short" term: 2 hours per week (In all, about 20 hours).

Percentage and interest	7 hours.
Ratio and proportion	9 ,,
Evolution	4 ,,

Third "short" term: 1 hour per week (In all, about 10 hours).

Method of the teaching of arithmetic	10 hours.

[In connection with the method of teaching are given useful advices concerning the use of the State text-books on arithmetic.]

Second Section: Algebra and Geometry.

First "short" term: 1 hour per week (In all, about 10 hours).

Introduction to algebra	1 hour.
Easy four operations	4 hours.
Easy equations	4 ,,
Negative number	2 ,,

Second "short" term: 2 hours per week (In all, about 20 hours).

Four operations	2 hours.

Fractional expression	6 hours.
Equation *(continued)*	10 ,,
Review	2 ,,

Third "short" term: 3 hours per week (In all, about 30 hours).

Angle and straight line	1 hour.
Triangle	2 hours.
Polygon —— quadrilateral, parallelogram, polygon, similar figures, area of rectangle and square	2 ,,
Exercises in extraction of square root ..	1 hour.
Area of rectilinear figures	2 hours.
Miscellaneous problems	2 ,,
Circle —— chord and arc, inscribed figure, circumscribed figure, perimeter and area of circle, ellipse	6 ,,
Solid —— Relation between two planes, relation between a straight line and a plane, regular hexahedron, cube, exercises in extraction of cube root, prism, circular cylinder, pyramid and circular cone, frustum of pyramid and of cone, sphere, miscellaneous problems	14 hours.

II. Method of Teaching.

The method of teaching mathematics usually consists of three steps, namely, preparation, teaching and review. That the teaching very often takes the form of questioning and answering, is only natural in view of the nature of the subject, but sometimes it takes the form very much like a lecture. The characteristic feature in contradistinction to

other subjects, is that very often students themselves explain, prove, and calculate.

The following give the details of the method of teaching.

(*a*) Preliminary preparation.

Precursory to the teaching, students are required to go through preparatory exercises regarding certain matters, of which notice is given beforehand.

Sometimes topics are given and students are required to consider them before the lesson begins. Such is customary in teaching compound numbers and percentage.

(*b*) Preparation.

Theoretical and practical methods already taught, which have relations with the matters to be taught presently, are reviewed and discussed. In arithmetic and algebra particularly, mental calculation is practised, and efforts are made so as to make the mind of students adapted to receiving the instruction to be forthwith given.

(*c*) Teaching.

Presentation of the subject-matter.

Explanation is done by questioning and letting students answer; or method of calculation is taught.

In teaching a theorem, first of all it is enunciated clearly, then its meaning explained by means of examples or diagram, and then proved.

In teaching a method of calculation, simple examples are taken and analyzed, and the steps of calculation explained fully; and then they are put together and given as a rule.

Matters already taught and those newly taught are compared, and the mutual relation is made to be understood clearly.

Propositions are classified according to the nature of theorems and rules, or according to the convenience of reference.

Students are required to point out important points and to read aloud the text-book, whereby proofs and explanations

are omitted.

Questions on difficult points are freely asked by students and answered by other students of the class, the teacher's explanation always coming last.

Students are to have memorandums. The parts which are hard to remember or apt to be mistaken, also the parts to which special attention is to be paid, and useful advices concerning the actual process of calculation, are to be recorded in the memorandum.

(*d*) Exercises.

General direction is given by the teacher; then, by means of questioning and answering, students are led to grasp the theory and practice; after that, students are required to find the solution independently.

Problems which can be solved with the knowledge of the rules already learned, are to be solved by a student. The rest of the class criticize, and correct it, if there be errors.

While students work on an exercise, the teacher inspects them and gives suggestions and advices to each individual. When various different answers were found for one and the same problem, they are shown to the class and discussed.

At times, a problem is solved entirely unassisted by the teacher. The papers or note-books are handed in, and corrected afterwards.

(*e*) Home work.

A suitable amount of home-work, not so much as will prevent the preparation for other subjects, is always given for further study and exercise.

The revision of home-work is done by the teacher sometimes in the class-room, and sometimes outside, by looking over the note-books.

III. Points Requiring Special Care in Teaching.

(*a*) In mathematics, the difference of aptitude among the students, is shown most distinctly; therefore great care should

be bestowed on this point in a class work.

(*b*) In the preparatory course of the normal school, the aim of teaching mathematics is to make more compact the knowledge of arithmetic, and to train the students in its practice. In the main course, three branches of elementary mathematics should be taught, due attention being paid to the correlations existing between them.

(*c*) *Soroban*-calculation in four operations is to be taught side by side with the figuring processes.

In the following are given some points to be particularly observed in connection with each individual branches.

Concerning arithmetic :

(*a*) Students are apt to make mistakes in calculation, because of the carelessness in writing down figures. For this reason, practice in writing figures should be given in the first year and the students should be accustomed to writing figures with ease and rapidity.

(*b*) As mental calculation is of great help for becoming skilled in calculation, mental exercises should be given during the first year either at the beginning or at the end of every lesson. Rapid calculation on *soroban* should also be practiced.

(*c*) In solving a problem, it is not enough to find only one solution. Emphasis should be laid on finding various different solutions and also on selecting from among them those suitable for the elementary school children.

(*d*) Purely theoretical reasoning is to be avoided as far as possible, and problems should be solved by considerations which appeal to common sense and ordinary understanding.

(*e*) In teaching weights and measures, bonds and shares, bank-note, and money-order, models or the things themselves should be shown, and efforts should be made so as to make the students come in contact with such matters as often as

possible.

(*f*) Such rough calculation and measuring as are done by taking steps, by observation, and by feeling, should be taught in the first year in continuation of the similar exercises taught in the elementary school.

Concerning algebra :

(*a*) Algebra given in women's normal schools should not be too far advanced, its aim being to simplify solutions of theoretical and practical problems which occur in arithmetic. Efforts should be made to make clear the connections and correlations with arithmetic.

(*b*) For those who study algebra for the first time, there is much more difficulty than the teacher usually expects. Therefore it is advisable to give them at the start arithmetical problems, and let them find the solution, then replace numerical values by letters and thus find algebraic formulae.

(*c*) Algebraical signs including those which occur in arithmetic, but are used in a somewhat different manner in algebra, should be carefully explained at the very beginning. For example, if the sign of multiplication in 3×4 be omitted, the result would appear as if it represents 34; but in the case of letters, there is no such trouble. Such peculiarities are to be particularly made aware of.

(*d*) Beginners are liable to find difficulty in understanding that a letter always represents an abstract number; so special attention is to be paid to this point.

(*e*) In algebra, as in arithmetic, mental calculation is to be encouraged.

(*f*) Important formulae should be committed to memory, so as to be ready for use whenever needed.

(*g*) In teaching equation, it is not necessary to collect together every thing pertaining to equation in one place. Easy equations may be given comparatively earlier. By so doing, students' interest in algebra is very often roused.

Concerning geometry:

(*a*) The early stage of geometry should be taught inductively by appealing to experiments and observations. Strictly logical training should be postponed till later; it will suffice, at the start, to give general geometrical conceptions, and then gradually proceed to more logical expositions.

(*b*) For those who have not as yet learned logic, or those who have learned logic as a part of pedagogics, but not as yet been trained in logical exercises, it will be necessary to give hints in making them prove theorems and solve problems.

(*c*) In case several theorems can be put together in the form of one theorem, they are to be done so in order to make it easy to remember, *e.g.*, such theorems as "In two triangles, if three sides, two sides and included angle, or one side and two adjacent angles, are equal, then the two triangles are equal."

(*d*) Similar theorems should be compared and contrasted, and made suitable for remembering, *e.g.*, such as the theorems concerning triangles and similar triangles.

(*e*) In proving a theorem or solving a problem, too much adherence to a particular diagram or figure in thinking out proof or solution, should be avoided.

(*f*) In connection with arithmetic, easy practice in measuring distance, area, volume by observation and also easy surveying should be practised; also with the help of the lessons in manual work, simple machines and models should be planned and made by the students.

Supplementary notes:

(*a*) Logical training —— Logical training is being given in connection with the teaching of geometry.

(*b*) Applied problems —— The selection of applied problems is one of the difficult tasks of the teacher. If necessary, the teacher modifies or alters the problems given in the text-

book, or himself makes problems which suit local conditions and the students' grade of intellectual development, and also problems which have to do with current events of the day. Sometimes problems are pointed, and copies distributed among the students.

Moreover, in training students for elementary school teachers, it is not wise to supply them always with problems ready made; aided by certain suggestions, they should be made accustomed to make problems by themselves.

(*c*) Method of exercise.

(i) Class-work.

Typical applied problems are given as class-work, and students are required to discuss and criticize one another's work. Especially, the students are expected to be able to explain clearly those problems which are given in the State text-books for the elementary school.

(ii) Home-work.

Typical applied problems or other important problems which the students were not able to solve in the class, as well as home-work.

As most students of women's normal schools are living in the dormitory, home-work in any subject can be conveniently inspected and necessary guidance given.

Both class-work and home-work are written down in the exercise-books. The teacher inspects them from time to time, and gives suggestions to each student as to proper manner of writing down, cleanliness, orderliness, etc. These exercise-books serve, in many cases, as a necessary means of individual instruction.

(iii) Exercise on the black-board.

In many schools, exercise on the black-board is looked upon as an efficient means of developing the

students' self-reliance, and in giving necessary training for becoming teachers.

IV. Instruments for Instruction.

The necessity of using specimens, hanging diagrams, models, etc. in teaching mathematics is obvious, but has only recently been sufficiently recognized by the teachers, and consequently there is not as yet sufficient equipment made for them. However, such as the following instruments can be seen in any of the women's normal schools nowadays.

Set of geometrical models.

Counting table, table of number.

Models and samples to be used in teaching percentage in arithmetic, such as samples of bonds, notes, receipts, etc.

Weights and measures, diagrams of coins.

Models of capacity measures and weights of principal foreign countries.

Drawing instruments.

Large *soroban* for class use.

Simple surveying instruments.

V. Text-books.

There are many text-books for use in women's normal schools, which were compiled by experienced teachers and others. From among those which were examined and approved of by the Department of Education, are chosen the text-books to be used and adopted, subject to the sanction of the governor of the prefecture.

VI. Reference Books and Collections of Problems and Exercises.

In schools where there is a library, the following books

are supplied for the use of the students:

Text-books other than those used in the school, books of similar contents.

Reference books written in expository form.

Books on the method of teaching arithmetic.

Mathematical magazines.

Scientific magazines.

Collections of problems and exercises.

Where there is no library, sometimes students are informed of the names of certain text-books, and told how to study them. Again, sometimes collections of exercises are recommended for private study. In all cases, however, the use of collections of examples with keys or with notes and explanations, and still more mischievous books known here in Japan by the name of "mathematical dictionaries," wherein examples with solutions are categorically given, should be deprecated. It will be difficult to over-estimate the harm and mischief done by such books to the real progress of the students.

CHAPTER V.

Preparation of Teachers.

Teachers of mathematics in women's normal schools are chiefly recruited from among the graduates of the higher normal schools for women. Besides, any one who has the qualification of becoming a teacher in the men's normal school, may also become a teacher in the women's normal school.

Appendix.

Conception of Numbers given to Children in Kindergartens.

In the new regulations for the normal school, the estab-

lishing of kindergartens attached to women's normal schools is particularly encouraged. But as yet, there are some normal schools where no accomodation is made for the training of kindergarten teachers. In some schools, this training is being given to the students in public kindergartens. So, although there are about five hundred kindergartens in Japan, those attached to normal schools count only fourteen.

As kindergartens have intimate relations with the women's normal school, a few facts about childrens' conception of numbers are added here as an appendix.

The work of a kindergarten is to bring up children from three years of age to their elementary school age.

The chief subjects in a kindergarten are sports and recreation, singing, conversation and manual work, all aiming at giving suitable physical and mental development to the children and to supplement what is lacking in home education. Therefore in kindergartens, the teaching of any difficult or complex matters is to be absolutely avoided.

Accordingly, any sort of instruction on geometrical figures, or giving them conception of numbers is not practicable; but when any numbers occur in plays, songs, etc., the chance is always taken advantage of for arousing their interest in counting. Usually they begin to have interest in counting from about four years of age.

There were found some children who could count abstract numbers up to forty-nine; but these were only a few out of the total of forty. It was interesting to observe that none of them could count up to fifty, which is only one more than forty-nine.

Of concrete numbers, only those that are necessary in children themselves are taught.

In manual exercise are used such as the following which fit the state of mental development and adapted for arousing interest of children.

Six spherical balls.

Blocks —— These are divided into the following three sets and distributed among classes.

(i) 4 cubes, 4 oblong blocks;

(ii) 4 cubes, 4 oblong blocks, 4 blocks;

(iii) 4 cubes, 4 oblong blocks, 4 blocks, 4 large triangular prisms, 8 small triangular prisms, 4 quadrantal prisms.

Tablet-laying —— The following kinds are used.

Square, one side white and the other side red;

Isosceles triangle, one side red and the other side green;

Non-isosceles right-angled triangle, both sides green;

Isosceles oblique triangle, one side reddish brown and the other side blue;

Equilateral triangle, one side yellow and the other side blue.

Stick-laying.

The sticks are made either of wood or of metal; the length varies from half an inch to three inches.

Ring-laying.

The rings are made of metal, and there are three kinds as regards their size. Besides, semi-circles and quadrants are also used.

Besides, all sorts of plays and recreations are given. The terms used for these playthings are derived from the shapes of objects that are familiar to the children. Sometimes chanting of euphonic phrases giving useful information are practised.

PART II.

Modern Tendencies Concerning the Instruction in Mathematics.

CHAPTER I.

MODERN TENDENCIES CONCERNING THE ORGANIZATION OF SCHOOLS.

It goes without saying that the normal school for preparing teachers for elementary schools, should be inseparably connected with the elementary education. Moreover, normal schools should be so organized as to meet the demand which the public makes for the elementary school teacher. The regulations for the normal school were accordingly revised and the organization of the school altered.

The instructions issued four years ago by the Department of Education, which accompany the revised regulations, show clearly the modern tendencies regarding the organization of the normal school. The following is an extract from the said instructions.

"With modern progress in education, much need is felt of improving the regulations for the normal school. Especially, the demand for competent elementary school teachers increased of late, after the prolongation of our compulsory education. These are the reasons for revising the old regulations. In the following are given the main object of the revision and some vital points on which special attention is to be paid in putting the revised regulations into practice.

"As for the courses in the normal school, the course

hitherto known as "easy course" is abolished, and the main course is subdivided into the first and the second section. In the first section, the period of study for women students is the same as that for men, namely, four years; and the period of study in the preparatory course is one year. In spite of the fact that women graduates were put in the same professional position as men, there has been, hitherto, a difference in the period of study of men and women students. This was an inevitable outcome of the slowness in progress of women's education in former days. But the present day has seen the raising of the intellectual standard of women, and the demand for women teachers is rapidly increasing. Consequently, in this present revision, the period of study for women students is made just the same as that for men."

"In the second section, the candidates are to be chiefly recruited from among the graduates of boys' middle schools or girls' high schools; and giving them, after admission, one or two years' training for elementary school teachers, it is hoped that they will become qualified as well as the graduates of the first section. Formerly, there have been many graduates of various schools who have gone into the teaching profession, but their lack of training for teaching was too often noticed. Of late, there have been, in some places, short courses of training attached to normal schools, but time being too short, they have not proved to be successful. To remove this difficulty, the second section is newly added to the main course."

"But such great demand for elementary school teachers as we are now experiencing, can not be supplied all at once; therefore it is advisable to work on, with the second section, steadily but not in too much hurry, for such a hurry in trying to get a large number of graduates, may necessarily cause to slight the first section, and so lower the standard of the properly qualified elementary school teacher."

"In case there is not a sufficient number of men and women candidates for two separate classes of the second section every year, it may be well to form men's class one year, and women's class the next year, and so on alternately, or the second section one year, and the short training course the next year, and so on alternately. At any rate, it is important that efforts should be made to enroll in the second section all those who do not enter the main course and yet desire to become elementary school teachers."

"To admit the graduates of elementary schools straight to the normal school is only a right course to take, judging from the nature of work given in those two kinds of schools. Moreover, this will greatly serve for the purpose of inducing many brilliant students to go into the normal school. These are the reasons for admitting to the main course those who have graduated from a higher elementary school of three years' course, and to the preparatory course those who have graduated from a higher elementary school of two years' course. And as there are not as yet many higher elementary schools with three years' course, governors of prefectures should encourage the establishment of the preparatory course."

"The supplementary courses for the training of elementary school teachers have for their purposes the giving of necessary additional training to those who are already qualified as such."

"After the revision of the regulations for the elementary school, those who have formerly been qualified as teachers of the lower grade, would find their scholarship not quite equal to the new requirements. For the benefit of such teachers particularly, supplementary courses should be opened at the earliest convenience."

"As for the change in the subjects of study, some explanations may be necessary. For men students, the new curriculum contains a new subject, "essence of law and economics," because of the great need of such knowledge in

the present day life. Again, for both men and women, manual work is also added. English is to be included in the curriculum of men's normal schools, and may be included in that of women's; in both cases, however, it is made optional."

"The essence of law and economics may be omitted for the present. There are several things requiring preparation in putting the revised regulations into force, and some more instructions will be given later." [The further continuation of the instructions is omitted here.]

As a whole, the main object of this revision of regulations consists in improving the work of the school which is the source of the elementary school education, in raising the qualifications of the directors and teachers of the elementary school, and in trying to induce as many men and women as possible to go into teaching profession in order to supply the want due to the insufficiency of regular graduates from normal schools. The schools are required to make earnest endeavours, within the limit of their available means, to train the greatest possible number of students in view of want of elementary school teachers most keenly felt nowadays. The number of students who pay their own expense and women students is to be increased as much as is compatible with attendant circumstances. In so far as, not too much strain is put on the work of teachers, day and night instructions may be encouraged. If it is necessary for the purpose of getting rooms, some of the students may be required to lodge out of the school. Such are some of the means to this end. Moreover, as the revision of regulations does not necessarily mean the actual improvement in education, the governor of a prefecture and the educators should make their combined efforts in realizing the work thus planned.

CHAPTER II.

Current Opinion on the Aim and Subject-matter of Instruction in Mathematics.

The most important branches of mathematics for the normal school education are undoubtedly arithmetic, algebra, and geometry. What are now being given in our women's normal schools are just these three, algebra having been newly added.

As a matter of course, a teacher must know a great deal more than what he teaches. Moreover, the teacher's personal interest in the study increases the pupil's interest considerably. For these reasons, those who aspire for becoming a successful teacher of arithmetic should have the knowledge of algebra and geometry.

Utilitarianism, too much prevalent nowadays, has found its way even into the corner of the normal school education. Too much attention being paid to the actual teaching in the elementary school, there is a tendency to overlook the importance of stimulating the student's interest in the subjects taught and of their acquiring the capacity of freely applying what they have learned. Keeping such considerations in view, it is ventured to suggest the following plan as giving the necessary subject-matters and minimum amount of mathematical instruction in the normal school.

Arithmetic —— Addition, Subtraction, Multiplication, and Division in Integers and Decimals; Fractions; Proportion; Percentage and Interest; Approximate calculation and Error; Square root and Cube root; *Soroban*-calculation (addition, subtraction, multiplication, and division).

Algebra —— Four Operations; Integral expressions; Fractional expressions; Equations of the first and second degrees; Simultaneous equations of the first and second degrees; Square root and Cube root; Logarithms.

Geometry —— Angle ; rectilinear figures ; circle ; area ; Proportion ; Locus ; Mensuration of plane figures and solids.

For a long time, there had been much discussion as to when decimals in arithmetic should be taught. And now the question has been finally answered to the effect that decimals should be taught simultaneously with integers.

CHAPTER III.

Examination.

Until lately, women as well as men students, had to pass a strict examination at the end of each term and each school year. But there came up a question as to whether this method was advisable for women students. Just then, in 1900, the Department of Education issued an instruction to the effect that the regular system of examination should be abolished in the women's normal school and girls' high school, where the students are in a critical stage of mental and physical development.

Since then, in all women's normal schools, the students' scholarship has been judged by the daily work. And this method seems to be particularly suited to mathematics, more than to any other subjects.

But after having had nearly ten years' experience in the above mentioned method, some teachers are now of the opinion that regular examinations are of much value and that the disadvantages of such examinations may be removed by improving the method of examining.

At present, womens' normal schools are giving stricter examinations than girls' high schools, for the following reasons :

(i) As the students of women's normal schools are to become future elementary school teachers, they are chosen by considering their physical strength and mental ability. Therefore, the normal school is more responsible than the girls' high school, for their intellectual development; and so it is necessary to know the exact condition of the students' progress and to direct the future course of instruction accordingly.

(ii) The habit of exertion and of hard work should be encouraged.

(iii) The physical welfare of the students may be guarded in a normal school, as they all live in the dormitory.

CHAPTER IV.

Method of Teaching.

I. Method of Teaching.

In spite of the recent progress and development of mathematics, and notwithstanding that everything in daily life is becoming more and more exact and mathematical, the work of teaching mathematics in the elementary and normal schools seems to have made but little progress. The blame may be put upon the unsatisfactory method of teaching, which does not seem to keep pace with the progress of mathematics. Though the teachers in women's normal schools have noticed this and endeavoured to improve the method of teaching, the result thus far has not been very satisfactory. Perhaps, in teaching mathematics to girls the teacher is apt to give too much importance to the psychological side of the work at the expense of endangering the basis of mathe-

matical instruction. For though it is necessary to use intuitive method at the start of the teaching of theoretical matters, it should not be kept on to the last. It is in infancy only and not in childhood that liquid should be used for daily food; for the same reason, it will be far better, after the girls have become sufficiently versed and interested in algebra or geometry, to follow rigorous methods.

II. Relation Between Different Branches of Mathematics.

There are various ways of grouping different branches of mathematics. We will notice only two of them, viz., in one, arithmetic is made the principal branch with other branches subordinate to it, and in the other, algebra is taken for the principal subject and other branches are associated with it as accessories.

Arithmetic taken as the principal subject, has the following advantages:

- (*a*) Direct connection with the elementary school arithmetic.
- (*b*) Convenience in teaching arithmetic after graduation.
- (*c*) Arithmetical calculations are of daily need.
- (*d*) Inductive method based on numerical examples is clear and easy to comprehend.

Algebra taken as the principal subject, has the following advantages:

- (*a*) Algebraic way of thinking and reasoning is necessary in the study of all other branches.
- (*b*) Algebraic operations admit of general application.
- (*c*) Algebraic methods are general and admit of wide application.

Arithmetic taken as the principal subject, has the following disadvantages:

(*a*) Relation with other branches is not very close.

(*b*) Application to other branches is limited.

(*c*) Method of reasoning is not general and often too circuitous.

Algebra taken as the principal branch, has the following disadvantages.

(*a*) It is too far removed from the elementary school arithmetic.

(*b*) Algebra is difficult for beginners to understand.

As the result of the above comparison, arithmetic taken as the principal branch, is largely in favour among women's normal schools. In these schools, although theoretical training is one of the objects in view, still the chief aim is to give sufficient mathematical common sense, and to make the students comprehend the method of teaching elementary school arithmetic.

III. Relation Between Mathematics and Other Subjects of Study.

(*a*) Between Mathematics, and Physics and Chemistry.

The closest relation exists between mathematics and these two branches of science. In physics, particularly in dynamics, optics, electricity, and in chemistry, particularly in the general theory, mathematics is largely used.

The study of chemistry and physics does not suffice with qualitative exposition, but should go on to quantitative measurement. It will be necessary to give a large number of problems requiring numerical calculation in the course of physics and chemistry. It will show to the student the utility of mathematics and at the same time will conduce to rousing fresh vigour for the further study of mathematics. In mathematics too, physical and chemical problems should be inserted here and there. Efforts should not be spared

to make progress side by side in these mutually dependent subjects of study.

(*b*) Between Mathematics, and Drawing and Manual Work.

Drawing is particulary related to constructions in geometry. The teachers of these two subjects should combine their efforts in order to make the instruction effective. This seems to have been noticed but recently, and not much has been accomplished in this direction as yet.

Relation between mathematics and manual work can only be established through the intervention of mechanical drawing. The diagram made in drawing may be proved and explained in geometry, then made over into a model in practising manual work. Such practice will arouse the pupils' interest, and, at the same time, give a sure knowledge of the matter. Therefore in geometry, the hours of practical work should be combined with those of manual work in order to utilize the time for both subjects. The introduction of manual work in the women's normal school is comparatively new, and we have not as yet had much experience.

(*c*) Between Mathematics and Domestic Economy.

Much importance is given to the course of domestic economy, or house-keeping and a large number of instruction hours is usually allotted to this subject. But in women's normal schools, there being so many subjects of importance, only a short space of time is given to domestic economy in the third and fourth years. The reason for this may be that it is entirely an applied study, and fundamental studies must come first of all. Now such subject as household economy should make progress along with mathematics; and on this much attention is being paid. In arithmetic too, something of economics should be taught, and some problems and exercises be made from matters relating to household affairs.

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CONTENTS.

FIRST PART.

Present State of the Organization and the Methods of Mathematical Instruction.

SECOND PART.

Modern Tendencies of the Teaching of Mathematics.

FIRST PART.

Present State of the Organization and the Methods of Mathematical Instruction.

CHAPTER I.

Historical Outline.

After a wide observation of the progress of the world, and realizing that the education of women should not be neglected, Fujimaro Tanaka, then Vice Minister of Education, asked for the sanction of the Prime Minister and founded the Tokio Normal School for Women on March 3, 1874. This was the beginning of the School.

In April 1876, the Normal Course was called the Regular Course and the preparatory work was first given separately.

In August 1883, the preparatory department was discontinued and the Girls' High School was organized.

In August 1885, the School was united with the Tokio Normal School, (now Higher Normal School) and called the ,, Women's Department of the Tokio Normal School."

In February 1886, the Accessory Girls' High School was separated from the School and named the "Tokio Girls' High School."

In April 1887, according to Normal School laws promulgated by Imperial Ordinances, the "Tokio Normal School" became the "Higher Normal School."

In March 1890, the "Girls' Department of the Higher Normal School" was set apart and became the "Higher Normal School for Women," including the "Tokio Girls' High School," the "Accessory Kindergarten of the Higher Normal School" and also the Elementary School which had then been established in connection with it.

In December 1897, according to ordinances promulgated by the Department of Education, the school curriculum was revised, and the whole course was divided into two courses—Literary and Scientific.

In February 1899, a new course, the Art Course, was added.

Besides the three courses mentioned above, with the purpose of supplying, when necessary, teachers for kindergartens and intermediate schools, a Training Department for Kindergarten Teachers and special courses in Household Affairs, in the Japanese Classics, in Geography and History, in Mathematics, Physics and Chemistry were to be opened. Of these, the special courses in Japanese Classics and Gymnastics, and in Household Affairs are now established.

In March 1908, to distinguish the school from a sister school which was then established at Nara and named "The Nara Higher Normal School for Women," the school was named "The Tokio Higher Normal School for Women."

In March 1910, each course of The Tokio Higher Normal School was subdivided into the first and second sections.

CHAPTER II.

Constitutions of the Schools.

The Higher Normal School for Women is the school where those persons who will become teachers in normal schools for women or girls' high schools are instructed, and is under the administration of the Minister of Education.

Those admitted are selected from among the graduates of the normal schools for women, from the women's departments of normal schools, and from girls' high schools having a four years course. They must have good health and good morals. They must also obtain their nominations from the prefectural governors, through the entrance examinations held

by the head-master of the school. The applicants must be between 17 and 22 years of age and not married. The average ages of the students at the beginning of the new school year are as follows:—

	Preparatory class	1st year class	2nd year class	3rd year class	4th year class
Tokio	—	19 Y. 1 M.	20 Y. 10 M.	21 Y. 9 M.	22 Y. 9 M.
Nara	18 Y. 10 M.	19 Y. 6 M.	20 Y. 10 M.	—	—

The courses cover four years (In the Nara School, four months of this time is given to the preparatory course) and are as follows:— in the Tokio School there are the courses of Literature, of Science, and of Art, each of which is divided into two sections—first and second sections—and in the Nara School there are the course of the Japanese and Chinese Classics, the course of Geography and History, the course of Mathematics, Physics, and Chemistry, and the course of Natural Philosophy and Household Affairs.

Mathematics is taught in both sections of the scientific course of the Tokio School, and in the preparatory course and the course of Mathematics, Physics and Chemistry in the Nara School.

CHAPTER III.

Aim and Subject-Matter of Mathematical Instruction.

The purpose of the course is to give the students mathematical knowledge and ability necessary for becoming teachers of mathematics in normal schools for women or girls' high schools and to cultivate the mathematical knowledge essential to the study of physics and chemistry as well as to give the necessary equipment for the study of higher mathematics. Therefore, the aim of the instruction is not only to train the student in logical thinking but also in the application of the knowledge to daily affairs and scientific study.

Subject-Matter of Instruction.

A. Tokio Higher Normal School for Women.

As was determined in April 1908, the mathematics in the Science Course comprised arithmetic, algebra, geometry (with geometrical drawing) and trigonometry. The course is as follows:—

year	1st	2nd	3rd	4th
hours per week	4	3	3	3
Subjects	Arithmetic, Algebra, Geometry.	Algebra, Geometry with Geometrical drawing.	Algebra, Geometry with Geometrical drawing.	Algebra, Trigonometry.

According to the revised regulation, issued in March 1910, the courses in mathematics, in both the first and second sections, comprise arithmetic, algebra, geometry with geometrical drawing, trigonometry, theory of equations, analytical geometry and the rudiments of calculus. They are distributed through the course as given below. In the second section, no exercise is given.

School year	Hours per week	Term	Subjects
1st	5	1st	Arithmetic and Algebra 3, Geometry 2, Exercise (1).
		2nd	Ditto
		3rd	Ditto with geometrical drawing.
2nd	4	1st	Algebra 2, Geometry 2, Exercise (1).
		2nd	Ditto with projective drawing.
		3rd	Algebra 2, Modern geometry 2, Exercise (1).

School year	Hours per week	Term	Subjects
3rd	6	1st	Algebra 2, Modern geometry with perspective drawing 2, Exercise (1).
		2nd	Theory of equations 2, Trigonometry 2, Analytical geometry 2, Exercise (1).
		3rd	Ditto
4th	5	1st	Analytical geometry 1, Rudiments of differential and integral calculus 4, Exercise (1).
		2nd	Ditto and methods of teaching.
		3rd	Teaching practice.

I. Arithmetic and Algebra.

Fundamental Laws.

The Number System; Numeration and Notation; Laws of Association and Commutation for Addition and Subtraction; Laws of Commutation, Association and Distribution for Multiplication and Division; Laws of Indices; Negative Number; Law of Signs; Numerical Values of Algebraic Expressions.

Polynomial Expressions.

Addition and Subtraction of Polynomials; Brackets; Product of two Polynomials; Continued Products; Division by a Polynomial.

Numerical Equations.

Equations with one Unknown Quantity; Equations with two or more Unknown Quantities; Arithmetical Solution of Problems.

Factors.

Monomial Factors; Factors found by comparing with Known Identities; Factors of Numbers; Applications of Factoring.

Highest Common Factors and Lowest Common Multiples.

Monomial Common Factors; Polynomial Common

Factors; Common Divisors of Numbers; Lowest Common Multiples of Monomials and Polynomials; Least Common Multiples of Numbers; Applications to Problems.

Fractions.

Reduction of Fractions to their Lowest Terms; Reduction of Fractions to a Common Denominator; Addition and Subtraction of Fractions; Multiplication and Division of Fractions; Theorems concerning Fractions; Numerical Values of Fractions; Arithmetical Solution of Problems.

Equations (continued).

Equations with Fractions; Simultaneous Equations with Fractions; Literal Linear Equations; Solution of Problems.

Powers and Roots.

Laws of Indices; Roots of Monomials; Square Roots of Polynomials; Square Roots of Numbers; Cube Roots of Polynomials; Cube Roots of Numbers; Rational and Irrational Numbers; Fractional and Negative Indices.

Decimal Approximations.

Approximate Operations with Numbers of Sufficient Decimals; Errors; Limits of Errors; Operations with Approximate Numbers.

Quadratic Equations.

Solution of a Quadratic Equation; Discriminant; Imaginary Numbers; Graphical Representation of Numbers; Continuity of Numbers; Solution of the General Quadratic Equation; Trinomials; Solution of Problems.

Equations which can be solved like Quadratics.

Equations Higher than the Second Degree; Equations with Fractions; Irrational Equations; Simultaneous Equations of Second Degree; Solution of Problems.

Ratio and Proportion.

Ratio of Numbers; Ratio of Magnitudes; Properties of Ratios; Properties of Proportions; Theorems; Direct and Indirect Variations; Theorems on Variations; Solution of Problems; Arithmetical Solution of Problems.

Arithmetical and Geometrical Progressions.

Arithmetical Progressions; Arithmetic Means; Theorems; Geometrical Progressions; Geometric Means; Infinite Geometrical Progressions; Arithmetic Solution of Problems in Percentage and Interest: Compound Interest and Annuities.

Numerical Calculations.

Short Methods; Calculating Machines; Use of Mathematical Tables; Logarithmic Tables.

II. ALGEBRA.

Integral Expressions.

Four Fundamental Operations; Rational Integral Functions of one Variable; Method of Detached Coefficients; Homogeneous Expressions; Symmetrical Expressions; Method of Undetermined Multiplier; Some Identities; Remainder Theorem; Factoring by means of Remainder Theorem.

Rational Fractions.

General Properties regarding Fractions; Examples of Operations with Rational Fractions; Expression of any Integer in any Scale of Notation; Radical Fractions; Rule for casting out the Nines.

Irrational Functions.

Index Laws extended; Examples of Operations with Irrational Forms; Rationalizing Factors.

Surds and Complex Numbers.

Algebraic and Arithmetical Irrationality; Theorems of Surds; Square Roots of Surds; Theorems on Complex Numbers; Rational Operations with Complex Numbers; Conjugate Complex Numbers and Modulus.

Solution and Discussion of Equations.

Equivalence of Equations; Solution and Discussion of Linear Equations; Graphical Discussions; Solution and Discussion of the General Quadratic Equation; Solution and

Discussion of Equations which can be solved by means of Quadratic Equations; Solution of Arithmetical and Geometrical Problems.

Inequalities.

Fundamental Principles of Inequalities; Solution of Conditional Inequalities; Theorems on Identical Inequalities; Maxima and Minima.

Permutations and Combinations.

r—Permutation of n Things; r—Permutation with Repetition; r—Combination of n Things; Vandermond's Theorem; Combinations with Repetition; Mathematical Induction; Examples of Permutations and Combinations.

Binomial and Multinomial Theorems for a Positive Integral Index.

Binomial Expansion; Greatest Coefficient and Greatest Term of the Expansion; Some Relations between the Coefficients of a Binomial Expansion; Greatest Coefficient of a Multinomial Expansion.

Summation of Series.

Summation of some Typical Series; Figurate Numbers; Polygonal Numbers; Method of Differences; Interpolation.

Determinants.

Definition; Elementary Properties; Simultaneous Equations of First Degree; Elimination; Sylvester's Method of Elimination.

Probability.

Definition; Direct Calculations; Mutually Exclusive Events; Mutually Independent Events; Compound Events; Inverse Probability; Expectation; Mortality Tables; Life Annuity; Life Insurance.

Indeterminate Equations.

Congruences; Power Rests; Circulating Decimals; Diophantine Analysis.

Indeterminate Equations of Second Degree.

Wilson's Theorem; Quadratic Rests; Pythagolian

Problem; Fermat's Theorem; Resolution of Composite Numbers into Prime Factors.

Continued Fractions.

Convergents; Law of Formation of Convergents; Reduction of any Rational Fraction to a Continued Fraction; Properties of Convergents; Reduction of Quadratic Surds to Continued Fractions.

Convergency and Divergency of Infinite Series.

Definitions of Convergency and Divergency; Necessary and Sufficient Conditions for Convergency and Divergency; Convergency and Divergency of Series whose Terms are all Positive; Binomial, Exponential, Trigonometric and Logarithmic Series.

Logarithms.

Exponential Theorem; Properties of Logarithms; Logarithmic Series; Calculation of the Logarithm of any Number; Common Logarithms; Proportional Parts; Exponential Equations; Convergency of Infinite Products; Binomial Theorem for any Index.

III. Theory of Equations.

General Properties of Polynomials.

The Value of Polynomial when the Variable takes large Values; The Value of Polynomial when the Variable takes small Values; Derived Functions; Continuity of a Rational Function; Graph of a Polynomial.

General Properties of Equations.

Real and Imaginary Roots; Equal Roots; Descartes' Rule of Signs; The number of Real Roots between any two given Values of the Variable.

Relations between the Roots and Coefficents of Equations.

Theorem; Application of the Theorem; Cube Roots of Unity; Symmetric Functions of Roots; Theorems relating to symmetric Functions.

Transformation of Equations.

Roots with Signs changed; Roots multiplied by any Quantity; Reciprocal Roots and Reciprocal Equations; Roots increased or diminished by any Quantity; Removal of Terms; Squared or Cubed Roots; Squared Difference of the Roots of a Cubic Equation.

Solution of Reciprocal and Binomial Equations.

Reciprocal Equations; Binomial Equations; the Special Roots of $x^n-1=0$; Solution of Binomial Equations by Circular Functions.

Algebraic Solution of Cubic and Biquadratic Equations.

The Solutions of the Cubic Equation; Criterion of the Nature of the Roots of a Cubic Equation; the Solution of the Biquadratic Equation; Criterion of the Nature of the Roots of a Biquadratic Equation; Application to Numerical Equations.

Properties of the Derived Functions.

Graphic Representation of the Derived Function; Maxima and Minima of a Polynomial; Rolle's Theorem; Constitution of the Derived Functions; Theorem relating to Multiple Roots; Determination of Multiple Roots; Theorem relating to the Passing of the Variable through a Root of the Equation.

Limits of the Roots of an Equation.

Theorems relating to the Limits of Roots; Practical Applications; Newton's Method of finding Limits; Inferior Limits.

Separation of the Roots of Equations.

Theorem relating to Commensurable Roots; Newton's Methods of Approximation; Horner's Method of Solving Numerical Equations; Lagrange's Method of Approximation.

Complex Numbers and Complex Variables.

Operations on Complex Numbers; Continuity of a Function of a Complex Variable; Cauchy's Theorem relating to

the Number of Roots comprised within a Plane Area; the Fundamental Theorem of the Existence of a Root of an Equation.

IV. Geometry.

(1) Elements of Geometry.

Introduction.

Geometrical Elements; Definitions; Axioms; Theorems and their Relations.

The Straight Line.

Angles; Parallel Lines; Triangles; Parallelograms; Some Loci of Points.

Circles.

Fundamental Properties of Circles; Central Angles; Arcs and Chords; Angles in a Segment; Tangents; Two Circles; Inscribed and Circumscribed Polygons; Constructive Geometry involving the above Principles.

Areas.

Comparision of Areas; Relation between Areas; Constructive Geometry.

Ratio and Proportion.

Fundamental Propositions of Proportion; Fundamental Geometrical Propositions; Similar Figures; Areas; Loci and Problems; Geometrical Drawing.

Applications.

Measurement of Length and Areas; Algebraic Applications; Measurement of Angles; Trigonometric Ratios; Trigonometric Ratios of certain Angles.

Planes and Straight Lines in Space.

Determination of a Plane in Space; Relative Positions of two Straight Lines; Relative Positions of a Straight Line and a Plane; Intersection of Planes; Normal to a Plane; Normals to the Same Plane or Parallel Planes; Projection of a Straight Line on a Plane; Theorem of three Perpendiculars; Dihedral Angles; Solid Angles.

Polyhedra.

General Theorems; Regular Polyhedra; Parallelopipeds; Prisms; Pyramids; Similar Polyhedra.

Solids of Revolution.

The Cylinder; The Cone; The Sphere; Spherical Angles; Spherical Triangles; Guldin's Theorems.

Projective Drawing.

Straight Lines; Polygons; Circles; Polyhedra; Cylinders; Cones; Sections of a Polyhedron; Sections of a Cone; Elementary Properties of the Conic Sections.

(2) Modern Geometry.

Fundamental Metric Properties.

Use of the Signs + and − in Geometry; Measurement of Length; Angles and Areas; Law of Continuity; Relations between the Segments of a Line; Centre of Mean Position.

Harmonic Ranges and Pencils.

Harmonic Division of a Straight Line; Pencil through a Harmonic Range; Sections of a Harmonic Pencil; Relation between the Segments of a Harmonic Range; Theorems relating to Harmonic Ranges and Pencils.

Involution.

Range in Involution; Double Points; Relations between the Segments of a Range in Involution.

Properties of Triangles.

Concurrent Lines drawn through the Vertices of a Triangle; Collineal Points on the Sides of a Triangle; Special Points connected with a Triangle; Some Circles connected with a Triangle.

Properties of the Complete Quadrilateral and Complete Quadrangle.

Properties of a Complete Quadrilateral; Properties of a Complete Quadrangle; the Principle of Duality.

Properties of a Circle.

Pole and Polars; Harmonic Properties of the Pole and Polar; Conjugate Points and Lines; Self-conjugate Triangles; Complete Quadrangle inscribed in a Circle; Complete Quadrilateral circumscribed about a Circle.

The Theory of Reciprocation.

The Principle of Duality; Harmonic Properties; Reciprocation applied to Metrical Propositions; the Reciprocal of a Circle.

The Theory of Inversion.

Inverse Points; The Inverse of a Straight Line; Inverse Circles; Corresponding Properties of the Inverse Figure.

Properties of Two Circles.

Orthogonal Circles; Power of a Point with respect to a Circle; Radical Axis of two Circles; Centre of Similitude of two Circles; Coaxial Circles; Limiting Points; Relations between the Powers of Coaxial Circles.

Cross Ratio.

Cross Ratios of Ranges and Pencils; Involution; Cross Ratio; Properties of a Circle; Homographic Ranges and Pencils.

V. Trigonometry.

The Trigonometric Ratios; Trigonometric Ratios of Complementary Angles, Supplementary Angles &c; Trigonometric Ratios of two or more Angles; Trigonometric Ratios of Multiple and Submultiple Angles; Relations between the Trigonometric Ratios and Circular Functions; The Inverse Notations; Solution of Trigonometric Equations; Elimination; Logarithms; Trigonometric Tables; Relations between the Sides and Angles of a Triangle; Solution of Triangles; Distances and Heights; Properties of Quadrilaterals and Regular Polygons; DeMoivre's Theorem; Expansion of some Trigonometric Functions; Outline of Surveying; Relations between the Trigonometric Functions of the Sides and the Angles of a Spherical Triangle;

Solution of some Spherical Triangles.

VI. Analytical Geometry.

Co-ordinates of a Point.

Cartesian Co-ordinates; Polar Co-ordinates; Distance between two given Points; Point which divides a given Segment in a given Ratio; Area of a Triangle; Locus of an Equation; Equation of a Curve; Transformation from Polar to Rectangular Co-ordinates or from Rectangular to Polar Co-ordinates.

The Straight Line.

Equation of a Straight Line; Different Forms of the Equation of a Straight Line; General Equation of First Degree; Equation of a Straight Line referred to Oblique Axes; Polar Equations of a Straight Line.

Problems on the Straight Line.

Straight Line which passes through a given Point; Straight Line which passes through two given Points; Straight Line parallel to a given Straight Line; Co-ordinates of the Point of Intersection of two Straight Lines; The Condition to be fulfilled in order that three Straight Lines may meet at a Point; The Angle between two given Straight Lines; The Straight Line which passes through a given Point and makes a given Angle with a given Straight Line; The Length of the Perpendicular drawn from a given Point to a given Straight Line; The Length of the Straight Line drawn from a given Point in given Direction to meet a given Straight Line.

Transformation of Co-ordinates.

To change the Origin of Co-ordinates; To change the Direction of Rectangular Axes; To change the Direction of Oblique Axes; To change from Oblique to Rectangular or from Rectangular to Oblique Axes.

The Circle.

Equation of the Circle referred to any Rectangular

Axes; Equation of the Tangent at any Point of a Circle; Equation of the Normal at any Point of a Circle; Tangents to a Circle from a given External Point; Pole and Polar; Equation of a Circle referred to any Oblique Axes; Polar Equation of the Circle; Radical Axes.

The Parabola.

Definition of Conic Sections; Equation of a Parabola referred to any Rectangular Axes; Tracing of the Parabola from its Equation; The Tangent and Normal to a Parabola; Two Tangents to a Parabola from a given External Point; Diameters; Equation of the Parabola referred to any Oblique Axes; Polar Equation of the Parabola.

The Ellipse.

Equation of the Ellipse referred to any Rectangular Axes; Tracing of the Ellipse from its Equation; The Tangent and Normal to an Ellipse; Two Tangents to an Ellipse from a given External Point; Diameters; Equation of the Ellipse referred to a Pair of Conjugate Diameters as Axes; Polar Equation of the Ellipse.

The Hyperbola.

Equation of the Hyperbola referred to any Rectangular Axes; Tracing of the Hyperbola from its Equation; The Tangent and Normal at any Point of a Hyperbola; Tangents from any External Point to a Hyperbola; Diameters; The Hyperbola Conjugate to a given Hyperbola; Equation of the Hyperbola referred to a Pair of Conjugate Diameters as Axes; Asymptotes; Equation of the Hyperbola referred to the Asymptotes as Axes; Polar Equation of the Hyperbola.

General Equation of the Second Degree.

Classification of the Loci represented by the Equation of the Second Degree; Equation of a Conic Section of a given Eccentricity and having a given Directrix; Equation of the Tangent and Normal of a Curve of the Second Degree.

VII. Analytical Geometry of three Dimensions.

Systems of Co-ordinates.

Cartesian Co-ordinates; Polar Co-ordinates; Change of Origin; Point dividing a Line in a given Ratio; Transformation from Polar to Rectangular or from Rectangular to Polar Co-ordinates; The Equation of a Surface; Equation of a Curve.

Direction Cosines.

Projection of a Segment; Projection of a Plane Figure; Direction Cosines (axes rectangular); Direction Cosines (axes oblique); The Angle between two Lines with given Direction Cosines; Direction Ratios; Relation between Direction Cosines and Direction Ratios.

The Plane and the Straight Line.

The Equation of a Plane; The Plane through three Points; The Distance of a Point from a Plane; The Plane bisecting the Angles between two given Planes; The Equations of a Straight Line; Intersection of a Plane and a Straight Line; Intersection of Planes; Intersection of Straight Lines; Shortest Distance between two given Lines; Problems relating to Planes and Straight Lines.

The Sphere.

Equations of a Sphere; Tangents and Tangent Planes; Section of a Sphere made by a Plane; Radical Planes of two Spheres.

Cylinders and Cones.

Equation of a Cylinder; Generating Lines; Equation of a Cone; Condition for the Tangency of a Plane and a Cone; Equation of a Cone with a given Conic for the Base.

Central Conicoids.

Equations of an Ellipsoid; Hyperboloid of one Sheet and Hyperboloid of two Sheets; Tangents and Tangent Planes;

Polar Plane; Normals.

Paraboloids.

Equations of Paraboloids; Tangent Planes; Normals.

Miscellaneous.

Circular Sections of Conicoids; Generating Lines of Ruled Surfaces; Confocal Conicoids; Reduction of General Equation of Second Degree to Normal Form. (Classification of the Surfaces); Tangent Plane to the Conicoid represented by the General Equation.

VIII. Introduction to Calculus.

Differential Calculus.

Constants and Variables; Functions; Limiting Ratios; Differential and Differential Coefficients; Differentiation of Products, Quotients, Powers and Function of Functions; Anti-differential or Integral; Differentiation of Direct and Inverse Trigonometric Functions; Differentiation of Logarithmic and Exponential Functions; Differentiation of Direct and Inverse Hyperbolic Functions; Successive Differentiation; Theorem of Leibnitz; Examples in Geometry, Physics and Dynamics; Taylor's and Maclaurin's Theorems of Expansions; Binomial, Logarithmic, Exponential Series; Expansion of $\sin x$, $\cos x$, $\sin^{-1}x$, $\cos^{-1}x$ &c.; Convergent and Divergent Series; Indeterminate Forms; Partial Differentiation; Total Differentiation; Euler's Theorem for Homogeneous Functions; Applications to the Error in Numerical Calculations, and to Isothermal and Adiabatic Expansions.

Maxima and Minima of Function of one Variable; Criterion of Maxima and Minima; Geometric and Algebraic Examples.

Maxima and Minima of Function of two or three Variables; Examples.

Tangents and Normals; Subtangents; Subnormals;

Inverse Curve; Reciprocal Polars; Asymptotes; Multiple Points on Curves; Convexity and Concavity; Points of Inflexion; Radius of Curvature; Curve-tracing.

Integral Calculus.

Connection between Primitive and Derivative; Elementary Integrals; Integration by Change of Independent Variable, by Parts, by Partial Fractions and by Successive Reduction; Some Typical Integrals; Definite Integrals; Integration regarded as Summation; Limits of Integration; Mean Values; Centre of Gravity; Length of Arc; Surfaces and Volumes; Moment of Inertia; Strength of Beams, Shafts etc.; Centre of Pressure; Practical Problems; Successive Integration—Double and Triple Integration; Deflection of Beam.

Differential Equations.

Formation and Classification; Typical Solutions; Exact Differential Equations; Linear Equations; The Integrating Factor; Symbolic Methods; Practical Problems involving Differential Equations;—Falling Bodies, Pendulums, Simple Harmonic Motion, Vibrations &c.

B. Nara Higher Normal School for Women.

I. Preliminaries.

During the first four months fifty hours are spent in studying advanced arithmetic, and algebra as far as linear equations.

II. Arithmetic and Algebra.

Three hours a week in first year, three hours a week in second year and two hours a week in third year are spent in studying algebra with *Sembon's* Elementary Algebra in 2 volumes as the text-book, and other books for reference.

The Syllabus is as follows:

(1)

(*a*) Four Fundamental Operations with the Natural Numbers; Three Fundamental Principles.
(*b*) Definition of Zero and Operations with it.
(*c*) Definition of Fractions and Operations with them.
(*d*) Definition of Negative Numbers and Operations with them.

(2)

(*a*) Addition, Subtraction, Multiplication and Division of Integral Expressions.
(*b*) Factoring; Highest Common Factors; Lowest Common Multiples.
(*c*) Addition, Subtraction, Multiplication and Division of Fractions.

(3)

(*a*) Principles of the Solution of Equations.
(*b*) The Solution of Linear Equations and Problems; Arithmetical Solution of the Problems.
(*c*) The Solution of Equations with Fractions, which can be reduced to Linear Equations.
(*d*) The Solution of Inequalities of the First Degree.
(*e*) Discussions of the Solutions of Linear Equations and Problems.

(4)

(*a*) Powers and Roots.
(*b*) Square Roots and Cube Roots of Numbers, and their Application to Problems.
(*c*) Fractional and Negative Indices.

(*d*) Definition of Irrational Numbers and Operations with them.

(5)

(*a*) Solution of Quadratic Equations.
(*b*) Relations between the Roots and Coefficients.
(*c*) Properties of Trinomials.
(*d*) Solution of Inequalities of Second Degree in one unknown Quantity.
(*e*) Equations of Higher Degree which can be reduced to Quadratics.
(*f*) Fractional Equations and Irrational Equations.
(*g*) Problems to be solved by Equations.
(*h*) The Change of Values of Integral Functions of Second Degree and Graphical Representation of them.
(*i*) Maxima and Minima.

(6)

(*a*) Theorems in Ratio and Proportion.
(*b*) Problems in Simple Proportion, Compound Proportion, Proportional Partition and Continued Proportions.
(*c*) Articles and Problems concerning Percentage and Interest.
(*d*) Variation.

(7)

(*a*) Arithmetical Progressions; Harmonic Progressions.
(*b*) Geometrical Progressions.
(*c*) Summation of some other Series.

(8)

(*a*) Permutations.
(*b*) Combinations.
(*c*) Binomial Theorem for a Positive Integral Exponent.
(*d*) Elementary Treatise of Convergency and Divergency of the Sums and Products of Infinite Series; Binomial Theorem for any Index.

(9)

(a) Theorems on Logarithms.

(b) Calculations with Logarithms; the Use of the Logarithmic Table.

(c) Exponential Equations.

(d) Compound Interest and Annuities.

(10)

(a) Continued Fractions.

(b) Properties of Numbers.

(c) Solution of Inequalities of the First and Second Degree.

(d) Alligation.

III. Geometry.

Time is given to Geometry as follows:—

Three hours a week in the first year, two hours a week in the second year and three hours a week in the third year.

The text-book—*Kikuchi's* Elements of Geometry in two volumes, plane and solid, with an appendix giving modern definitions and theorems of ratio and proportion and the proofs of the fundamental theorems of proportion.

IV. Trigonometry.

Two hours a week in the second and third terms of the third year is given to trigonometry. The text-book to be used is not yet fixed.

(a) The Measurement of Angles.

(b) Trigonometric Functions of the Angles from 0^0 to 90^0, and Relations between them.

(c) Trigonometric Functions of General Angles.

(d) Trigonometric Functions of the Sum and Difference of two Angles.

(e) Trigonometric Functions of Multiple and Submultiple Angles.
(f) The Solution of Trigonometric Equations.
(g) The Use of Logarithmic Tables.
(h) The Solution of the General Triangle.
(i) Simple Surveying.
(j) Chain Surveying.
(k) Plane Table Surveying.
(l) The Outline of Transit Surveying.

CHAPTER IV.

Examinations.

One school year is divided into 3 terms, at the end of each of which an examination is to be held. In the Nara School, however, examinations are held twice a term on the first and second terms, and at the end of the third term, an examination covering all the year's work is given.

There is no "graduation examination" but instead, the students are judged by their work in "practical teaching" in the third term of the fourth year.

The problems may be classified as those which test the students' ability in (1) memorizing, (2) reasoning, (3) application and (4) calculation. These four ought to be mixed up adequately. The kinds of problems shall be carefully chosen so as to enable the students to show their attainments and too difficult questions should be avoided. Both oral and written examinations are held, the latter exclusively for the term examinations and the results of daily-lessons and of home-exercises are referred to, also, for works.

Much care should be given to choosing suitable problems, since it exercises great influence on the students' method of studying.

CHAPTER V.

Methods of Teaching.

The students of the Higher Normal School for Women are expected to teach afterwards in secondary schools; therefore, they must not only clearly understand what they learn but actually make the knowledge their own in order to be able to teach others later. Therefore, for daily lessons, higher theories are not required, but in the proper order and by simple means the truths must be made clear, and especially in the elementary courses of arithmetic, algebra, and geometry these truths must be carefully stated. Moreover, students must learn to make out good schedules and to choose suitable problems.

In teaching mathematics particular attention shall be paid to the following points:—

(1) to encourage students in the preparation of the next lesson by assigning it at the end of the lecture.

(2) to encourage students in practice in solving problems either by the classs as a whole, or by individuals in the class room or by home exercises.

(3) to accustom students to the correct expression of their own thoughts by having them prepare papers at home to be given to the teacher.

(4) to have students select the problems and make out a schedule for the teaching of a given grade of work.

(5) to give to students the names of reference-books on the subjects of the lectures and the problems discussed and to provide such books either in the class room or a library.

I. Arithmetic.

(1) Calculation is the essential part of arithmetic, and the degree of skill in it has much effect on the general

progress of the student in mathematics, and yet, calculation is often undervalued and neglected. This must be especially emphasized.

It is the object of calculation to obtain accurate results as quickly as possible, through skill in the handling of numbers and this may be attained by constant practice, by mastering the rules of calculation and by using short methods.

The dislike of calculation as something troublesome on the part of students and the consequent slow progress is to be traced to uninteresting instruction such as the giving of complicated rules for the mechanical working out of problems. Therefore when a method is given, it must be compared with the usual one and its expediency in abridging the labour shown to the students thus by letting them apprehend its necessity, and finally have them discuss it and try it themselves.

The results of calculation can never be utilized unless their correctness is assured, and it is most desirable that if any error be found, re-calculation should be done immediately. By showing students that errors in calculation make great errors in the results in some practical cases, particularly in navigation and astronomy let them become so careful in their work that they may be able to rely firmly upon the results of their own calculation.

It is necessary to check all calculation work and also examine the result obtained.

To avoid miscalculations, it is necessary to be careful in the writing of figures as well as in the method of working. Students must be told that mistakes in the writing down figures cause more errors than miscalculation.

Complicated problems shall be left until algebra is studied when they may be concisely solved by using letters. However, the mathematics in most of the girls' high schools, with the exception of some, where elementary algebra and

elementary geometry are given, is limited to arithmetic and the use of algebraic demonstration is inadmissible and therefore, the students who will teach in such schools must become skilful in arithmetical demonstrations.

Arithmetical demonstration is synthetical and every step of a calculation has its meaning while algebraic or literal demonstration is analytical and symbolical, consequently each stage has no particular meaning. Thus, as these two have their own characteristics, the methods deduced by algebra are scarcely comprehensible to those who are not acquainted with algebra. Therefore, persons who will be engaged in teaching in the secondary girls' schools are required to be particularly careful about methods of demonstration.

The problems shall be such as have connection with everyday life or have reference to geography, physics, household affairs or statistics, and also to the prices of commodities, that the students may cultivate common sense about these subjects.

Particular emphasis must be placed on the conversion of units from one system to the other, *i.e.*, the units of weight, measures, coins, some physical quantities, and those commonly used in commerce and industry.

Such topics as life annuities and life insurance of practical value to the everyday life of the student shall be given due place in algebra.

The following are the essential points in the teaching of arithmetic :—

(1) Mental calculations are important.

(2) The common sense of the student must be developed by having her make estimates before a problem is solved.

(3) Comparison between units such as those of weights and measures shall be made.

(4) Economical questions dealing with everyday life

must be considered at suitable time.

(5) Theoretical points shall be explained in connection with algebra.

(6) Drill in approximations, according to the ideas of irrational numbers in algebra, and mensuration in geometry, shall be given.

II. Algebra.

Algebra is begun immediately after arithmetic and there is no definite distinction between them. The object of elementary algebra is to increase ability in calculation by extending the theory of arithmetic and the method of its calculations, and to study the problems of arithmetic more carefully by using equations; and also to demonstrate various problems which have to do with geometry, geography, physics and other topics; and finally to give the student the knowledge preparatory for study in higher mathematics and higher physics.

In one way algebra is a general arithmetic, because the arithmetical theorems can only be simple and completely worked out by substituting letters for numbers; therefore, in elementary algebra each step must be compared to arithmetic and it is a great mistake to consider that the study of arithmetic ends with the beginning of algebra.

Not only in the discussion of theories but also in the actual work of calculation, elementary algebra and arithmetic must be closely correlated. The letters in elementary algebra must be, at first, treated as signifying certain numbers as a matter of convenience. And step by step the real significance of the letters must be impressed upon the students. At first, let the students substitute numerical values for the letters and calculate the value of the algebraic expression and test the results of the literal or algebraic calculations.

The elemetary algebra has, as its foundation, the solution of equations and their applications. Along with equations, identities ought not to be neglected. At the same time the solution of equations is taught, by the change in value of the algebraic expressions the conception of a function and its variation is to be taught and by the graphs of algebraic expressions the relation between the variables and their functions must be shown.

The elementary algebra treats chiefly, as above stated, numerical calculations and the solution of equations, but in the advanced course careful discussions and strict demonstrations and drill in mathematical induction must be given.

In algebra, contrary to geometry, strict demonstrations are not given from the first, therefore there is a tendency toward the inaccurate use of words and inexact proofs of theorems, which ought to be carefully avoided.

The following points should be carefully observed by teachers.

(1) Problems shall be taken from everyday life as in arithmetic.

(2) Comparison shall be made between the algebraic and the arithmetical solutions of the problems in the equations of the first degree.

(3) Abundant drill in operations on algebraic expressions shall be given.

(4) Uniformity in the ways of writing down and expressing one's thought is to be aimed at.

(5) Exact distinction between the necessary condition and the sufficient condition in demonstration.

III. Geometry.

Geometry is the science of reasoning and gives most valuable mental training and cultivates accurate thinking, careful use of words and thoroughness in study. At the

same time, the studying of the practical problems of space with the aid of algebra, physics, geography &c., gives adequate materials for exercise in calculation.

Geometry is the most abstract subject among the pure sciences since it deals with things which do not enter into other spheres but in its relation to secondary education, the object must be, on the one hand, mental training, and on the other hand, practical applications. Therefore it is desirable to apply its definitions and theorems to the solution of problems in other spheres. Moreover, the theorems which have already been proved, must be tested from all points of view and practical applications made of them.

A preliminary course might be given under the name of elementary, experimental, or inventive geometry before the systematic study of demonstrative geometry is begun but students of this school are expected to have, already, some knowledge of elementary and therefore the demonstrative geometry is systematically begun at once.

In the last century the study of geometry made certain advances in new direction and not a few theorems, such as the principles of continuity, duality, inversion, harmonic ratio, anharmonic ratio &c., were discovered and are taught under the title of Modern Geometry as a supplementary course to Euclidean geometry.

However, the theorems and discussions of Modern Geometry are too complex and difficult to be given to students of this degree of advancement. Moreover, there is an advantageous method of explaining the definitions and of demonstrating the theorems by the translation and rotation of the figures, but this method is not taught as it does not agree with the former proofs and theorems.

Not only is it very convenient and helpful to the correct understanding of geometry to denote geometrical quantities by numbers and to apply algebraic principles to the geometric theorems but it also provides further opportunity

for drill in the use of equations, especially irrational or quadratic equations.

Geometrical drawing and geometry are closely related to one another, and the proper arrangement of subjects will help the progress in each of them, *e. g.*, on the one side, by practically expressing what is designed in geometry and on the other side, by that practical expression leading the pupils to discover some relations essential in the investigation of the theorem under discussion.

Graphical representation is not as easy in solid geometry as in plane geometry, and those, especially, who have, as yet, no knowledge of solid geometry can scarcely understand the diagram of a solid figure. Therefore, some models of solid figures ought to be provided.

Care should be taken to the following points in teaching geometry:—

(1) The distinction between what is defined and what is to be proved.

(2) To make the nature of a theorem and the methods of proof thoroughly understand. Lessons in logic would be helpful.

(3) Geometrical models should be provided in the class room since the close observation of them will greatly help students to comprehend the figures discussed.

(4) At first, rulers shall be used to draw perpendicular and parallel lines, the foot rule to divide straight lines, and the protractor to divide angles; later, in constructive geometry, the straight edge and compasses only shall be used while in geometrical drawing other instruments may be used.

(5) After finishing a geometrical drawing it must be measured and the results compared with those obtained from the algebraic calculation.

(6) In connection with geometrical drawing the outlines of graphic calculation may be given.

(7) A diagonal scale and a scale of chord may be made with card board and so also models of solid figures.

IV. Trigonometry.

Trigonometry is the application of algebra and geometry and can not be as entirely separated from other subjects as arithmetic, algebra and geometry are, because the trigonometric functions are no more than the ratio between two sides of a right angled triangle. Therefore the properties of the functions and their relations to each other are studied according to the principles of geometry, the transformation of trigonometric functions and the solution of trigonometric equations are treated according to the theory of algebra, and the solution of any triangle is only the application of algebra to theorems of geometry. From this point of view, it would seem that trigonometry should be included within geometry; yet, trigonometrical functions have their characteristic properties, and it seems better to teach trigonometry as a separate branch.

Trigonometry is a subject which can, very easily, be made interesting to students. Its functions though simple, are, nevertheless, most widely applied, *e. g.*, in physics, to the composition and decomposition of forces, to the coefficient of friction and to some problems in the refraction of rays. Moreover, practical application of the solution of triangles is made in surveying, a subject which greatly interests the students.

Practical applications of the theorems of trigonometry should be made as soon as possible. Before the logarithmic solution of a triangle is given the solution of a triangle by the table of natural functions should be given, with some simple applications to surveying. This will reach the purpose given above and, at the same time, make the meanings and the applications of the functions more thoroughly understand.

It is advisable to use trigonometry in the demonstration of some of the theorems of solid geometry, which can be simply proved by the aid of trigonometry and also to the harmonic ratio of pencils in modern geometry.

Actual practice in surveying shall be given.

SECOND PART.

Modern Tendencies of the Teaching of Mathematics.

CHAPTER I.

School Organization.

Formerly, all the students of the Higher Normal School for Women studied only the higher common subjects, but the progress in the education of girls made necessary the better training of teachers. Therefore in 1907 and 1908 all subjects taught in the school were classified in the three courses of Literature, of Science and of Arts, and in the sister school, which was established at Nara in 1908, all subjects taught were classified into the Courses of the Japanese Language and Chinese Classics; of Geography and History; of Mathematics, Physics and Chemistry; of Natural Philosophy and Household Affairs, and subsequently in 1911 in the Tokio School all the courses were subdivided into the first and second sections. Thus the domain of any special course is gradually made narrower but greater emphasis is placed upon the subjects in the course by increasing the number of hours of lectures.

Instructors in mathematics, particularly, must have higher training and greater skill in teaching than those in other subjects. And it is generally acknowledged to be true that mathematics is most difficult to teach and in order to do it well, sufficient training and talent in teaching are required.

CHAPTER II.

Aim and Subject-Matter of Mathematical Instruction.

(1) Subjects recently added to the Course.

Formerly, arithmetic, algebra, geometry and trigonometry were given, but recently, higher algebra, analytical geometry and elements of differential and integral calculus were added to this list.

(2) Topics to be Omitted.

Those difficult articles and problems of arithmetic, which can only be solved with the aid of algebra, are given in algebra.

In elementry geometry the treatments of Euclid's ratio and proportion are omitted and are given as an article in the historical discussions.

The following are the points about which particular care should be taken :—

The conception of a function is not only necessary in mathematics but also in practical questions, therefore by the teaching of the "graphs" of algebraic expressions &c. students should be given a clear conception of a function.

Mental arithmetic, estimations and contracted calculations have heretofore often been undervalued in spite of their practical value. This mistake must be carefully avoided.

The substitution of numerical values for geometrical quantities not only makes the subject practical, but also makes its connection with the principles of arithmetic and algebra more close and thus gives training in both of these as well. This sort of drill should always be given.

CHAPTER III.

Examinations.

The object of an examination is to find out how the students are studying and to force a review of their work. The value of education lies not in the amount taught, but in the amount understood by the students; therefore the efficiency of teaching can only be judged by the progress made by students and the method of teaching must be based on the result of examining actual progress made by

students. For these two reasons, it is necessary to know the results obtained by their study, and though this will partly be accomplished in the class room, it is impossible to find out how much of the work discussed during a certain period was comprehended by each student. Therefore, regular examinations are necessary in our schools in their present stage of development. Moreover, in this school the equipment of the students must be compared with a fixed standard in order to ascertain whether they are fitted to be teachers in the intermediate schools for girls and this can be done by the regular examinations.

Moreover, the regular examinations on the lessons studied during a certain period, will cause the students to review them, and thus gain a better understanding of them. For these reasons the examinations may be considered expedient and can not be abandoned merely because they have some defects. One thing that should be guarded against in examination is the overexertion on the part of students before the examination. Especially among girls, mental and physical development is hindered by it, and for this reason in the elementary schools, and in the intermediate girls' schools no regular examinations are held. But in our school for the reasons above mentioned, they can not be omitted.

Further, the selection of problems for the examinations shall be carefully made.

In mathematics, more than in other subjects, there is often considerable difference in the results obtained by students caused by the wrong selection of problems. Problems that require some special talent or training or that can be solved by some accidental flash of thought should be avoided and those, that allow the students to show their real attainments, chosen. Moreover, sufficient time should be given to solve and discuss the problems completely.

Though the examination is, on the one hand, a test, on

the other hand, it is a real part of the lessons, and in the case of written examinations, papers shall be inspected and criticized in detail, and after the mistakes have been corrected the papers shall be sent back to students to let them know how they made mistakes.

CHAPTER IV.

Methods of Teaching.

(*A*) Psychological observations and precautions of students are still required in this school. The fundamental characteristic of mathematics is the accuracy and exactness of its demonstrations and its treatments of numbers, and unless demonstrations are satisfactorily made, the object of mathematics is not attained. If, however, work which is beyond the mental grasp of the students is expected at the beginning, the subject will become uninteresting to them and their progress will be slow. Therefore, those articles which are difficult of demonstration may simply be explained by the teacher or assumed as being true, and the proof postponed until later. Thus as the students progress in power the work may increase in difficulty. For example, in algebra, we may dispense with systematic treatment at the start, and as to theorems and laws, they shall first be given in concrete numerical form and the theoretical demonstration later. The importance of teaching by intuition diminishes as the degree of attaiment of the student increases and consequently no particular emphasis is placed on it in this school except in the following:—

(1) Measure and weights in their models shall be used in the class room.

(2) To let the students observe geometrical figures.

(3) To give concrete calculations before proceeding to theoretical explanations.

(4) To let the students make the "graphs" of the functions.

(*B*) Examples of Practical Application and of Experimental Teaching.

(1) Surveying.
(2) Use of slide rule and mathematical instruments and graphical calculations.
(3) Drill in estimations.
(4) Tabulation of various units; prices of commodities and statistical articles.
(5) Construction of diagonal scales; models of solid bodies; and others that may be referred to the teaching of mathematics.

A mathematics exercise room shall be provided for the purpose.

(*C*) To make the Popular Knowledge of Mathematics more widely spread, It is advisable to publish such Books as will, by combining Literature and Mathematics, interest their Reader. We have a Few of these, *e.g.*, the "Mathematical Cards," "Mathematical Club," &c.

Before the great political change in the beginning of Meiji in Japan, a book of popular arithmetic, named "Jin-ko-ki," or "Book of Numbers," *Jin* meaning one ten-billionth, *Ko*, ten billions, and *Ki*, the books or note, was most widely used and almost every family was provided with it as well as with the calendar. But the modern mathematical books published are limited to text-books, reference-books for its study, or books on higher mathematics for professional men, which are not of interest to the whole family.

Though mathematical plays are one of the best means of increasing popular education in mathematics, it is not yet widely carried out.

Of course the mathematics is an abstract subject, but if it is well comprehended and well applied, it will gradually become interesting. The causes of the general dislike of mathematics are as follows:

(1) The giving of abstract subject from the very

beginning.

(2) The improper arrangement and treatment of subject.

(3) Unsuitable choice of subjects for the equipment of students.

(4) Incomplete comprehension of the properties of numbers.

(5) The bringing out more clearly than in other subjects distinction in the ability of the students.

(*D*) The Way to remove these Causes of the General Dislike of Mathematics.

(1) By having the students work out many concrete problems and keep in close contact with practical questions.

(2) By selecting the subjects suitably and arranging them and their treatment properly.

(3) By insisting on reviews.

(4) By calling out the interest of students in the deductions of mathematics by making their understanding of them clear.

(5) By finding out the cause of the defect in poor students, and to remove the cause or give them additional special instruction.

(6) By showing students the necessity of having mathematical knowledge in order to accomplish any enterprise in life.

(*E*) Relation between different Branches of Mathematics.

(1) Elementary algebra and elementry geometry shall be kept separate. Though the proof of some of the theorems of elementary geometry may be simplified by the application of numbers, in our school logical training, as well as the practical application, is highly esteemed, and therefore, the geometry is given as a pure science; and the close connection between the

articles must be carefully preserved. If geometry is given with algebra the connection between theorems will become confused and inexact demonstrations will result. Therefore elementary geometry is given separately, not with elementry algebra. And after the course in plane geometry is finished the measuring of geometrical quantities is given and then the application of algebra follows. The same is true of solid geometry.

(2) As to the teaching of plane and solid geometry together, there is a great advantage in placing the simpler propositions of solid geometry before the most complex ones of plane geometry and in grouping subjects which are similar in both of them such as loci, symmetry etc, but it will be best for the strictly logical training to keep them separate.

(3) There is also an advantage in giving the definitions and properties of trigonometric functions and the relations between them, when teaching the similar figures in elementary geometry. However, there is no necessity of removing the conventional limits between them as it would put some obstacles in the way of progress in each of them.

(4) As to elementary geometry and elementary algebra, the latter, which has the closest connections with arithmetic, ought really to be given first. In this school both are given simultaneously but no trouble is found.

(5) There is no need of removing the boundary line between synthetical and analytical geometry.

(6) As for infinitesimal calculus, both the integral and differential calculus shall be given together and after the differentiation of a function, the integration is given as the reverse operation, thus the relation between the differential coefficients and the original

function will be well apprehended. Applications to geometry and dynamics may be easily made by reducing the equations obtained by differentiation.

(*F*) Relation between Mathematics and other Subjects.

(1) In drawing, the geometrical drawing is based upon the principles of geometry and the conceptions of distance, of shades and shadows in free hand drawing come from geometry.

(2) Ranges of mathematics applicable to physics :—

Application of algebra and trigonometry are made in dynamics, *e.g.*, in the discussion of falling bodies the solution of equations of second degree is necessary, and in decomposition of forces &c., the trigonometric functions are used. Logarithmic tables are used in the quantitative experiments, and in wave theory, electricity, &c. the expansion of binomial theorems and trigonometric functions of sum and difference of two angles are used, and in order to express the relation between vapour tension and temperature, or other diagramatical expressions such as "Stress-strain diagrams," &c. the fundamental conception of analytical geometry is applied. To deduce the critical pressure and the critical temperatures from the "Van der Vall's" equation the properties of equal roots of cubic equations may be used.

On the whole, instead of the application of infinitesimal calculus, certain algebraic means should be substituted and those articles which are not suitable for students below the third year class, shall be given in the last term of the fourth year under the topic of Miscellaneous Theories of Physics.

(3) In chemistry, also, mathematical calculations are

necessary in the laboratory work, and the solution of quardratic equation is necessary in the theory of chemical dynamics among the general theorems of chemistry.

(4) Geography requires the aid of mathematics in the following cases:—

Measurement of direction on maps; the reducing of maps; measurement of distance on maps; sight measurement of distance and direction; measurement of distance on foot; orthogonal projection; enlarging and reduction of maps; use of proportional compasses for above object; stereographic projection; measurement on maps with compasses and with map measures; simple conic projection; the Banne conic projection; the measurement of surfaces with gelatine paper; Mercator's projection; contour maps; isothermal and isobaric lines; average height of mountain range; topographic sections; charts of profile; charts of rainfall; use of clinometer; square measure with cross section paper and with balance; plane table surveying; sea sounding; charts of the density of population; hypsometrical maps; observation of temperature and barometer; use of currentmeter; square meter with planimeter; charts of distribution of products; charts of races of mankind; charts of imports and exports; relief maps; use of sextant; use of pantograph; isochronic chart; observation of temperature of water; use of terrestial globe.

(5) Arithmetic and algebra have close connection with applied sciences and problems of everyday life, such as: transmission and commutation; astronomy and meteorology; calendar; agricultural industry; hygiene and necessities of life; civil engineering; archtecture and surveying; population and ages;

labour and wages; taxes and insurances; bills and exchanges; banks and companies; measure and weight; coins; longitude and latitude; difference of time &c.

CHAPTER V.

Preparation of Teachers.

(1) Institutions for this Object.

Among the institutions for the preparation of women as teacher in the secondary schools for girls, those following are under the administration of the Minister of Education: the Tokio Higher Normal School for Women, the Nara Higher Normal School for Women, the Tokio Fine Art School, the Tokio Musical School, the Special Course in Tokio Higher Normal School for Women, and the Sixth Temporary Institution for Training of Teachers. Moreover, the honour graduates of those private schools which have been acknowledged by the Minister of Education, and those who have passed the examination for teachers, which is held once a year, may become teachers in the intermediate girls' schools.

The positions as teachers of mathematics are taken by the graduates of the first and second sections of the mathematical course of the Tokio Higher Normal School for Women, of the course of Mathematics, Physics and Chemistry, of the Nara Higher Normal School for Women, and of the Special Course of Mathematics, Physics and Chemistry of the Tokio Higher Normal School for Women. But the special courses are only established when it is considered necessary, and at present no special course in mathematics exists.

No graduates of private schools are approved as teacher of mathematics without passing the qualification examination.

A qualification examination is held in the subject desired by the applicant. An applicant for a position as teacher

of mathematics shall be examined in arithmetic, algebra and geometry, after she has passed the examination in pedagogy, and after those are passed she may, if she wishes, be examined in trigonometry, analytical geometry or infinitesimal calculus.

The degree of the qualification examination is equal to that of the attainments of the graduates of Higher Normal Schools and no great difference is made between women and men.

Applicants for arithmetic, algebra and geometry shall pass, first, the preparatory examination, second, the main examination, and third, the oral examination.

(2) Attainments of Teachers.

Teachers of the girls' school of the middle class are expected to have some knowledge of trigonometry, analytical geometry and infinitesimal calculus besides arithmetic, algebra and geometry as well as pedagogy. It is often said that it would be advantageous to have men teachers instead of women for all branches, except household affairs and sewing, and particularly for natural philosophy and arithmetic because the women teachers are said not to have sufficient attainments. However, the necessary womanly training cannot be had without women teachers.

The incomplete preparation of the women teachers is due to the fact that the course for teachers included too many subjects and it may be considered as an advance in the right direction that the course has been limited to subjects closely connected with a given branch. Good results from this may be expected in a short time. The women teachers have less time than the men to spend in study because of household affairs and they seldom make study their life work. It cannot be denied that they are under some disadvantages in the pursuance of their studies and the daily progress in knowledge. Therefore it is most desirable that meetings be held where lectures are given at

some time of leisure such as the summer vacation. At present, besides that under the auspices of the Department of Education, many such meetings are held by educational societies &c. in every part of the country.

Besides this the establishment of societies, in which the results of the study of each member may be given the knowledge of each imparted to the others, will serve to greatly encourage study. There are such meetings, but, to our regret, they seem to be chiefly supported by men and few women who have to do with the education and bringing up of children attend these meetings.

In the Tokio Higher Normal School for Women, there are three such societies, *i. e.*, The Society of the Literal Course, The Society of the Scientific Course and The Society of the Art Course, all composed of the students and graduates of the respective courses. They meet once a term, at which time their studies are discussed.

This does not give, as yet, satisfactory results, but after making gradual progress, we belive, it will, to some degree, serve to attain the objects mentioned above.

CHAPTER VI.

Exercise Room.

Just as a laboratory is necessary in physics, chemistry, natural philosophy and geography, so is the exercise room in mathematics. In this room black boards, sufficient to accommodate all the class at once, are to be provided. The following equipment is essential.

(1) Spherical black board.

(2) Some cross section boards.

(3) Hinged dihedral black boards.

In addition the following instruments are to be provided.

(1) Rulers and compasses for black board and an elliptical disk.

(2) Wooden cubes of a cubic decimetre, a cubic centimetre &c.
(3) Rectangular and cylindrical measures of *Sho* and cylindrical measures of a litre and a gallon.
(4) Brass weights from 500 grammes to 1 gramme, and balance.
(5) Measures of *Shaku*, *Kujira Shaku*, a metre and a yard.
(6) Foreign coins or their models.
(7) Instruments for measuring distances (Tape, pole, skewer).
(8) Instruments for measuring anlgles (Sextant, theodolite).
(9) Patterns of bills, documents, certificates of stocks and bonds.
(10) Slide rule; Arithmometer.
(11) Geometrical figures and solids.
(12) Sections of cones and models of surfaces of second degree.
(13) Drawing instruments.
(14) Copy of mechanical drawings.
(15) Models and apparatus made by students to illustrate the various mathematical concepts in the education of the elementary and secondary schools.

Books to be provided are as follows:—

(1) Text-books.
(2) Reference-books and exercise-books.
(3) Current text-books in the elementary and secondary schools.
(4) History and pedagogy of mathematics.
(5) Logarithmic tables from 3 to 7 places, and tables of squares, cubes, square roots, cube roots, reciprocals, compound interest.
(6) Reference-books on teaching.

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CONTENTS.

First Part.

Present state of the organization and the methods of mathematical instruction in commercial colleges and schools.

Second Part.

Modern tendencies of the teaching of mathematics in commercial colleges and schools.

FIRST PART.

PRESENT STATE OF THE ORGANIZATION AND THE METHODS OF MATHEMATICAL INSTRUCTION IN COMMERCIAL COLLEGES AND SCHOOLS.

(*a*)

COMMERCIAL COLLEGES.

The Types of Schools.

The Higher Commercial Schools or Commercial Colleges, founded subject to the regulations relating to Special Schools (*Semmon Gakkō*) and to Technical Schools (*Jitsugyō Gakkō*), provide a higher course in commercial education.

At present, five colleges are established and maintained by the Government, and one by the Osaka municipal authorities, viz :—

Tōkiō Commercial College,
Kōbe ,, ,
Yamaguchi ,, ,
Nagasaki ,, ,
Otaru ,, ,
Ōsaka Municipal ,, .

In the first two Colleges, the course covers four years — a one-year preparatory course and a three-year principal course. In the others, the course covers three years.

Besides these, there are some six private Commercial Colleges of similar standing.

Requirements for admission are generally as follows :—

1. Candidates must be over 17 years of age, healthy, and of good moral character.
2. They must be graduates of Middle Schools or of Commercial Schools of Class A, or must present certificates of the same degree of scholarship as graduates of Middle Schools.
3. They must either pass the entrance examinations, or be qualified to have the privilege of free admission according to the regulations of respective colleges.

Average Age of the Students.

Tōkiō Commercial College (Oct. 1910)		Yrs. mos.
First year Class	(Preparatory Course)	20.00
Second year ,,	(1st yr. Princ. Course)	20.10
Third year ,,	(2nd yr. ,, ,,)	21.09
Fourth year ,,	(3rd yr. ,, ,,)	22.10
Yamaguchi Commercial College (June 1910)		
First year Class	....	20.00
Second year ,,	....	21.00
Third year ,,	....	22.03

Remark 1. To the course in the Tōkiō Commercial College is added the Professional Course extending for two years, to which the graduates of the Tōkyō Commercial College and those of the Kōbe Commercial College are admitted. But in this course, the curriculum includes no subject pertaining to mathematics.

Remark 2. The Commercial Teachers' Training Institute, attached to the Tōkiō Commercial College, will be dealt with later on.

Aim and Material of Mathematical Instruction in Commercial Colleges.

In the Commercial Colleges, the principal aim of mathematical instruction is to impart knowledge of the principles and methods of the calculations required in the different branches of commerce and to drill the students in practical accounting. Along with this, attention is paid to commercial usages, to the correct and concise statement of accounts, and to the fostering of quick understanding, accurate ideas, and good habits, such as keeping things in an orderly manner.

Generally, this is given under the name of Commercial Mathematics or Commercial Arithmetic, except in the Tōkiō Commercial College where the course is divided into three parts, viz., Arithmetic and Algebra, Commercial Mathematics, and Higher Mathematics, of which the second mentioned subject naturally occupies the greatest number of hours.

At present, different colleges have different time-tables and syllabuses, some of which are shown below.

Time-Tables of Mathematics.

Commercial Colleges having a Four-Year Course.

	Tōkiō Commercial College	Kōbe Commercial College
First Year (Preparatory Course)	hrs. per week 3	hrs. per week 4
Second Year (Principal Course, 1st year)	2	2
Third Year (Principal Course, 2nd year)	3	2
Fourth Year (Principal Course, 3rd year)	1	—

Commercial Colleges having a Three-Year Course.

	Yamaguchi Commercial College	Nagasaki Commercial College	Otaru Commercial College	Ōsaka Commercial College
1st year	hrs. per week 2	hrs. per week 2	hrs. per week 2	hrs. per week 2
2nd year	1	2	2	2
3rd year	2	1	1	—

Syllabus of Mathematics in The Tōkiō Commercial College.

Arithmetic and Algebra.

First Year.

(Preparatory Course)

Three hours per week.

1. Mental calculation.
2. Soroban-calculation.
3. Written calculation :—
   Foreign weights and measures.
   Foreign monetary systems.
   Practice, simple and compound.
   Percentage.
   Interest.
   Logarithms.
4. Algebraical subjects :—
   Summation of series.
   Probability.
   Graphical representation of functions.

Allotment of hours :—To Nos. 1 and 2, one hour per week is allotted; to No. 3, two hours per week in the first term, and one hour in the second and third terms (each

academic year being divided into three terms); and to No. 4, one hour per week in the second and third terms.

Commercial Mathematics.

This course is given in English, the whole subject being treated in such a way as to constitute a *mathematical interpretation of commercial usages, methods and principles.* In the majority of subjects, the theory is treated algebraically and is followed invariably by arithmetical applications based on actual business data.

Second Year.

(Principal Course, 1st year.)

Two hours per week.

1. Introduction :—
   Scope of Commercial Mathematics.
   Their relative position in a course of business training.
   Their principal aim.
   Precautions to be taken in order to ensure reliability.
2. Time-saving Rules.
3. Approximation.
4. Reduction, ascending and descending, of denominate numbers. Practical study of international commercial metrology.
5. Chain rule.
6. Simple and compound conversion.
7. Valuations.
8. Percentage.
9. Profit and Loss.
10. Miscellaneous percentage problems.
11. Interest.
12. Discount.
13. Common due dates.
14. Equation of time.

15. Compound equation of time.
16. Current accounts.
17. Problems in accounting.
18. Alligation.
19. Tares, etc.
20. Freightage.
21. Customs duties & fiscal dues generally.
22. Charges on goods generally.

Third Year.

(Principal Course, Second Year.)

Three hours per week.

23. Bullion and Specie :—

Gold bullion. Silver bullion. Partings. Money. Generalities on monetary systems and Mint regulations. Standards. Remedy allowances. Seigniorage. Least current weights. Standard coins. Subsidiary coins, Trade coins. Different values of coins. Arithmetical illustration of the usages of the leading bullion markets (London, Paris, New York, Berlin) and of the Far Eastern centres (Japan, China, Hongkong, Philippines, French Indo-China, Siam, Dutch East Indies, Straits, British India). Rapid survey of the monetary conditions of other countries.

24. Foreign Exchange :—

Generalization. Definition of the rate of exchange. Fluctuations of same. Favourable and unfavourable rates. The par of exchange. Theoretical par. Practical par. Export and import bullion and specie points. Diversity of estimates in this respect.

25. Direct Exchange :—

Position of the calculator. Modes of quotation. Types of bills quoted. Methods for calculating

value of a given bill on the basis of the quotation for a different type of bill. "Tel quel" rates. Arithmetical peculiarities of certain courses of exchange (London, Paris, New York, Berlin and Far Eastern banking centres)

26. Bills in foreign currency drawn on Far East :—
Method of settlement.

27. Comparison of direct reciprocal rates :—
(*a*) between markets giving fluctuating prices to each other.
(*b*) between a giving and a receiving market.
(*c*) between markets quoting by agios.

28. Indirect exchange :—
Its usefulness and limitation.
Arbitrage. Arbitrated pars. Compound and circuitous arbitrated pars.

29. Arbitrage expenses :—
Methods of treatment. Percentage links.

30. Fire insurance :—
Generalities. Premiums.
Claims for losses under a specific policy.
Claims for losses under an average policy.
Apportionment of fire losses under concurrent existence of a number of policies (specific and/or average).

31. Marine insurance :—
Computation of the premium.
Fiscal and other insurance charges.
Calculation of amount to insure on all classes of insurable interests.
Valued and open policies.
Claims against insurers for returns of premium on cancelment of policy, for short distance, and for short interest.
Claims for total loss, salvage losses, general average

contributions, particular average losses, and expenses.

Particular average "franchise" and "series."

General average adjustments according to York-Antwerp, British, and "foreign" Rules.

32. Foreign Trade :—

Standing-in accounts; simple and compound.

Computation of "Scales" based on a differentiation between specific and advalorem items.

Limit calculations.

Net result accounts of consignments.

Calculation of margins.

Invoices and account sales.

Higher Mathematics

as applied to Financial, Actuarial, Statistical and Economical Problems.

Fourth Year.

(Principal Course, 3rd year.)

One hour per week.

1. Financial subjects :—

Annuities.

Instalments.

Redemption with premiums.

2. Actuarial subjects :—

Mortality tables.

Premiums.

Premium reserves.

Joint life insurance.

3. Statistical subjects :—

Averages, median and quartiles, percentile methods, and mode.

Theory of errors.

Interpolation.

4. Economical subjects :—

Utility of commodities.

Demand curves and supply curves.

Exchange and other economic problems.

In other colleges, the syllabus of mathematics is similar to that of the Tōkiō Commercial College, except that some subjects belonging to Higher Mathematics are only briefly treated or entirely omitted, more stress being laid on soroban-calculations.

Examinations.

Examinations are of two kinds, viz: annual examinations and term examinations.

The annual examination is held at the end of the academic year in all the subjects studied during that period.

The term examination is generally held at the end of each of the first two terms.

The Year's mark is based on the marks of the annual and term examinations as well as on daily marks.

In some colleges, occasional examinations take the place of term examinations.

Generally, no special graduation examination is held, but the annual result at the end of the last academic year decides the graduation.

Methods of Teaching.

Instruction is given in various ways:—Lecture, practice in calculation, solution of problems, and the making out of business forms and documents.

Text books are not generally used, but formulae, problems and summaries of lectures are reduced to prints or papyrographic impressions and distributed to the students to aid their memories.

Preparation of Teachers.

Up to the present, there is no special organization in existence for the preparation of teachers of mathematics in the Commercial Colleges.

(*b*)

Present State of the Organization and the Methods of Mathematical Instruction in the Commercial Schools of Class A and B.

The Types of Schools.

The Commercial Schools which are established in accordance with "the Commercial School Regulations," are divided into two Classes, viz., Class A and B.

The aim of these is to give instruction necessary for those who enter into commerce.

The course of the Commercial Schools of Class A covers from three to five years. The requirements for admission are as follows: — The candidate must be over 14 years of age and a graduate of a Higher Elementary School having a two-year course, or have equivalent preparation.

The course of the Commercial Schools of Class B covers three years or less. The requirements for admission are: — The candidate must be over 12 years of age and a graduate of an Elementary School, or have a higher preparation.

To most commercial schools of Class A, a preparatory course of one or two years is added. Graduates of an elementary school are admitted to the first year of the preparatory course covering two years. Where the preparatory course consists of one year only, the graduates of

elementary schools are admitted upon their making additional preparation.

Average age of boys in Commercial Schools of Class A. (Oct. 1910)

Course	Class	Yrs. mos.
Three-year course	1st year class	15.11
	2nd ,,	17.00
	3rd ,,	18.00
Four-year course	1st year class	15.07
	2nd ,,	16.09
	3rd ,,	17.09
	4th ,,	18.09
Five-year course	1st year class	15.03
	2nd ,,	16.04
	3rd ,,	17.00
	4th ,,	18.00
	5th ,,	19.00

Average age of boys in Commercial Schools of Class B. (Oct. 1910)

Course	Class	Yrs. mos.
Two-year course	1st year class	13.02
	2nd ,,	14.03
Three-year course	1st year class	13.08
	2nd ,,	14.09
	3rd ,,	15.09

Tōkiō Municipal Commercial Evening School.

Course	Class	Yrs. mos.
Two-year course	1st year class	16.03
	2nd ,,	17.02

Remark. Among the Girls' Commercial Schools, besides those for training clerks and accountants, there are some schools whose express object is to prepare diligent and intelligent wives for traders. In the former, the mathematical course is the same as in a Commercial School of Class A or B, but in the latter, in addition to the ordinary

arithmetic taught in girls' schools, lessons in commercial arithmetic are given.

Aim and Material of Mathematical Instruction in Commercial Schools.

The aim of mathematical instruction in the Commercial Schools is the training in the practical use of commercial calculations and the development of mathematical mental power.

Generally speaking, in the Commercial Schools of Class A, the mathematical curriculum consists of the following subjects:—

1. Soroban-calculation and Mental calculation,
2. Arithmetic,
3. Algebra,
4. Geometry,
5. Commercial Arithmetic.

In most of the schools which have a preparatory course, arithmetic is taught only in that course.

Under the present regulations, Commercial Schools of Class A may be divided into three kinds according to the number of years over which the course extends, viz., three years, four years and five years; and again into another set of three kinds from their having a preparatory course of either one or two years or no preparatory course at all.

We have, therefore, so many types of Commercial Schools of Class A, and moreover schools of the same type may have different courses of studies, giving rise to a very large variety of time-tables and syllabuses. The following tables are specimens of some typical ones.

Time-Tables.

(No. of hours per week.)

Commercial school with a three-year course and a two-year preparatory course.

		Soroban-cal. & mental cal.	Arith.	Alge.	Geom.	Com. arith.
Preparatory Course	1st yr.	1	3			
	2nd yr.	1	3			
Principal Course	1st yr.	1		3		
	2nd yr.	1		1	2	2
	3rd yr.	1				2

Commercial school with a three-year course and a one-year preparatory course.

Prep. course		1	4			
Principal Course	1st yr.	1	2	2		
	2nd yr.	1		2	1	2
	3rd yr.	1			1	2

Commercial school with a four-year course and a two-year preparatory course.

		Soroban-cal. & mental cal.	Arith.	Alge.	Geom.	Com. arith.
Preparatory Course	1st yr.	1	4			
	2nd yr.	1	4			
Principal Course	1st yr.	1	2	2		
	2nd yr.	1		2		3
	3rd yr.	1		1	2	2
	4th yr.	1			2	

Commercial school with a five-year course and without preparatory course.

1st yr.	2	3			
2nd yr.	2	3			
3rd yr.			2		2
4th yr.			1	1	2
5th yr.	1			1	1

Syllabus of Mathematics in the Commercial Schools of Class A.

Mental Calculation.

1. Addition.
2. Subtraction.
3. Multiplication.
4. Division.

Soroban-Calculation.

1. Addition.
2. Subtraction.
3. Multiplication.
4. Division.

Arithmetic.

In most schools, this subject is given in the preparatory course.

1. Integers and decimals.
2. Compound numbers.
3. Measures and multiples.
4. Fractions.
5. Ratio and proportion.
6. Percentage.
7. Square root and cube root.
8. Mensuration.

Commercial Arithmetic.

1. Introduction :—

   Scope and nature of the subject. Specialities of the subject compared to ordinary arithmetic.

   Mathematical applications and their practical uses.

   Statement of accounts.

   Relative position of the *soroban* in arithmetical calculation.

2. Time saving rules :—
   Rapid calculation.
   Approximation and contraction.
   Testing.
3. Weights and measures, monetary systems, time.
4. Trading, profit and loss :—
   Prices, Rates.
   Discount.
   Tares and drafts.
   Profit and loss, and rate per cent.
5. Trade charges and office expenses :—
   Freight.
   Insurance premiums.
   Storage.
   Customs.
   Commission and brokerage.
   Taxes.
6. Interest calculations :—
   Interest.
   Discount.
   Current deposit accounts.
   Accounts current.
   Equation of time.
   Investments. (Public bonds, stocks, immovable property, long term loans.)
7. Proportional division :—
   Apportionment.
   Average settlement.
8. Foreign exchange.
9. Invoices, account sales, &c.
10. Standing-in accounts.
11. Calculations in accountancy.
12. Annuities and instalments.
13. Miscellaneous exercises.

ALGEBRA.

1. Introduction.
2. Four rules for integral expressions.
3. Simple equations.
4. Measures and multiples.
5. Fractions.
6. Fractional equations.
7. Square root and cube root.
8. Quadratic equations.
9. Irrational quantities.
10. Proportion.
11. Progressions.
12. Logarithms.

GEOMETRY.

1. Introduction.
2. Straight lines.
3. Rectilinear figures.
4. Circle.
5. Proportion.
6. Solids.

N.B. In many schools, some of the subjects in the syllabuses on algebra and geometry are not given, and in a few, geometry is entirely omitted.

Syllabus of Mathematical Instruction in the Commercial Schools of Class B.

ARITHMETIC.

1. Mental calculation.
2. Soroban-calculation.
3. Written calculation.
4. Compound numbers.
5. Fractions.

6. Proportion.
7. Percentage.
8. Commercial calculations.

Generally, four or five hours per week are given to this subject throughout the years over which the course extends.

In some schools, this subject is divided into three branches, viz:—

1. Soroban-calculation and Mental calculation.
2. Ordinary Arithmetic.
3. Commercial Arithmetic.

And in that case, the syllabuses of these subjects in the Commercial Schools of Class A are gone through briefly.

Examinations.

There are three kinds of examinations:—

1. Annual examination,
2. Term examination,
3. Occasional examination.

Some schools hold all the three examinations, some the first and the second only, while others the first and the third only.

Besides the marks obtained from these examinations, daily marks are taken from daily exercises.

The annual mark which decides the promotion or graduation is based on all of these marks.

Methods of Teaching.

Text books being used, the method of teaching is similar to that in the Middle Schools, excepting (i) much stress is laid on the practice of calculations (mental, *soroban* and written), (ii) applications and examples relating to commerce are given profusely, and (iii) special attention is paid to the following points:—

Commercial usages.

Correct and concise statement of accounts.

Ready application of mathematical notions.

Quick understanding, logical reasoning, exact and clear thinking.

Good habits such as the keeping of things in an orderly manner.

Generally, the following models and instruments are made use of :—

Models of geometrical figures,

Scales, weighing machines,

Japanese measures, metric measures, English measures, &c ;

and in some schools calculating machines.

Preparation of Teachers.

For the preparation of commercial teachers, there is a special organization called the Commercial Teachers' Training Institute (*Shōgyō Kyōin Yyōseijo*) in connection with the Tōkyō Commercial College. The course of study covering four years, is nearly the same as that of the College and is as follows :—

		No. of hours per week.			
		1st year	2nd year	3rd year	4th year
1.	Commercial Ethics	1	1	1	1
2.	Japanese Penmanship . ..	1			
3.	Japanese Composition	2	1		
4.	Mathematics	3	2	3	1
5.	Commercial Geography		2	2	
6.	Commercial History				3
7.	Book-Keeping	3	2	2	1
8.	Applied Physics	1			
9.	Applied Chemistry	3			
10.	Mechanical Engineering		1		
11.	First principles of Law	2			
12.	First principles of Economics ..	1			
13.	Commercial Products		3		

14.	Economics		3	3	3
15.	Finance				2
16.	Statistics			1	
17.	Private Law		3	3	4
18.	Constitutional Law				2
19.	English	9	6	6	6
20.	Commercial Science		2	7	
21.	Commercial Practice				9
22.	Pedagogics & Methods of Teaching			1	1
23.	Psychology & Logic	1	1		
24.	Gymnastics	3	3	2	
	Total No. of Hours per week ..	30	30	31	33

The conditions for admission are the same as for the Tōkiō Commercial College, except that the Normal Schools are added to the schools whose graduates are entitled to sit for the entrance examinations.

The syllabus of mathematics is exactly the same as that of the above College.

Though this institute can not be looked upon as strictly an organization for providing teachers of commercial arithmetic, some of the graduates of this institute may be chosen for the positions as teachers of mathematics in the Commercial Schools.

SECOND PART.

MODERN TENDENCIES OF THE TEACHING OF MATHEMATICS IN COMMERCIAL COLLEGES AND SCHOOLS.

Modern Ideas Concerning the Organization of Commercial Schools.

According to circumstances, people may enter upon a business career at any stage of their existence, and at any period of their education. In most of the large cities, besides many regular schools of commerce, there are a number of evening schools and even morning schools, which are attended by eager persons who are free only before or after the usual business hours. In such schools, there sit, side by side, boys of say 13 years of age and adults of 30 years or more. If a boy, having left the elementary school, enters into a supplementary commercial school for one year, he receives only one year's instruction in commerce, while if he begins his studies in the preparatory course of a commercial school of Class A and continues them until graduation in the professional course of the Tōkiō Commercial College, he spends 12 years in study. Between these two extremes we have courses of every length, namely; $1\frac{1}{2}$ years, 2 years, $2\frac{1}{2}$ years, etc. And even courses covering the same number of years may be different in the different schools. Hence, as mentioned in the first part of this paper, numerous types of commercial schools have come into existence — which may be looked upon as the embodiment of modern ideas concerning the organization of commercial education.

Modern Tendencies Concerning the Aim and the Material of Instruction.

The application of mathematics to commerce has been greatly extended of late; and moreover, it is found that mental training in mathematics should be made really effective in its results, people having experienced many grave consequences which must be attributed to their lack of the mathematical sense.

Thus the subject-matter of mathematical courses having been increased both in their extent and depth, serious attention must be given to the economical use of time allotted to the courses. But in rearranging or amalgamating the subjects for this purpose, careful deliberation should be applied lest their essential character be impaired.

The separation of arithmetic and algebra which had long been looked upon as inconvenient by many educators may be considered to have been dismissed *de facto*, though not formally, by the regulations pertaining to the syllabus of study in the Middle Schools issued by the Department of Education a few months ago, in which the advanced part of arithmetic is combined with algebra. This change has extended the subject and the meaning of elementary algebra in the Japanese educational nomenclature, and as a matter of course, it will exert some influence on the mathematical course in the Commercial Schools of Class A.

Examinations.

Although examinations can not be taken as showing the real attainment of students, they are still convenient means of obtaining some idea of their proficiency. They serve, moreover, as a means for making the pupils more diligent, and for giving them a good opportunity to summarize all the subjects previously studied.

Under such considerations, the marks obtained from

examination papers, in almost every school, are combined with the daily marks in order to get the annual marks, in such a way that the daily marks count for about one-half.

Methods of Teaching.

In the whole mathematical course, it is not only necessary to make the different subjects harmonize, but also to use to advantage the knowledge obtained in other courses.

Thus, care should be taken primarily to keep the mathematical course in close touch with kindred ones, such as book-keeping, accountancy, commercial science and commercial practice; also, to a certain extent, with lessons on commercial products, physics, chemistry, economics, statistics, etc.

Invoicing or billing is one of the subjects to be particularly emphasized in commercial arithmetic. Even in the lowest class, the drafting out of simple invoices is a good exercise, and in advanced classes, this may be extended to the working out of more complicated commercial calculations. Hence, as an effective method of teaching commercial arithmetic, the making out of invoices and other documents by pupils themselves may well deserve the special attention of teachers.

One of the causes for the dislike of mathematical subjects on the part of some pupils, is the fact that they consider calculations to be hard work. In order to remove this impression it will be advisable, first of all, to assist their progress so as to render the work easy for them.

It has been suggested by some experts, that in order to stimulate practice in calculations, a competitive meeting should be held once or twice a year. Good results may be expected, provided all the boys in the class, or at least a majority of them, take part in it.

Relations Between Commercial Arithmetic and Other Branches of Mathematics.

The relations of the different subjects of commercial arithmetic (or mathematics) to the various parts of the other branches of mathematics are of very complex nature; if they are examined minutely, various degrees of relationship would be found, so that some have a high degree of importance whilst others are comparatively less important, and moreover they may have quite different values according to the opinions of instructors. We shall here try to examine briefly some of the essential points.

Arithmetic being the foundation of mathematical instruction, every part of it is used in commercial calculations.

Positive and negative quantities, the reduction of algebraic expressions, and the solution of equations are of value for general culture as well as for the solution of mathematical problems generally.

Geometry is valuable for the training of logical reasoning and for general culture.

Arithmetical and geometrical progressions, and logarithms are useful in the solution of problems in compound interest, annuities and instalments.

Summation of series is made use of in advanced problems in annuities and instalments.

Theory of probability is used in the solution of problems in insurance.

Graphical representation of functions is used in the illustration of economic theories and phenomena.

Theory of equations is used in finding rate of interest in problems pertaining to annuities and instalments.

Differential calculus and integral calculus are used in the advanced parts in actuarial and statistical calculations.

The relations of commercial arithmetic to various

branches of commercial science and the correlation between its parts are too obvious to be dwelt upon further in this paper.

Preparation of Teachers.

For a teacher of commercial mathematics or arithmetic, not only mathematical knowledge and talents, but also sufficient preparation in commercial science and transactions as well as pedagogical science and practice are required, so that we need a special institution for each kind of school of commerce. If this be difficult to realize, either of the two following plans may be suggested.

(1) Giving an additional preparation of study on advanced branches of mathematics and pedagogics to those who have received a higher commercial education.

(2) Giving a sufficient knowledge of commercial science and transactions to those who have studied mathematics and pedagogics.

It may also be remarked that there is some possibility of obtaining efficient instructors by teachers' certificate examinations under proper regulations; but this means should not be looked upon as the only source of supply.

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Article XII.—The Teaching of Mathematics in Technical Schools and Colleges. Prepared by T. Ōtashiro, *Lecturer at the Tokio Technical College*, under the supervision of M. Mimori, *Professor at the Tokio Technical College*, and S. Yokota, *Professor at the College of Engineering, Tokio Imperial University.*

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CONTENTS.

[Unfinished Manuscript.]

Introductory Remarks.

In order to find the mathematical courses appropriate, at the present, to our technical schools, it is necessary that we should investigate what branches and subject-matters of mathematics are applied in the study of the other sciences to be taught in such schools, and how they are taught.

The Reference Matter consists of the extracts from the lectures on the other subjects, in which mathematics finds its application. We have made these extracts, because we think it to be the only means by which we may know to what extent and how mathematics is applied. They are those from the lectures given at the Tokio High Technical School during the last three years.

The essential part of the present report consists of the Reference Matter, as we shall be able to draw many important conclusions, if we study them thoroughly. We regret that we were not able to discuss them in detail on account of lack of time. And for the same reason we have been unable to pay sufficient attention to our way of exposition.

Whether the lectures from which the Reference Matter has been extracted, are the most appropriate for the object of the Technical School, and whether the present system of education is a proper one,—the questions which lead to a fundamental investigation into the mathematical course appropriate to the Technical School—can not be answered in such a short time as was granted us. We have therefore assumed that the Reference Matters here given, are sufficient for the object of the Technical School under the present system of education.

This report is divided into three parts, of which Part I contains the Reference Matter, Part II the report on the mathematical instruction in the Technical School of Middle Grade, and Part III that of the High Technical School.

PART I.

Reference Matters.

CHAPTER I.

Application of the Four Rules of Algebra.

1. System in which each rope has one end fixed, passes round a pulley, and is fastened to the preceding one (geometrical progression).

2. Tractive resistance and effort:—
   (*a*) HP at uniform speed on level load.
   (*b*) HP at uniform speed on gradient.
   (*c*) HP for accelerating.

3. Sander's formula and Wellington's formula in Architecture (work).

4. Quantity of raw materials lifted up per minute by the elevator and creeper (volume of cylinder).

5. Oil testing machine (friction, condition of equilibrium of forces, trigonometrical function).

6. Buckton machine for testing tensile and compressive strength (theorem of lever).

7. Impact testing machine (work).

8. Tractive effort of an engine (work, friction, perimeter of circle, volume of cylinder).

9. Strength of boiler shell (areas of rectangle and circle).

10. Relative volume of steam (density of gas, Charles' law).

11. Horse power of a steam engine (volume of cylinder).

12. Babcock's empirical formula for the determination of size of pipe (an example for calculation of number).

13. Pucking ring (composition of force, moment of force, area of rectangle) :—

To prove $t = t_1 \sqrt[3]{\sin^2\frac{\theta}{2}}$, where t_1 is the thickness of the thickest part and t that of any part of the ring.

14. Diameter of piston rod (area of circle).

15. Velocity ratio of crank and connecting rod, and determination of pressure on the cross head pin (resolution of velocity, similar triangles, trigonometrical function).

16. Inertia of the reciprocating part in a steam engine (centrifugal force).

17. Pendulum governor (centrifugal force, condition of equilibrium of forces, perimeter of circle).

18. Safety valves :—

(*a*) Lever safety valve.

(*b*) Dead weight safety valve.

(*c*) Spring safety valve.

19. Depreciation (interest).

20. Jaw crusher (friction, perimeter of circle, resolution of force, trigonometrical function).

21. Brake dynamometer (friction, perimeter of circle).

22. Centrifugal dryer (perimeter of circle).

23. Cutting horse power in Theory of Cut.

24. Increase of temperature of material when all the cutting work done by a cutter during the interval required to travel a certain length of the material is converted into heat (the first law of thermodynamics, specific heat).

25. Definition of shearing strain (radian).

26. Stress due to temperature (linear expansion by heat).

27. Ultimate shearing stress, etc. (number).

28. Height of pin head and diameter of neck (area of circle, area of cylinder).

29. Number of bolts corresponding to the total pressure upon the cover of a pipe (area of circle).

30. Stress at a point on a cross section of a beam is proportional to the distance of the point from the neutral axis (rectangle, parallelogram, trapezoid, similar triangles).

31. Shaft for the transmission of power (work, perimeter of circle).

32. Angle of twist in round shaft (radian).

33. Stress of wire rope (area of circle).

34. Strength of draught of chimney (Charles' law, variation).

35. Sectional area of chimney (Torricelli's law, volume of cylinder).

36. Stability of chimney (condition of equilibrium of forces, trigonometrical function).

37. Power of water wheel.

38. Pump (theorem of lever, volume of cylinder).

39. Suction and delivery pipes (Torricelli's law, volume of cylinder).

40. Centrifugal pump (Torricelli's law, perimeter of circle, volume of cylinder).

41. Effective head and lost head energy, Torricelli's law).

42. Fluid friction (variation).

43. Circular conduit partly full (perimeter of circle, areas of triangle, sector and circle, radian, inverse trigonometrical function).

44. Barrel or tank calorimeter.

45. Heat quantity produced from fuel (calorimetry).

46. Calculation of the number of interlacing in Technology of Textile Design (combination).

47. Ohm's law (variation).

48. Energy loss due to electric resistance (variation).

49. Relation between the flux and reluctance in a magnetic circuit (variation).

50. Specific resistance, conductivity, reluctivity, permeability (variation).

51. Relation between the induced electro-motive force and magnetic lines of force (variation).

52. Efficiency of dynamo (percentage).

53. Formula with respect to a lamp.

54. Life of electric lamp (percentage).

55. Empirical formula showing the relation between the strength of current passing through a conductor and its sectional area, the maximum rise in temperature being given (an example for calculation of number).

56. Weight of wire is inversely proportional to the square of voltage.

57. Relation between strength of current and voltage, voltage and number of turn in a transformer.

58. Formula with respect to conductor size.

59. Calculation of the out put corresponding to given number of cars.

60. Calculation of size of trolley wire (arithmetical progression).

CHAPTER II.

Application of Elementary Mathematics to Mechanism and Drawing.

Elementary mathematics are applied to the following problems on Mechanism:—

Double point or centre of similitude.

Crank and rocker (lever crank).

Relation between crank angle and displacement of cross head.

Inversion of double cross slider crank chain mechanism.

Peaucellier's straight line motion.

Hart's straight line motion.

Bricard's straight line motion.

Approximate straight line motion.

Pantograph.

Length of belt (crossed belt, open belt).

Cord friction.

And elementary mathematical problems used to treat them are as follows :—

In Algebra—Factorisation. Index. Proportion. Variation. Binomial theorem with any index.

In Geometry—Congruency of triangles. Relation between the sides and angles of a triangle. Isosceles triangle. Parallelogram and Rhombus. Proportion. Similar figures. Locus.

In Trigonometry—Radian, $\sin\theta \div \theta$. Some fundamental formulae. Common and natural logarithms.

Elementary mathematics are applied to the following problems on Drawing :—

To find the length of a limited straight line and angles between the line and two planes of projection.

To draw a straight line through a given point and parallel to a given straight line.

To draw a perpendicular from a given point to a given plane.

To draw a plane passing through three given points.

To find the angle between two planes.

To find one of the projections of a point which lies on a given plane, the other being given.

To find the intersecting point of a straight line and a plane.

To draw a plane through a given point and perpendicular to a given straight line.

To find the angle between two straight lines.

To draw a straight line whose length is equal to a circular arc.

To draw a circular arc whose length is equal to a given straight line.

To draw regular polygons, one side being given.

To draw an involute of a given circle.

To draw cycloids.

And elementary mathematical problems used to treat them are as follows:

There is only one straight line passing through two given points.

There is only one straight line passing through a given point and parallel to a given straight line.

Pythagoras' theorem. Similarity of plane figures.

There is only one plane passing through three points which do not lie on a straight line.

The intersection of two planes is a straight line.

There is only one plane containing two intersecting lines.

If there are two planes parallel to a given straight line, then their intersection is a straight line parallel to the given straight line.

CHAPTER III.

Application of Analytical Geometry.

Graphical Representation of the relation between two Physical Quantities.

1. Displacement and time in a simple harmonic motion.
2. Variable speed and time in a motion of a particle.
3. Path of a particle with constant acceleration and a certain initial velocity.
4. Strain-diagram.
5. Cutting stress and cutting angle.
6. Discharge curve in Hydraulics.
7. Oval diagram and harmonic diagram.
8. Temperature and boiler pressure, draught, feed water etc.
9. Isothermals and adiabatics.
10. Indicator diagram.
11. Entropy-temperature diagram.

12. Solubility curve in Physico-chemistry.

13. Luminosity curve, absorption curve in Colour Mixing and Colour Matching.

14. Voltage of a secondary battery and time; the density of electrolyte and voltage; voltage and temperature; concentration of electrolyte and its electric resistance and temperature.

15. Electric resistance and temperature.

16. Hysteresis curve.

17. Thermo-electric curve.

18. Thermo-electric power and temperature.

19. Instantaneous induced E.M.F. and time ($e = \mathrm{E}\sin\omega t$).

20. Probability curve.

Equation of a Plane Curve, and Geometical Properties of Conics.

21. Determination of the path of a projectile.

22. Equations of bending moment and shearing force.

23. Stress figure.

24. Inversion of double cross slider crank chain mechanism (equation of ellipse and the relations between the rectangular and polar co-ordinates).

25. Explanation of the elliptic toothed wheel (geometrical properties of an ellipse).

26. Revolving fluid in Hydraulics (equation of a parabola and its geometrical properties).

27. Calculation of the discharged quantity of water from a circular orifice in Hydraulics (equation of circle and the relation between the rectangular and polar co-ordinates).

28. Calculation of the time of emptying a semi-spherical vessel when water discharges from the bottom (equation of circle).

29. To express that a plane curve obtained by the experiment is an algebraic equation. e.g. If the relation between the linear expansion by heat or electric resistance of a substance

and temperature is expressed by a straight line, its equation takes the form $y=a(1+bx)$, or thermo-electric curve is shown by the equation of a parabola.

30. Boyle's law (equation of hyperbola).

31. To prove the wave front after reflection by a parabolic mirror is a plane, incident rays being parallel to the axis (geometrical property of parabola).

32. In cases where we prove that the two polars of a point on the boundary of an icelandspar and air with respect to the ellipse and circle on a cross section of the wave surface coincide.

33. Equation of the second degree shewing the characteristic of an alternator represents an ellipse.

34. Length of transmission line, whose span and sag are given (equation of parabola.)

35. Power of motor and generator connected together in series (equation of hyperbola $xy=$constant).

36. In Drawing, meaning of the locus of a point is frequently used.

37. In Drawing, construction of *a*) Ellipse.
b) Parabola.
c) Rectangular hyperbola.

Equation of a Plane. Co-ordinates of a Point.

Definitions of Some Surfaces and Space Curves.

38. To prove that the magnitude of pressure at a point in liquid is independent of the direction of the plane on which the pressure acts (equation of a plane, and relation between direction cosines).

39. Fundamental formulae in Spherical Trigonometry (relations between the rectangular and polar co-ordinates).

40. Composition and resolution of vectors.

41. In Drawing, the definitions of the following terms are given:

Helix, Ellipsoid, Hyperboloid, Paraboloid, Sphere etc.

CHAPTER IV.

Application of Differential, Integral Calculus and Differential Equation.

The Meaning of Differential, Differential Coefficient and Integral. Properties of an Integral.

1. Centre of gravity.

2. Moment of inertia.

3. Pressure at a point in liquid.

4. Principle of Archimedes.

5. Elementary volume of liquid which flows through an elementary area dA at a point in dt, $dQ = V \sin\theta \cdot dA \cdot dt$, where V denotes velocity and θ the inclination of dA to the stream line passing through the point.

6. Torricelli's law.

7. Bernoulli's theorem.

8. Discharge curve in Hydraulics (geometrical meaning of $\int y dx$).

9. Total vertical component of the pressure upon a boiler shell $(pr\int \sin\theta d\theta)$.

10. Work done by gas (geometrical meaning of $\int y dx$).

11. Heat quantity determined by means of entropy-temperature diagram (geometrical meaning of $\int y dx$).

12. Acceleration in the motion of crank and connecting rod $\left[\frac{dV}{dt} = a\left(\cos\theta + \frac{\cos 2\theta}{b}\right)\frac{d\theta}{dt}\right]$.

13. Probability curve (geometrical meaning of $\int y dx$).

Derived Functions. Differential Equations and their Solutions.

14. Pressure due to head upon an elementary area immersed in liquid.

15. Pressure on a vertical plane and its moment in Hydraulics.

16. Bernoulli's theorem.

17. Change of pressure along the normal at a point on a stream line

$$\frac{wv^2}{gr}=\frac{dp}{dr}.$$

18. Time of liquid emptying a vessel, an orifice being at the side or bottom of the vessel:

$$dQ=Ady, \quad dQ=a\,(c_0\cdot v)\,dt$$
$$-Ady=C_0\cdot av\cdot dt$$
$$\frac{C_0\cdot a\sqrt{2g}}{A}dt=-\frac{dy}{\sqrt{y}}.$$

19. Time of liquid emptying a hemi-spherical vessel, an orifice being at the bottom $\left(dy=\frac{ax-bx^2}{c\sqrt{x}}dx\right)$.

20. Discharged quantity of water which flows from a vertical orifice:—

(a) Rectangular orifice $(dy=a\sqrt{x}\,x\,d\,x)$.

(b) Circular orifice $(dy=a\sin^2\theta(1-b\cos\theta-\cos^2\theta-\,.\,.)d\theta)$

(c) Triangular orifice $(dy=a(b-x)\sqrt{x}\,dx)$.

21. In a tube whose surface is bounded by stream lines of water,

$wdy+dp+\xi w\frac{B}{A}\cdot\frac{V^2}{2g}\cdot dl=0$, where ξ denotes coefficient due to friction, A cross section of the pipe, B perimeter of the pipe, and l length of the pipe.

22. Relation between the cross sectional area of water course and its gradient $\left(\frac{\Delta h}{h}=a\frac{\Delta A}{A}\right)$.

23. Cord friction $\left(ady=\frac{dx}{x}\right)$.

24. Maximum cutting force and its moment of a twist drill $(dy=adx.\ \ dy=bxdx)$.

25. Work done by a plane milling cutter ($dy = a \sin x dx$).

26. Distribution of load and relative motion between the shaft and its support ($dy = a \cos^2 x dx$).

27. Work of elongation or resilience ($dy = ax dx$).

28. Stress on the inner surface of thick cylinder ($x dy + 2y dx = 2a dx$).

29. Moment of inertia and theorem about it.

30. Relation between the bending moment and vertical shear: $\frac{dM}{dx} = V$.

31. Stress figure (we find some application of moment of inertia).

32. Differential equation of elastic curve.

33. Deflection of cantilever beams:—

(a) A load at end

$$M = -Px = EI\frac{d^2y}{dx^2}.$$

(b) Uniform load

$$M = -\frac{1}{2}wx^2 = EI\frac{d^2y}{dx^2}.$$

Deflection of simple beams:—

(a) A single load at the middle

$$M = \frac{Px}{2} = EI\frac{d^2y}{dx^2}.$$

(b) Uniform load

$$M = \frac{wl}{2}x - \frac{w}{2}x^2 = EI\frac{d^2y}{dx^2}.$$

34. Elongation of a spiral spring ($dy = a dx$).

35. Euler's formula for a long column.

36. Tension of a thick pipe $\left(dy = a\frac{dx}{x^2}\right)$.

37. Crushing force of a jaw crusher $\left(dy = a\frac{dx}{x}\right)$.

38. In Thermodynamics:

$$\frac{dT}{T}+(\gamma-1).\frac{dV}{T}=0,$$

$$\frac{dT}{T}-\left(\frac{\gamma-1}{\gamma}\right).\frac{dP}{P}=0,$$

$$\frac{dP}{P}+\gamma\frac{dV}{V}=0,$$

where γ is the ratio of specific heats.

29. Work done by a gas during its isothermal change $\left(dy=a\frac{dx}{x}\right)$.

40. Work done by a gas during its adiabatic change $\left(dy=a\frac{dx}{x^m}\right)$.

41. When wet steam is made from water at the temperature T_0 the entropy

$$\varphi=\int_{T_0}^{T_1}\frac{dh}{T}+\frac{q_1T_1}{T_1}=\int_{T_0}^{T_1}\frac{dT}{T}+\frac{q_1T_1}{T_1}.$$

42. To find the condensing surface of a surface condenser $(dx=a.\quad a(y+b)dx=cdy)$.

43. In a steam turbine, the horizontal and vertical components of the centrifugal force which steam along the vane undergoes $(dx=a\sin a da.\quad dy=a\cos a da)$.

44. In Physico-chemistry, the relation between the reaction constant and temperature is shown by the following equation,

$$\frac{d\log k}{dT}=-\frac{Q}{2T^2},$$ where k denotes reaction constant,

T absolute temperature, and Q heat of reaction.

45. Velocity of a solution.

46. In Electro-chemistry, Helmholz's formula.

47. Relation between induced electro-motive force and the rate of change of flux is expressed by the following equation :—

$$e = -\frac{dN}{dt}.$$

If $N = \varphi \cos \omega t$, then the average power lost by the rotation of a coil

$$\frac{1}{\pi}\int_0^{\pi} r \,.\, L^2 . \sin^2(\omega t) d(\omega t) = \frac{I^2}{2} . r,$$ where r denotes the resistance of the coil.

48. Induced electro-motive force due to self-induction

$$e = -L\frac{d\,i}{d\,t}.$$

In a circuit with ohmic resistance R,

$$E - L\frac{d\,i}{d\,t} = iR.$$

The electric energy given to the circuit in dt is

$$E\,i\,dt = i^2 R dt + Li\frac{d\,i}{d\,t}dt.$$

$Li\frac{d\,i}{d\,t}\,dt$ is accumulated in the circuit. The total energy accumulated until the strength of current becomes from i to I

$$\int_0^I Li di = \frac{L}{2} I^2.$$

When a circuit whose strength of current is I is broken,

$$0 = Ri + L\frac{d\,i}{d\,t},$$

and the energy lost

$$\int_0^{\infty} i^2 R dt = \frac{LI^2}{2}.$$

And we can find from derived functions of $i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}.t}\right)$ and $i = Ie^{-\frac{R}{L}.t}$ how change of strength of current i takes place according as time elapses.

49. Electro-motive force in a circuit in the preceding article, the current being $i = I \sin \omega t$.

50. Induced electro-motive force in a coil due to the current of another coil.

$$e = -M\frac{d\,i}{d\,t}.$$

In the first coil, $E_1 - L_1 \cdot \frac{di_1}{d\,t} - M \cdot \frac{di_2}{d\,t} = R_1 i_1$, and in the second $E_2 - L_2 \cdot \frac{di_2}{dt} - M \cdot \frac{di_1}{dt} = R_2 i_2$.

51. Relation between the current and electro-motive force of a condenser

$$i = \frac{dq}{dt} = C\frac{de}{dt}.$$

And to apply this relation for the cases $e = F \sin \omega t$ and $i = I \sin \omega t$.

52. Calculation of the power loss due to eddy current in sheet iron of a transformer ($dy = ax^2 dx$).

53. Magnetic flux in a conductor ($dy = axdx$).

54. In the Theory of Alternating Current, when a motor and generator are connected together in series, it is found the change of their powers according as the variation of the phase difference of their electro-motive forces by differentiating the following functions

$$f_1(x) = a + b\cos(c - x),$$

$$f_2(x) = a' \cos x - b'.$$

55. From the fundamental equation about generator and synchronous motor, it is found by differentiating it the change in current according to the change of the electro-motive forces of generator and synchronous motor, and maximum and minimum of current can also be found.

56. When two alternating current generators are in series, of which if the electro-motive force of one generator is equal to $E \sin(wt + a)$ and the other $E \sin(wt - a)$, then

$$L\frac{di}{dt} + Ri = 2E \sin wt \cos a.$$

And in the case of parallel running of alternators,

$$L\frac{di_1}{dt}+ri_1=E\sin(wt+a)-\left[L_L\frac{d(i_1+i_2)}{dt}+R_L(i_1+i_2)\right],$$

$$L\frac{di_2}{dt}+ri_2=E\sin(wt-a)-\left[L_L\frac{d(i_1+i_2)}{dt}+R_L(i_1+i_2)\right].$$

57. Determination of the equation of the probability curve.

58. List of differential equations, whose solutions are required in the foregoing subject-matters:

(1) $\frac{dy}{dx}=a.$

(2) $\frac{dy}{dx}=ax.$

(3) $adx+bdy+cdz=0. \quad axdx+bdy+dz=0.$

(4) $\frac{dy}{dx}=a\sqrt{x}.$

(5) $\frac{dy}{dx}=\frac{a}{x}. \qquad \frac{dy}{dx}=\frac{a}{x-b}.$

(6) $\frac{dy}{dx}=ax^2.$

(7) $\frac{dy}{dx}=\frac{a}{x^2}$

(8) $\frac{dy}{dx}=\frac{a}{x^m}.$

(9) $\frac{dy}{dx}=a(b-x)\sqrt{x}.$

(10) $\frac{dy}{dx}=a\sqrt{(b-x)(c-x^2)}.$

(11) $\frac{dy}{dx}=a\sqrt{y}.$

(12) $a\frac{dy}{dx}+by=0. \qquad a\frac{dy}{dx}+by+c=0.$

(13) $\frac{dy}{dx}=a\frac{y}{x}.$

(14) $\frac{dy}{dx}=a \sin x.\quad \frac{dy}{dx}=a \cos x.$

(15) $\frac{dy}{dx}=a \cos^2 x.\quad \frac{dy}{dx}=a \sin^2 x.$

(16) $\frac{dy}{dx}+by=c \sin x.$

(17) $a\frac{dy}{dx}+by=a' \sin(wx+\alpha)-\left\{b' \frac{d(y+z)}{dx}+c'(y+z)\right\},$

$a\frac{dz}{dx}+bz=a' \sin(wx-\alpha)-\left\{b' \frac{d(y+z)}{dx}+c'(y+z)\right\}.$

(18) $\frac{dy}{dx}=ae^{-ax}$

(19) $\frac{d^2y}{dx^2}+bx=0.$

(20) $a\frac{d^2y}{dx^2}+bx^2=0.$

(21) $a\frac{d^2y}{dx^2}+bx^2+cx=0.$

(22) $a\frac{d^2y}{dx^2}+Py=0.$

Integration and Change of the Variable of Integration.

59. The followings occur in the foregoing subject-matters :

(1) $a\int x^m dx.$

(2) $a\int\frac{dx}{x}.\qquad a\int\frac{dx}{b-x}.$

(3) $a\int\frac{dx}{x^m}.$

(4) $a\int x^2(b-x)dx.$

(5) $a\int(b-x)\sqrt{x}\, dx.$

(6) $a\int\sin a da.\qquad a\int\cos a da.$

(7) $a\int \cos^2 \alpha d\alpha$.

(8) $a\int \sin \alpha \sin(\alpha+\varphi)d\alpha$,

(9) $a\int \sin^2\alpha(1-\frac{\cos \alpha}{b}-\frac{\cos^2 a}{b}\ldots.)d\alpha$.

(10) $a\int e^{-bx}dx$.

(11) Determination of the constant in the equation of probability curve. Formulae for probable and mean errors.

Differentiation. Maxima and Minima of Functions.

60. When energy is supplied to a water wheel by impulse, the amount of it depends upon the velocity of the wheel, the relation between energy E and velocity V is expressed as follows :

$$E=a(b-V)V.$$

We can find the value of V for which E is maximum.

61. Maximum value of the acceleration (Art. 12 Chap. (IV)).

62. To find the maximum value of bending moment due to centrifugal force of the connecting rod from the formula

$$M=a(bx-x^3).$$

63. Determination of the speed of a belt by which maximum power is transmitted (maximum value of $y=a(b-cx^2)x$).

64. Art. 55 Chap. (IV).

65. To cut a rectangular beam of a timber whose diameter is given, so that the beam may have maximum strength (maximum value of $y=ax(b-x^2)$).

66. Determination of maximum value of tangential stress

on a cross section of a beam (maximum value of $y=a\sin x\cos x$).

67. In Hydraulics, to design a channel, it is necessary that its perimeter must be small and sectional area large, since the friction of water is proportional to the perimeter and inversely proportional to the area :—

(a) Rectangular channel $\left(\text{value of } x \text{ in order that } y=a(b-x)x \text{ may have a maximum value}\right)$.

(b) Trapezoidal channel $\left(\text{value of } x \text{ in order that } y \text{ given by the equation } \frac{y}{x}+ax=y^3 \text{ may have a maximum value}\right)$.

68. Determination of the value of a which makes $\Sigma h^2(x-a)^2$ minimum in the Method of Least Squares.

69. List of functions, whose differential coefficients are necessary in the above mentioned subjects :—

(1) Rational integral functions of x.

(2) Trigonometrical functions and rational integral functions of them.

(3) $y=\dfrac{a}{x}+bx$.

(4) $f(x,y)=ay^3-by-cx=0$.

(5) $dQ=d\phi+PdV$, when $\phi=f(T, V)$.

Expansion of Functions. Tracing of Curve. Plane Curve, and others.

70. Expansions of the following functions :—

(a) $y=\sqrt{1-x}$, $y=\sqrt{1+x^2}$, $y=\log_e\sqrt{1+x^2}$,

$y=\log_e\left(1+\dfrac{x}{\sqrt{1+x^2}}\right)$,

and Fourier's series (in the Theory of Alternating Current).

(b) $y = \log_e (1+x)$ (in Electro-chemistry).

(c) $y = a\left(1-\frac{x}{a}\right)^{\frac{1}{2}}$ (in Hydraulics).

(d) Reduction of observation equations to the linear forms.

71. Tracing of curves :—

(a) It is necessary to trace the curve $y = \frac{a}{\log_e x}$ to explain a dryer which is one of chemical machines.

(b) To trace the following curves in the Theory of Alternating Current;

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right),$$

$$i = Ie^{-\frac{R}{L}}.$$

72. Definitions of the following terms :

Cycloid, Epicycloid, Hypocycloid, Involute of a circle (in Drawing).

Logarithmic spiral (on the explanation of mill stone).

Catenary (on the explanation of transmission line in Electrical Engineering).

Spiral curve and parabolic curve (on the explanation of curve of rail).

Parabola of the third and fourth degree (on the explanation of the equation of elastic curve).

CHAPTER V.

List of fragmental subjects regarding the Elementary Mathematics applied to several articles mentioned above.

In Arithmetic :—

(1) Numerical calculation.

(2) Percentage.

In Alegebra

(3) Fraction.

(4) Factorisation.
(5) Equations of the first degree.
(6) Power and index.
(7) Ratio and proportion.
(8) Inequality.
(9) Arithmetical progression and geometrical progression.
(10) Combination.
(11) Interest.
(12) Maxima and minima.
(13) Variation.
(14) Limit.
(15) Infinity.
(16) Binomial series with any index and approximate calculation.
(17) Elimination.

In Geometry :—

(18) Triangle.
(19) Parallel lines.
(20) Circle.
(21) Ratio and proportion.
(22) Similar triangles and figures.
(23) Pythagoras' theorem.
(24) Areas of a triangle, rectangle, circle and sector.
(25) Circumference of a circle.
(26) Area of a right cylinder.
(27) Volume of a parallelopiped, prism, right cylinder, and sphere.
(28) Projection.
(29) Locus.

In Trigonometry :—

(30) Definitions of trigonometrical functions.
(31) Trigonometrical functions and values of them.
(32) Fundamental trigonometrical formulae.
(33) Radian.

(34) $\mathrm{tg}\theta \doteqdot \sin\theta \doteqdot \theta$ for small angle θ.

(35) Natural and common logarithms.

(36) Inverse trigonometrical functions.

(37) Solution of a trigonometrical equation.

(38) Summation of simple trigonometrical series.

PART II.

The Technical School of Middle Grade.

CHAPTER I.

Aim of Technical Schools of Middle Grade.

Aim of these schools is the same as that of High Technical Schools.

CHAPTER II.

Aim and Material of Mathematical Instruction.

1. Aim. The object of mathematical instruction in the school is to enable the students to understand the lectures on engineering and other like subjects, and also to train them in practical applications.

2. Materials of instruction. From the Reference Matter, we see that the branches of elementary mathematics, preparatory to other sciences, especially in industrial ones, are as follows:—

(1) Arithmetic.
(2) Algebra.
(3) Geometry.
(4) Trigonometry.

As a graduate or one who has the same attainments as a graduate of an elementary school is admitted to enter the school, the course of mathematical instruction must begin from just the same starting point as in a middle school. In addition to the syllabus of instruction in mathematics in a middle school issued by the Department of Education, it will be desirable in the technical schools to give the following:

In Algebra:

(1) Inequality.

(2) Permutations and combinations.

(3) Maxima and minima.

(4) Variation.

(5) Limit. Infinity.

(6) Binomial series with any index.

In Geometry:

(1) Value of π.

(2) Projection.

In Trigonometry:

(1) Radian.

(2) $tg\theta \div \sin\theta \div \theta$ for small angle θ.

(3) Inverse trigonometrical functions.

(4) Natural logarithm.

(5) Solution of trigonometrical equations.

(6) Summation of simple trigonometrical series.

The combination is used only once in the technology of textile design except the application in the method of least squares. But the variation and the binomial theorem with any index have a wide use. Particularly the terms in the variations "two quantities vary in direct proportion or in inverse proportion or a quantity varies as the square of.... etc." are often used, so that they should be carefully explained, by giving actual examples.

In the theory of alternating current, the summation of certain trigonometrical series is used.

From the Reference Matter, it is to be seen that the problems of arithmetic which frequently occur are those of numerical calculation and of percentage. The numerical calculations would be of great importance for students in helping understanding of problems and their practical applications. Especially for engineers, it is very important to be skilled in numerical calculations.

In teaching the problems of percentage, special attention

should be paid to make the students fully understand the meaning of percentage, by giving actual examples.

It appears from the Reference Matter that nearly all of the applications of algebra are problems of the four rules. Thus in teaching arithmetic and algebra, special attention must be paid to make students familiar with applications of the four rules. To become skillful in calculations of this kind, a student must go through many exercises. And for exercises, we would advise teachers to choose such problems as are connected with the Reference Matter. This remark also applies to the High Technical School. A student of a High Technical School, asked us about the solution of a differential equation, which notwithstanding he has already learned in the lecture of mathematics, but it appeared to him to be a new problem owing to different notations. Mathematical skill of graduates of the High Technical School is sufficient to understand the mathematics used in industrial sciences, but they make often mistakes when instead of symbols numerical values are given. These facts plainly show that mathematics and their practical applications to industry are taught quite apart. A method of relieving such defect, we think, is to make students more familiar with the relations between mathematics and their applications, by giving them such problems as are connected with the Reference Matter.

3. Hours. Duration for instruction in the school is defined by the educational authorities to be three or four years and also two years' preparatory course may be added.

In fact, many schools have no preparatory course at present, and instruction of three or four years is given according to the conditions of the place or kinds of industrial sciences. Last year course has no mathematics at some schools, while mathematics are taught throughout the course at others. Total number of hours for mathematics varies from 200 to 650.

Average age of students of the first year being only about fifteen, it will be necessary to teach mathematics step by step in as long course as possible, and make the foundation for their applications to industrial sciences. On the other hand, the principal aim of mathematical instruction is to make students fully understand how mathematics are applied in industry; hence it will be preferable that the course of mathematics would be finished at least one year back their graduation. Thus, for a school of four years, mathematical instruction should be held during first three years, and last year left for its practical applications.

According to the program described in preceding article, we will recommend the allotment of hours of Okayama Technical School as an appropriate distribution of hours to mathematical branches. The course of this school is four years, and there is no mathematics in the highest class. The allotment of hours is as follows:

Class / Branch	1st year	2nd year	3rd year	4th year	Total
Arithmetic	145	0	0	0	145
Algebra........	78	156	39	0	273
Geometry......	50	78	39	0	167
Trigonometry ..	0	0	78	0	78
Total	273	234	156	0	663

From this table, it seems that ratios of hours for arithmetic, algebra, geometry and trigonometry are as 2 : 3·5 : 2 : 1. We believe, that the hours for algebra are far greater than for other branches, depends upon the fact that lessons to be taught in that branch are greater than others, and also it is on account of the fact that algebra is most frequently used in practical problems as shown in the Reference Matter. Hours for arithmetic are comparatively few, because the students have already learned it in elementary school.

CHAPTER III.

Compilation of Text Books and Examinations.

If there is no dogma on the method of teaching, the only method of teaching according to the program described in preceding section, we think, is to rely upon text-books. The use of text-books in the school should be more necessary than in higher schools, if we observe the difference of ages as well as attainments of students in those schools. Text-books at present used in the school are those for a middle school. But it will be better to compile a text-book proper for students of the school. In respect to this we have the same opinion as in the case of the High Technical School.

As in other schools, written examinations are made in this school, with the objects that the students' knowledge may get well arranged and the stage of their progress in learning may be known. The marks are given, taking into account also the students' skill shown in usual exercise hours. Examinations are divided into three kinds of special, term and annual. Annual examination takes place once a year, and term examination once in a term (there are three terms in a year). Number of special examinations is not settled, and there is none in some schools (see also Chap. IV, Part III).

PART III.

The High Technical School.

CHAPTER I.

Aim of the High Technical School.

The institutions for engineering and industrial education in Japan may be grouped into four classes, namely: Colleges of Engineering in the Imperial Universities, High Technical Schools, Common Technical Schools and Apprentices' Schools. The Common Technical and the Apprentices' Schools comprise the primary courses only and the mathematics there taught is but a simple rudiment. The High Technical School takes up more advanced curriculum and differs not significantly in the variety of courses, from the College of Engineering, except in the extends of time. But sound difference between these two institutions exists in this that the former aims to build up practical and handy engineers, while the latter undertakes to train the students equally on theories and practice. As a natural consequence, there is a corresponding distinction between the methods of teaching mathematics; the practical application is chiefly taught in the school, whereas, in the college, the theories carry as much weight as practice.

CHAPTER II.

Object of Mathematical Instruction in the School and the Lessons given.

1. Object. The only difference between High Technical School and that of middle grade is in degree and kind.

2. Lessons given. By the Reference Matter, we see that the branches of mathematics, preparatory to engineering and other sciences, are as follows:

(1) Arithmetic.
(2) Algebra.
(3) Geometry (Descriptive Geometry is treated here as not a branch of mathematics).
(4) Trigonometry (Plane and Spherical).
(5) Analytical Geometry (Plane and Solid).
(6) Differential Calculus.
(7) Integral Calculus.
(8) Differential Equation.

Now we proceed to discuss the scope of teaching the elementary mathematics. This school admits those to enter, who have completed a Middle School course or those who are regarded as equally qualified. According to the Reference Matter and the syllabus of instruction in mathematics in a middle school issued by the Dapartment of Education, we see that the most parts of arithmetic, algebra, geometry and trigonometry, the indispensable knowledges in this school, are already taught in the Middle Schools and need not to be repeated in this school.

The lessons to be given here are:—

(1) The knowledges to be presupposed in engineering, but which could not be obtained in a middle school.
(2) Those which are taught in a middle school, but require a repetition on account of importance for engineering.
(3) The knowledges to be presupposed in higher mathematics.

We shall discuss this somewhat in detail. (1) We have already described in our report of the Technical School of Middle Grade. (2) Now we look into those which require repetition or renewal on account of their importance in en-

gineering. According to the Reference Matter, those parts of arithmetic, algebra, and geometry which find their application to engineering are almost included in middle school's curriculum and are of such a nature as to require no repetition or renewal. But those subjects, as the determination of a locus, the area of a circle, the volume of a circular cylinder and in particular the use of the trigonometrical functions and formulae and logarithms must be somewhat novel and imperfect knowledge to the graduates of a middle school. An excellent graduate from the machinery course of this school asserted that although he has learned already in the Middle School the trigonometrical functions and formulae appearing in the lectures in this school, but a great labour was needed to make himself accustomed to the practical applications of them. In an hour of exercise on analytical geometry, we met with an example of a student in the first year class of the machinery course, who has forgotton the formulae expressing the tangent of the sum of two angles in terms of tangent of each angle. There are also not a few students, who do not like the troubles of taking recourse to the logarithmic tables in the physical laboratory. Besides the Reference Matter, taking these facts into account, we set down the followings as the subjects which require repetition and renewal :—

a) Exercises on the calculation of surface areas especially of circles.

b) Exercises on the mensuration of solid volume especially of circular cylinder.

c) Exercises on the applications of trigonometrical formulae.

d) On the occasion of all these exercises, to make the students accustomed to the logarithmic tables.

e) Greek letters.

We may drop here a few remark on the slide rule. That much benefit is derived from the use of the slide rule, which

is especially handy in the operations of multiplication and division of numbers, is widely admitted by chemists, physicists, astronomers and particularly by engineers; and it is commendable to be used in the school hours, but it must dispensed with for the present on account of its high price. It is true, that the slide rules may be placed at the students' disposal in school rooms, but this is not sufficient to make them familiar to the application of the implement, which can be attained only by the constant use. Moreover, the principle and the use of slide rules can be comprehended by the students themselves, if they wish, by reading the directions, which require only some knowledge of trigonometry to understand. For that reason the slide rule is not adopted in the curriculum, but the students are directed to use the logarithmic tables as a means for calculation.

The first four of the above mentioned subjects, which are given with a view to renew the students' knowledge once acquired, differ in nature from those, which are to be given on account of their necessity to the subsequent study of either engineering or higher mathematics. The omission of the foregoing lessons will require some labour on the part of the students to understand the lectures on the other subjects. Indeed some of the students may be ready to make this effort, but it is safer to give the above lessons, taking for granted that most of the students do not like the trouble to do so, if we consider their age (the average age being about eighteen) as well as their behaviour so far as we know. (3) Lastly we will enumerate among the subjects of elementary mathematics those which are necessary to the study of higher mathematics, and are not taught in middle school:—

In Algebra.

a) Imaginary numbers.

b) Inequalities.

c) Permutations and combinations.

d) Maxima and minima.

e) Limit, infinity and indeterminate forms.

f) Convergency and divergency of series.

g) Determinant.

[*c*) and *g*) especially in preparation for the study of the Method of Least Squares].

In Trigonometry

a) Inverse trigonometrical functions.

b) Natural logarithm.

c) Solution of trigonometrical equations.

In fine, the subjects of elementary mathematics which are to be taught in this school, are as follows :—

a) Exercises in the calculation of surface areas, especially of circles.

b) Exercises in the mensuration of solid volumes, especially of circular cylinders.

c) Exercises in the application of trigonometrical formulae.

d) The use of logarithmic tables in the foregoing exercises.

e) Greek letters used as symbols of angles.

f) Radian.

g) Inverse trigonometrical functions.

h) Natural logarithm.

i) Solution of trigonometrical equations.

j) Summation of trigonometrical series.

k) Imaginary numbers.

l) Inequalities.

m) Permutations and combinations.

n) Maxima and minima.

o) Variation.

p) Limits, infinity and indeterminate forms.

q) Binomial series with any index.

r) Convergency and divergency of series.

s) Determinant.

Next we proceed to consider the scope of teaching the higher mathematics in this school. The first requisite in teaching analytical geometry, differential and integral calculus, and differential equation is always to keep in mind the prescribed object of the school and not to miss out, in lectures, the general characteristics of science. To obtain this effect, according to the Reference Matter, we choose the followings as the materials to be given in the mathematical lessons.

Analytical Geometry.

Plane Geometry.

- Co-ordinates. Straight line. Transformation of Co-ordinates. Circle. Parabola. Ellipse. Hyperbola. General equation of the second degree.

Solid Geometry.

- Co-ordinates. Projection. Straight line. Plane. Certain surfaces of the second order.
    - Sphere. Cone. Cylinder. Ellipsoid. Hyperboloids. Paraboloids.

Differential and Integral Calculus.

Limits and continuity.

Differentiation.

- Derived functions of some special functions.
    - Logarithmic functions.
    - Exponential functions.
    - Trigonometrical functions.
    - Circular functions.

Applications of Derived Functions.

Successive Differentiation.

Taylor's Theorem and its Applications.

Partial Differentiation.

Applications to Plane Curves.

Integration.

- Properties of an integral.

Fundamental integrals.

Change of variables in integration.

Partial integration.

Integration of Elementary Functions.

Applications of Integrals.

Differential Equation.

Differential Equation of the First Order.

Separation of variables.

Homogeneous equation.

Linear equation.

Riccati's equation.

Exact differential equation.

Integrating factors.

Simultaneous equations of the first order.

Differential Equations of the First Order and Higher Degree.

Solution by separation into factors.

Solution by differentiation.

Clairaut's equation.

Differential Equation of the Higher Order.

Equation $\frac{d^n y}{dx^n} = \varphi(x)$.

Equation $F\left(\frac{d^n y}{dx^n}, \frac{d^{n-1} y}{dx^{n-1}}\right) = 0,$

$$\frac{dp}{dx} = f(p), \qquad p = f\left(\frac{dp}{dx}\right).$$

Linear equation of the second order

$$\frac{d^2 y}{dx^2} + P\frac{dy}{dx} + Q\,y = R.$$

By the Reference Matter we know that the most commonly used co-ordinates system in engineering is the rectangular, the next being the polar. Special care must be taken in the lessons of analytical geometry, to bring home to the students' minds the following three points:

(*a*) The relations between two physical quantities may

be represented by a curve on a plane.

(b) A resultant or component vector may be represented by the distance of two points and their coordinates.

(c) Any given algebraic equation may be interpreted to represent a curve or curves and vice versa.

Of various topics of solid analytical geometry above mentioned, the equations of surfaces seldom appear in the lectures in this school, except in those of pure mathematics, and may be held dispensable. But as the analytical geometry is a preparatory subject for calculus, it is to recommend not to omit any part of it as far as the time admits.

According to the Reference Matter, the partial differentiation and integration of elementary functions should be taught in some outlines. And also such applications of these operations on curved surfaces or lines in space as, for instances, expressions for tangents or normals to a curve in space or of tangent planes or normals to curved surfaces or their mensuration etc. may be left out. But as they are supplementary to the solid analytical geometry, it is advisable to include them in the lectures as much as possible in the fixed time limits.

In general terms, as previously explained, we undertake in this school to teach mathematics, not as a pure science, but only as a means for practice, so that some free scope with respect to the scientific exactitude and rigor may be allowed, as for example to explain Rolle's theorem graphically.

According to the Reference Matter, the differential equation of the elastic curve in the Strength of Materials is the only one of the second order, all others being simple equations of the first order. Moreover, we know that, although much use is made of calculus and differential equations in the Theory of Alternating Current, Hydraulics and the Strength of Materials, the process of applying them are, in a large measure, alike. To speak more concretely, we have to express, first of all, the relations among physical quantities by means

of the differential equations, and next to solve them. Doing this is not so difficult. In this school the students must be equipped with all mathematical branches previously mentioned, as early as possible, in order they may comprehend the lectures which contain those applications. It is very difficult to adequately set the orders and the proportion of hours for these branches.

That which we have spoken above is mainly one side of the aim of the mathematical instruction, as to the other side we have the same opinion as that mentioned in the report of the Technical School of Middle Grade. By the Reference Matter we see that the most fundamental and the most important application of mathematics is to make qualitative ideas, especially physical ones, quantitative by means of mathematical expressions. Numerous cases of such kind of applications of arithmetic and algebra appear in engineering, but they are comparatively simple ones of the four rules, which have been learned for a long time, so that the students perhaps have had many opportunities to train themselves thoroughly to them. But as for differential and integral calculus, they have only a shorter period allotted to the study than arithmetic and algebra, and besides, the symbols used in the former are different from those which occur in the study of the latter. Thus the infinitesimal calculus being applied in the lectures on industrial science, before it is mastered by the students, we may be sure that they are not thoroughly skilled in representing physical ideas by means of it. Therefore attention must be taken on the side of application in teaching mathematics in this school. For instance, if we suppose that we have to express in the most general formulae such laws as, "the acceleration caused by gravity in a certain place is constant," or, "The strength of induced current is proportional to the number of magnetic lines of force cut off by the conductor in a unit time," then it must be well taught that the former can be expressed by

a linear differential equation of the second order, and the latter by a differential equation of the first order. Although special hours can not be given to the exercises in such applications, but we may share them a part of the time fixed for the exercises in mathematics, or we can give such examples in the course of the lecture on physics. We believe that it is not impossible to give them in physics, even before the students have no knowledge of differential and integral calculus. Nay, this may be one of the most important services physics should do to other sciences, especially to engineering.

3. Allotment of time. According to the prescription of the Educational Department, the course of instruction of this school extends over three years. In fact the course in every school of the same kind is just three years, and the mathematical instruction is not found in the highest class, and the total number of hours for mathematical branches varies about from 70 to 300, and depends upon the different schools and different kinds of engineering which are found in those schools. In the Tokyo High Technical School, the oldest one of this kind, the true number of weeks for which lessons are given during one year is about 33, and the allotment of hours for mathematical branches as follows:

Course / Branch	Architecture	Textile Technology	Electrical Engineering	Mechanical Engineering	Chemical Engineering	Engineering Drawing
	First Year.					
Algebra	66	66	66	66	—	—
Analytical Geometry	99	99	99	99	—	—
Mathematics	—	—	—	—	66	0

	Second Year					
Calculus & Diff. Eqn.	66	66	66	66	—	—
Sum	231	231	231	231	66	0

From this table, we set down the following as the minimal number of hours needed for teaching each of the branches of mathematics stated in the foregoing article:

Analytical Geometry 100 hours or thereabouts a year

Differential Calculus, Integral Calculus and Differential Equation 100 hours or thereabouts a year

Elementary Mathematics 40 hours or thereabouts (in the first term of the first year).

As the lectures on the strength of materials, in which differential calculus, integral calculus and differential equation are oftenest applied, begin in the first term of the second year, and moreover these three branches are most closely connected with one another, the lesson on these subjects must be completed in the first year. This is not imposssible, if 240 hours or so, the total number of hours for the course of these lessons together with that of elementary mathematics in the first year, are properly divided and allotted to each course per week. As its actual instance we may take the allotment of hours in the Sendai High Technical School: Mathematical branches assigned in this school are the same as those of the Tokyo High Technical School, and number of hours for the Electrical Engineering or Mechanical Engineering course is 6 a week and 200 a year, and the mathematics is only found in the first class.

CHAPTER III.

The Necessity of the Compilation of the Text-Books.

The most common practicable methods of teaching mathematics are as follows:

(1) To dictate and explain the definitions, propositions, and demonstrations in the most accurate terms.

(2) To dictate only the essentials of the definitions, propositions and demonstrations, and to amplify them by explanation.

(3) To use the text-books,

(a) specially compiled for the use in this school or

(b) edited not for the special use in this school.

The method (1) has a defect therein that it takes too much time; the method (2) is not recommendable as a method of teaching mathematics, which requires accurate ideas and expressions, to the students whose minds are not sufficiently developed, and whose language is poor in adequate expressions. Besides, by these two methods, it is unavoidable to cause sound confusion as well as some waste of time in giving exercises. The method (3) is free from those reproaches; but in case of (*b*) we may feel some difficulty in selecting the subjects to give out of the text-books, and sometimes miss the opportunities to teach subjects appropriate to this school, because we must follow the order of propositions and demonstrations by the author of the books. Taking these facts into account we may say that the method (3) (*a*) is the best. But we know that the method oftenest adopted in our polytechnical schools is (2) or (3) (*b*). Although the method (3) (*a*) is accepted generally as the most perfect, it is not adopted, we think, on account (i) of the difficulty of fixing the subjects in mathematics appropriate to the school, and (ii) of the labour, time and expense needed for the

compilation of the text-books. But we believe that these obstructions can be removed by the proper managements of the educational authorities concerned or the director of the school, and the compilation of the text-books can be effected somehow. In this connection we must remark, that the educational authorities concerned or the school director should stand in managing the text-book compilation on the more fundamental stand-point than that upon which we set down the subjects in mathematics necessary for the study of engineering science as shown in the Reference Matter. They should reflect on the state of things in this country, and the object and stage of progress of this school in fixing the subjects to be given in the lessons of engineering sciences, to the study of which mathematics should be taught in preparation. As to the relative degree of importance which should be ascribed to the compilation of the text-books on mathematics, as compared with other tasks, only the authorities concerned or the school director can decide, but we may say that this task is of no small importance, for it provides us with the means to make mathematics, which requires accurate ideas and expressions, memorized, understood, and applied satisfactorily by the students, whose minds are not sufficiently developed.

CHAPTER IV.

Examinations.

As in other schools, written examinations are made in this school, with the same objects as that mentioned in our report of the Technical School of Middle Grade and the marks are given, taking into account the students' skill in usual exercise hours. Though the number of times and method of examinations differ a little in each school of the same grade, but the examinations with notice shall generally be held from three to ten times in a year. In some schools no

notice is given of an examination, and in others notice is sometimes given and sometimes not. The former class of schools aims at keeping the students from over work, and the latter thinks that opportunities are given to the students to systematise their knowledge by their preparation for the examination.

Similar methods of examinations are held at all the schools in our country.

In short these facts show that it is difficult to find other appropriate means than such methods of examination to test how far the students understand their lessons and can apply them practically.

CHAPTER V.

Training of Teachers.

A class of training teachers for technological schools of middle grade is attached to the High Technical School in Tokyo. As to mathematics the same lessons as in the principal course are given to this class, and special lessons are not given, because the students of this class are few and most of them are graduates of the normal schools, where they have acquired sufficient knowledge of the method of teaching and other branches of pedagogy and they must only acquire in addition the knowledge necessary to the teachers for the technical schools.

[THE END]

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CONTENTS.

PREPARATORY MILITARY SCHOOLS

FIRST PART.

Present State of the Organization and the Methods of Mathematical Instruction.

CHAPTER I.

Two Kinds of Preparatory Military Schools.

There are two kinds of preparatory military schools, namely, the Central Preparatory Military School and a number of Local Preparatory Military Schools. The former which is in Tokyo contains the two courses, the main course of two years and the preparatory course of three years. There are five of the latter, namely one each in Sendai, Nagoya, Osaka, Hiroshima, and Kumamoto, and they contain only the preparatory course of three years. Pupils who have finished their course of study in the preparatory course of the Central or local preparatory military school, are admitted into the main course of the Central Preparatory Military School.

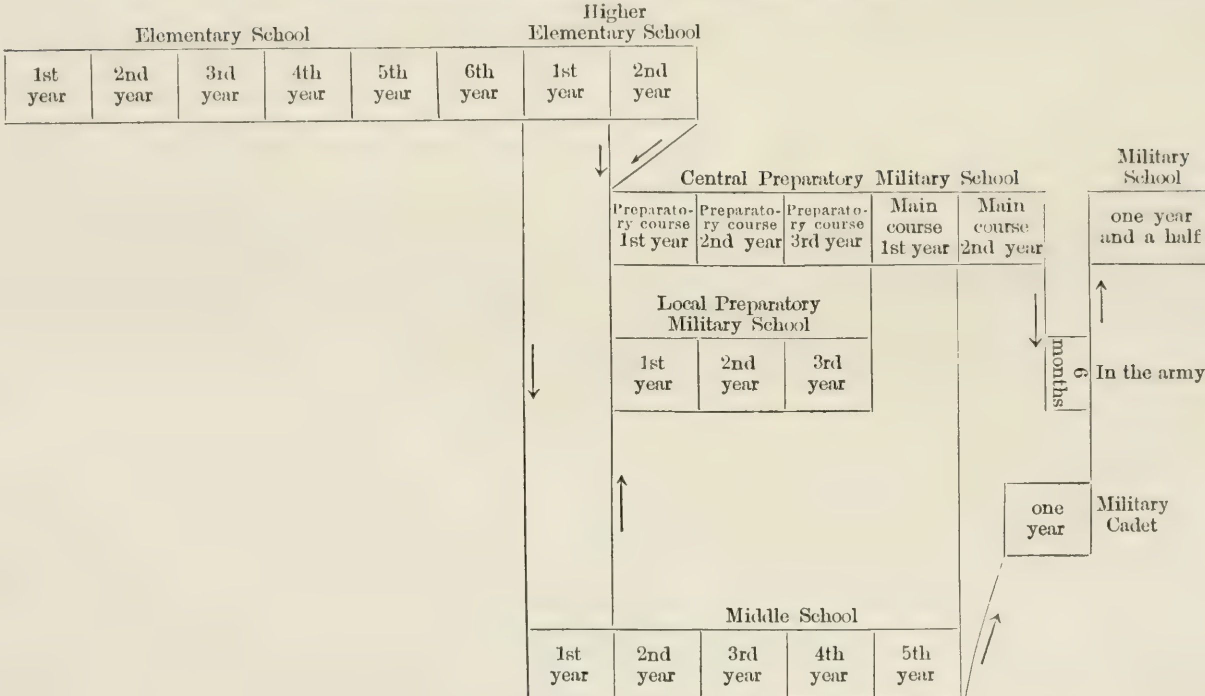
The Aim of Preparatory Military Schools.—The aim of preparatory military schools is to impart pupils general knowledge which will be necessary for them to become military cadets, and also the general preparatory education for military training.

Relations with Other Schools.—Those who apply for admission to the first year of the preparatory course in the Central or any local preparatory military school must pass through an entrance examination, the standard of which is the completion of the first year's course of the middle school, and consequently, those who have passed the examination

and been admitted therein possess about the same attainment as the second year pupils of the middle school. Thus the pupils of upper classes are of about the same qualification as the corresponding upper class pupils of the middle school, save that in the second year of the main course pupils have lessons in trigonometry, analytical geometry, mechanics, and descriptive geometry, *géométrie côté* while in the middle school they go only so far as trigonometry.

The following table is appended to show the relations of the preparatory military schools with middle and elementary schools as well as military cadets schools :—

Table showing grades and relations with different schools.



The Average Age of Pupils.—The average ages of the pupils in the main and the preparatory courses of the Central Preparatory Military School are, according to the inquiries made in 1910, as follows:—

14 years 9 months	....	1st year of the preparatory course
15 ,, 10 ,,	....	2nd ,, ,, ,, ,, ,,
16 ,, 6 ,,	....	3rd ,, ,, ,, ,, ,,
17 ,, 8 ,,	....	1st ,, ,, ,, main course
18 ,, 5 ,,	....	2nd ,, ,, ,, ,, ,, .

The average ages of the pupils in local preparatory military schools are about the same as those of the corresponding grade in the Central Preparatory Military School.

CHAPTER II.

Aim and Subject-matter of the Mathematical Instruction.

Aim of Mathematical Instruction.—As has been stated, the aim of the preparatory military school being to impart general knowledge to the pupils and at the same time to give them the preparatory military education, mathematics is taught to give them mathematical knowledge as a part of general education, as well as such mathematical knowledge as are especially useful from the military stand-point of view. To cite an instance or two: in algebra, probability is not omitted, as it helps the pupils in learning technology of arms in military schools in higher grades; in analytical geometry, the properties of conic sections are particularly well explained to the pupils, as they will have frequent opportunities to apply them to technology of arms and fortification; in mechanics, many problems of mathematical application are given as a preparatory study of technology of arms; and lastly *géométrie côté* is given to aid the study of topography.

The aim of mathematical instruction in the preparatory military school is, not only to teach the pupils formal opera-

tions and logical reasoning, but also to teach the actual facts contained therein and to foster the common sense of pupils. For instance, practical problems are selected for exercises, and too hypothetical data are avoided. For example, the quantities and the price of a thing are proportional to a certain limit, beyond which premium or discount comes into play, so that negligence of such factors in giving problems would be counted as a lack of common sense. The second aim of mathematical instruction in the preparatory military school is to train the pupils in practical calculations and to develop their mathematical talents. Nevertheless, the training of memory is not to be under-estimated and the important mathematical formulae should be committed to memory by the pupils and also the multiplication table not only up to 9^2 but up to 19^2. Furthermore, in the beginning of elementary geometry, strictly logical inferences are not attended to, but logical drill is left until logic is given as an independent lesson in the Central Preparatory Military School.

Subject-matter of Mathematical Instructions.—The syllabuses and number of lessons on mathematics and other subjects closely connected with it are given in the following tables. Hereby it is to be observed that one hour of a lesson means fifty minutes of actual teaching.

Preparatory Course in the Central Preparatory

Subjects \ Year / Term / Details & Number of lessons		I								
		I (Sept. 1—Dec. 24).			II (Jan. 8—July 10).			I (Sept. 1—Dec. 24).		
		Details	Number one week	Number one term	Details	Number one week	Number one term	Details	Number one week	Number one term
Mathematics	Arithmetic	Integers, fractions, decimals, compound number.	6	96	Compound number (Continued), proportion, square root.	6	78			
	Algebra				Introduction, integral expressions (definitions, addition, subtraction, multiplication, division), equations of the first degree (one unknown quantity).	6	54	Equations of the first degree (continued, i.e., more than one unknown quantity), integral expressions (continued, i.e., formulae of multiplication and division, factor, greatest common divisor, least common multiple).	3	48
	Geometry (plane)							Introduction, straight lines.	3	48
Drawing								Geometrical drawing.	1	16
Remark		Algebra begins in the second term of the first year in con-								

Military School and Local Preparatory Military School.

II			III						
II (Jan. 8—July 10).			I (Sept. 1—Dec. 24).			II (Jan. 8—July 10).			Sum
Details	Number		Details	Number		Details	Number		
	one week	one term		one week	one term		one week	one term	
									174
Fraction, equations of the first degree (continued).	3	66	Equations of the 2nd degree (one unknown quantity), ratio and proportion, equations of the 2nd degree (continued, i.e., equations containing fractional expressions, of higher degree with one unknown quantity, of the 2nd degree with more than one unknown quantity).	3	48	Equations of the 2nd degree (continued, i.e., with more than one unknown quantity continued, equations containing radical expressions), powers and roots.	3	66	282
Circles.	3	66	Area, proportion	3	48	Area and proportion continued.	3	66	228
Geometrical drawing.	1	22	Projective geometry	1	16	Projective geometry.	1	22	76

tinuation of arithmetic.

The Main Course in the Central

Year / Term / Details and number of lessons / Subjects		I					
		I (Sept. 1—Dec. 28).			II (Jan. 6—July 10).		
		Details	Number		Details	Number	
			One week	One term		One week	One term
Mathematics	Algebra	Variation, progression, inequality.	3	48	Permutation and combination, binomial theorem, probability, logarithm.	3	69
	Geometry	Straight lines and planes.	3	48	Polyhedrons, solid of revolution.	3	69
	Trigonometry						
	Analytical geometry						
	Mechanics						
Science		Physics, chemistry and mineralogy.	4	64	Physics, chemistry and mineralogy.	4	92
Logic		Elements of logic.	1	16	Elements of logic.	1	23
Drawing		Perspective drawing, projective drawing, quick sketching in pencil.	2	32	Projective geometry (actual drawing), water color.	2	46

Preparatory Military School.

II						Sum
I (Sept. 1—Dec. 28).			II (Jan. 6—May 27).			
Details	Number		Details	Number		
	One week	One term		One week	One term	
						117
						117
Acute angles, general angles, multiple and submultiple angles, triangles in general.	6	96				96
			Straight lines, circles, conic sections, general equations of the second degree.	6	96	96
			Linear motion, forces acting on a rigid body, various motions.			
Physics, chemistry and mineralogy.	3	48	Physics, chemistry and mineralogy.	3	48	252
Colouring, projective geometry (actual drawing).	1	16	*Géométrie côté.*	1	16	110

In reference to the distribution of subject-matter, much attention has been paid towards the relations that exist between different branches. Thus in those branches which are alternately given, as algebra and geometry, or analytical geometry and mechanics, or in those which succeed each other, as arithmetic, algebra, trigonometry, and analytical geometry, any subject which is found in different branches is treated on a proper occasion, or, if necessary, as an application of the facts already given in another branch, so that no distinct line can be drawn between the boundaries of the different branches, as far as such a subject is concerned, but the full knowledge of each mathematical branch is obtained after the pupils have finished their five years' course in the preparatory military school. Concerning the details of what has been stated, more minute explanations will be given under the heading of Text-books in Chapter IV.

CHAPTER III.

Examinations.

There are two kinds of examinations, namely, daily examinations and annual examinations held at the end of each school year, the last annual examination being at the same time the graduation examination. There is no term examination.

The Daily Examination.—This is of two kinds, the oral and the written examinations, and is held in each lesson by asking pupils questions of what they have learned, in order to make them grasp the matter more accurately and correctly. Besides, the answers of extemporary questions written by a pupil on the blackboard, the performance of home exercises, and the answers which all the pupils are required to write down, the time allotted being taken out of the ordinary teaching time within the limit of from ten to thirty minutes,

are also taken into account as a part of the daily examination.

The written examination is one to which the time for one lesson (50 minutes) is entirely assigned and is held at least twice in one term on one subject, the material being taken from what they have learned during the term. But, as it will be seen under the heading of the Method of Teaching, it is the object of all preparatory military schools to make the pupils repeat and repeat what they have learned, so that problems or questions dealing with what they had learned are sometimes given on this occasion. In the main course of the Central Preparatory Military School the written examination is held either with or without forewarning; but in the preparatory course in both the central and the local schools, it is held always without any previous notice.

Annual Examinations and Graduation Examination.—These are held at the end of every school year after giving notice thereof, two hours being allowed for each subject. The final mark in each subject is determined by combining the average mark of daily examinations and the mark of the annual one in the ratio of two to one.

CHAPTER IV.

Method of Teaching.

Outlines of the Method of Teaching common to all subjects at the Preparatory Military School.—In order to improve the methods of teaching at the preparatory military schools and make it uniform, a summer course was held in 1901 for instructors of mathematics in the Central Preparatory Military School and local preparatory military schools and "Outlines of the Method in Teaching at the Preparatory Military School" was discussed and approved. This was put into practice during the following seven years, and, after various inquiries and experiences, it was revised by the summer course of 1908. As a result of this revision, not only the

methods of teaching have become uniform in all preparatory military schools, but also various branches of mathematics were taught effectively to all the pupils, so that no distinction can hardly be made as to proficiency in mathematics among the graduates of the preparatory course in different schools, when they turn out to be pupils in the main course in the central school.

Outlines of the Methods of Mathematical Instruction at the Preparatory Military School.—

I. Cares should be taken before and after lessons.

1. The teachers of mathematics should make sufficient preparations for what they are going to teach, they should hold meetings to discuss and consult with each other how to teach, and they should see and criticize the teaching of others. They should thus aim at the progress and unity of the methods of teaching.

2. Each teacher should make a table of what he is going to teach before the school year begins.

3. Each teacher should prepare printed matters to be distributed among the pupils as occasion requires.

4. Each teacher should examine from time to time the note-books of pupils, and give necessary remarks on them.

II. Methods of Teaching.

1. At the beginning of each lesson, the teacher should try to ask questions concerning the recently taught matters as a part of the daily examination.

2. Each teacher should give a review of important matters which had been taught as a part of the daily examination.

3. Explanation and reading of the text, if they are necessary, may be made in any order, but a clear distinction should be made between them in order to concentrate the attention of the pupils.

4. Questions or problems out of the text-book or from other sources should be given to the pupils as either extemporary or home exercises as a part of the daily examination. In the latter case, the teacher should take care not to overtask them, considering the time which has been placed at their disposal.

5. Let a pupil solve a problem in the class in the following ways:—

(*a*) If the problem is simple or similar to one just finished, the teacher should make the pupil solve it orally, instead of writing it down on the blackboard.

(*b*) When there are various ways of solving a problem, the teacher may ask some pupils to state orally the essential points of the different methods; or picking out two or three pupils, and let them write down on the blackboard the different solutions which should be corrected or criticized by himself or by other pupils.

(*c*) When the teacher wishes to show the pupils a typical solution, he should himself write it down on the blackboard or he may make some pupil write it down on the blackboard and then criticize and correct it, and make the pupils compare their own with it and correct their mistakes, if there be any.

6. When the teacher wishes to try a certain pupil, he should first give the problem to the whole class, giving all pupils time to consider, and then name the pupil he wanted to try.

7. The teacher should pay attention to pupil's pronunciation or writing and make him accustom to do them correctly and clearly.

8. When a pupil is to stand on the platform or some other place to make an explanation or to answer a question, he has to use polite words suitable to addressing his teacher, but, at the same time, he must speak, as far as possible, to the whole class.

9. When the teacher states something to call the attention of the whole class, he should also make it known to the pupils who are working at the blackboard, if there are any.

10. At the end of each lesson, the teacher should give to the pupils the summary of what has been taught that day, whenever it is deemed necessary to do so.

11. The teacher may sometimes give the outline of the next lesson as a preparatory rehearsal, give problems as home exercises with hint, if necessary, for the solutions, or give order to prepare the text-book or to review lessons.

III. Miscellaneous.

1. Though it is needless to say that every subject in mathematical instruction should be made to be understood rightly and clearly, yet that which is comparatively less important may be taught briefly, and the whole energy of the pupils should be directed toward more important points. The pupils should be thoroughly drilled in the following subjects:—

In arithmetic: operations in the calculations of fractions and decimals, weight and measure, and square root.

In algebra: factors, fractions, equations, and logarithm.

2. To help the pupils to remember what are important in mathematics and to make it convenient to apply them, they should be made to prepare the synopsis of mathematics, the contents of which should be the same as that of "Synopsis of Mathematics" given in a separate pamphlet. Copying of the synopsis should be commenced when a pupil is in the second year of the preparatory course and be completed by the time he graduates that course. Afterwards, it should be supplemented by him to the mathematical branches which are to be given in the main course of the central school.

3. To make the pupils skillful in calculation and also able in mathematical application, the teacher should adopt the following rules:—

(*a*) Let the pupil consider the order of operations and adopt as simple a method as possible.

(*b*) Let him train himself in mental as well as quick calculations.

(*c*) Let him in most cases make approximate estimates whenever he makes calculation.

(*d*) Let him measure length, breadth, weight, etc., by sight or by hand.

4. To make the pupils interested in the mathematical study, the teacher's skillful art of teaching should of course be relied upon, but the teacher should particularly observe the following rules:—

(*a*) He should explain how far each branch of mathematics can be applied and how it is useful in other respects, in order to let the pupils conceive the great importance of its study.

(*b*) Materials other than those of regular lessons should be moderately chosen so as to suit the ability of pupils.

(*c*) Let not the pupils forget any fundamental facts.

(*d*) The pupils should be made skilled in operations.

(*e*) The teacher should give practical problems or other interesting ones.

(*f*) Reference books should be told, if there be opportunity.

(*g*) Let the pupils play mathematical games occasionally.

5. The following signs will be used, and besides, other signs used in text-books: $\backsim$, $\doteqdot$, $A\hat{B}C$, $\triangle ABC$, $\frown ABC$, $\odot O$ (O being the centre), $\odot ABC$ (A, B, C being three points on the circumference), $\geqq$ ($\measuredangle$), $\square ABCD$, $\square AC$, $\hat{R}$.

6. The matters to be noted down:—Notes are to be

taken only of such matters as are supplemented by teachers and not given in text-books. They should be pointed out by teachers for beginners until the pupils can do it for themselves.

7. In arithmetic and algebra, simple operations should be made mentally; and in geometry, as pupils advance in knowledge, solutions should sometimes be made, without making use of a part or the whole of figures.

8. If there be a pupil who is excellent in every lesson given in the school and is especially so in mathematics, the teacher should sometimes help him for his further progress by giving him more difficult problems or reference books for self-study.

9. Inferior pupils should be improved, giving them extra-problems to be solved by themselves, and answering their questions in a separate room, if necessary, either by trying them questions, or by some other means.

10. The following errors which are liable to be committed by inferior pupils, should be corrected in such ways as to impress them deeply:

(*a*) $a^2+b^2=9, \quad \therefore \quad a+b=3$

(*b*) $\sqrt{6.75}=2.5\sqrt{0.5}$, or $153+54\sqrt{3}=207\sqrt{3}$

(*c*) $\sqrt{a-3ab+b^2}=a-\sqrt{3ab}+b$

(*d*) $a(b+c)^2=(ab+ac)^2$, or $a\sqrt{b+c}=\sqrt{ab+ac}$

(*e*) A square millimetre = a millimetre square.

(*f*) $5\pi+14\pi=19\pi^2$

(*g*) $$\frac{\frac{9.8}{4}}{4\pi^2}=\frac{4\pi^2\times 9.8}{4}=9.8\pi^2$$

(*h*) $$\frac{\cancel{a}\times\cancel{b}\times\cancel{c}\times\cancel{d}}{\cancel{c}\times\cancel{a}\times\cancel{b}\times\cancel{d}}=0$$

(*i*) $$\frac{y+z}{x}-\frac{x+z}{y}=\frac{(y+z)-(x+z)}{xy}$$

(*j*) $$\frac{x}{2}+\frac{x}{3}=3 \quad \therefore \quad 3x+2x=3$$

(*k*) $3 \times 8 = 24$, $24 + 5 = 29$. This is often written
$$3 \times 8 = 24 + 5 = 29.$$

(*l*) $8x - 3x = 15 = 5x = 15 = x = 3$,
$$\frac{a}{b} = \frac{c}{d} = \frac{a+b}{b} = \frac{c+d}{d}$$

(*m*) Omission of equation marks.

Text-books.—Excepting algebra in the preparatory course and trigonometry, all the text-books of mathematics in preparatory military schools were compiled by the teachers and have been used in common at different schools. Each text-book is so compiled as not to accomplish all its object independently of one another, but to complete the study of each mathematical branch after all the branches have been finished. To give an example, in the text-book of arithmetic many subjects are omitted to be appropriately treated in algebra, geometry, trigonometry, analytical geometry, or mechanics, so that there are not a few places in the latter branches where the pupils are taught as in mere arithmetic. More detailed accounts are given below :—

Difficult problems in arithmetic are referred to equations in algebra, complicated operations in recurring decimals to geometrical progression in algebra, compound interest to logarithm in algebra, area and volume to plane and solid geometry. Further, all problems that have close relations to arithmetic, such as the application of the binomial theorem and logarithm in algebra, the estimation of height, distance, and area in trigonometry, and the practical problems in mechanics that require arithmetical calculation, are treated as the higher parts of arithmetic. In algebra, theoretical part of equations is briefly treated, but the subject is taken up again in analytical geometry, where the analytical interpretations of equations and their solutions are given at the same time. Incommensurable quantities are slightly touched, while progression and probability are discussed more fully.

In geometry, a special plan has been adopted as the result of inquiries: If theoretical explanations are given from the beginning, as in any ordinary text-book of geometry, so as to adopt a method of strict reasoning to what common sense easily recognizes, some of the pupils may possibly be surprised and misled to study geometry by recitation only. To avoid this, the pupils' power of understanding has been constantly kept in view in compiling text-books, so that even the names of theorem and corollary have been omitted in the initial part of plane geometry, and theorems such as, for instance, vertical opposite angles are equal to one another, are not proved, but are taught simply as facts. On the other hand, special care is taken for the application, so that the properties of straight lines and angles are so treated, as far as possible, as to be applicable to geometrical drawing and surveying, triangular rulers containing angles of 30° or 45° and protractors being also allowed to be used under certain conditions. After a while the pupils will advance a little higher and begin to take interest in the study, and then the formal methods of theorems, corollaries, proofs, and so on are introduced. As regards the distribution of theorems and exercises, ordinary text-books of geometry are so compiled as to make the teacher, not in few cases, instil into the pupils' minds many theorems in succession and then give them lots of exercises at a time. In following this method, it will sometimes tax the brains of pupils too much, and so in the text-book used at the preparatory military schools, what are to be taught as theorems by the teacher and what are to be given to pupils as exercises are mixed up together and consequently instead of expressing theorems in the form of propositions, they are often given in the form of questions, which are to be studied and solved by the pupils themselves. Inconveniences as naturally arise in applying important theorems given in the form of questions will be wiped away by preparing a sepa-

rate pamphlet entitled "Synopsis of Mathematics," in which the theorems are of course expressed in the form of propositions.

As regards proportion, it is taught in geometry immediately after ratio and proportion in algebra have been finished. Lines and angles are considered therein as mere quantities and their respective ratio an algebraical quantity, so that the study of geometrical ratio is not separately treated. Lastly, it must be added that many complicated problems on loci are left to analytical geometry.

In trigonometry, the trigonometrical functions of small angles are amongst others very important as a preliminary military education, so that many problems concerning it are given outside the text-book. The application of logarithmic table is first taught in algebra, next in the calculation of area and volume in geometry, and lastly it is repeatedly drilled in trigonometry, together with trigonometrical tables, while the use of slide-rule is taught at a proper place. In this way, the instruction concerning numerical calculations is completed.

In analytical geometry, straight lines and circles are discussed at first as in any ordinary text-book, and then comes to the proper part of conic sections. Every opportunity is taken for discussing those subjects which were only slightly touched in other elementary branches, full discussions on loci and equations being given in this way.

In mechanics, screw motion and projectile are treated more or less minutely, as they are very important as preparatory military education. Here the discussion of the quadratic equations and the arithmetical progression in algebra and of the problems relating to the solutions of triangles in plane trigonometry find their proper places of application in treating the uniformly accelerating motion and the parallelogram of forces respectively.

"Synopsis of Mathematics."

This pamphlet is compiled in order to help the pupils to remember fundamental facts in mathematics and to give them facility in applying them in theoretical as well as practical studies. Each pupil is made to take this pamphlet with his text-book, putting the former in the pouch attached to the underside of the cover of the latter. The synopsis contains important formulae, theorems and other things which are arranged synthetically, the tables of compound numbers being added at its end. The matters concerning the preparatory course are given in print, blank pages being left beyond. The latter part of the pages is used by the pupils to write down equally important matters which are to be found in their higher course in the central school.

Object Lesson and Mathematical Apparatus.

To let the pupils form correct idea as regards the number of an assemblage at a glance, the following method has been devised: To make the pupils form correct ideas of large numbers, such as, 1000, 10000 and so on, a hanging tablet on which 1000 or 10000 small circlets are described, is held up before them and they are made to guess the number of the circlets, these circlets being arranged in different manners such as in a curved line or in a zigzag line. These guessings are very important for cadets in order to be trained in calculating the number of persons in a large crowd at a glance. Further the pupils are educated to measure distance, area, height and the angle of inclination of an ascent, the time they spend in taking walks in the country or in going out for excursions being used for the purposes. The area of the school compound, the distances between different objects in it, and the inclination of the ascents within the enclosure are measured, the values thus obtained being put on boards or wooden pegs in the immediate neighbourhood. In the recreation room, appliances

for weighing and measuring are provided, giving pupils opportunities for being practised in weighing and measuring. Besides, a slide-rule is given to each pupil, and its use taught during the last two weeks of summer vacation, which are spent at the sea side for swimming exercise. The time is considered to be opportune, as it is just after the pupils have been trained in the use of logarithmic table in the latter part of the first year of the main course.

Interest taken in Mathematics.—To get rid of the aversion of pupils to mathematics and induce them to take interest in its study, various means are used such as, giving interesting problems to the pupils from time to time, letting them play mathematical games, and so on. With regard to reference books, those in Japanese, such as, the *Jinkōki*, the *Kanja-otogizōshi*, the *Sanpōdōjimon*, etc., which were in use before the Restoration, have great influence in exciting the curiosity of the pupils for mathematics, while those in foreign languages being well known are not given here. History of mathematics promotes also the interest of pupils for mathematics and makes them put more weight to the study; consequently foot-notes are appended to the text-books, where the names of Thales, Pythagoras, Archimedes, Kōwa Seki (a famous Japanese mathematician, 1642–1708) etc. are to be found.

Relations between Mathematics and Other Branches.—Plane geometry and geometrical drawing are two subjects which are for the first time taught side by side in the second year of the preparatory course. As has been stated, the text-book of plane geometry is so compiled in its beginning as to be taught including geometrical drawing, so that the theoretical part of drawing is explained as far as possible in plane geometry, only the practical part of geometrical drawing and methods of solving certain problems being left untouched.

As to the relations between mechanics, as a branch of

applied mathematics, and science (in preparatory military schools physics and chemistry are not taught as separate subjects, but these two branches of science and mineralogy are united into one subject under the name of science), only qualitative explanations of dynamical part of the text-book in science are given in the first year of the main course, quantitative discussion being given in mechanics as a branch of applied mathematics in the second year of the same course.

As the study of mathematics in preparatory military schools should also have the general character deemed necessary for preparatory military education, their text-books contain matters which have close connections with topography, fortification, and technology of arms, consequently problems of this nature being much added to it, as problems of probability and logarithms in algebra, problems of height and distance in trigonometry, and the motion of projectiles in mechanics.

Regarding the relations existing between mathematics and logic, strict logical drill is dispensed with in the beginning of geometry as it is to be treated fully in logic. For this purpose, the text-book of logic is so compiled as not to contradict with the explanations given in mathematics and to make up for what have been left untaught in the latter, mathematical explanations and demonstrations, as the pupils advance in their knowledge, being made, as far as possible, according to strict logical reasoning.

As to the relations between mathematics and foreign languages, principal technical terms found in the text-books are translated into French, German, and Russian, and each pupil is expected to remember the equivalents in the language he is studying.

The reading of integers, decimals, fractions and other expressions and the names of certain geometrical figures are also taught in foreign languages, whereby use is made of hanging tables prepared for this purpose.

CHAPTER V.

Instructors of Mathematics.

Instructors of mathematics in the preparatory schools should be the graduates in mathematics in the Imperial University or those persons who possess licenses for teaching mathematics in the middle school. Besides them, persons who have been especially recognized by the school are sometimes employed.

SECOND PART.

Present Tendencies in the Teaching of Mathematics.

CHAPTER I.

Ideas concerning School Organization.

As has been stated in Part I, the mathematical instruction in the preparatory military school has an aim which differs in some respects from that of any other schools under the direct control of the Department of Education, and has been adapted to accord with its aim. Improvements having been made from time to time in its text-books and method of teaching in the past decade, it is now believed that there is no more need for reforming the present status of organization.

CPAPTER II.

Present Tendencies concerning the Aim and Subject-matter of Instruction.

As has been pointed out in Chapter II of the first Part, the Subject-matter of teaching has been adopted by dispensing with those unnecessary and uninteresting parts of each subject; but those parts which are professionally im-

portant and interesting have been included, in as much as the present tendencies have been realized in actual practice.

CHAPTER III.

Examinations.

There is no necessity whatever of dispensing with or modifying the existing methods of examinations.

CHAPTER IV.

Methods of Teaching.

As has been remarked, any plan which is believed by teachers to improve the existing methods of teaching, having been recognized by the authorities, it will be better to reconsider what has been stated in Chapter IV of the first Part, as it would reflect the present tendencies concerning the methods of teaching.

Owing probably to the result of the late reform in the teaching of arithmetic in elementary schools, the attainments in actual calculation of the pupils who were admitted into the preparatory military school, have been gradually on the decrease, and consequently, although much time has already been allotted to arithmetical calculation, it would be desirable to increase the time for the same purpose still more. The logarithmic calculation is at present taught in the first year of the main course, but there is a tendency to commence it one year earlier, when powers and surds in algebra have been finished, that is, in the last term of the third year of the preparatory course, and it has actually been fixed to teach it in the beginning of the first year of the main course. As regards the use of the slide-rule, it is much desired to enlarge the field of practical application.

The change in the way of expressing decimal sign and the form of operations in division in elementary schools have brought their effects upon the military school also, and the teachers in the latter are experiencing not a little incon-

venience, so that they are expecting for an opportunity to discuss on this subject.

CHAPTER V.

Instructors.

The school has no provision for training its own instructors, nor has it any such hope or chance ; but in order to improve the method of teaching in the sense of making it more practical and interesting, it is desired that teachers of mathematics should not only be acquainted with differential and integral calculus, but also be well versed in physics and dynamics, and particularly, in machines and machinery.

THE MILITARY ARTILLERY AND ENGINEERING SCHOOL.

FIRST PART.

PRESENT STATE OF THE ORGANIZATION AND THE METHOD OF MATHEMATICAL INSTRUCTION.

CHAPTER I.

The Nature of the School and its Organization.

This school has been established by the Imperial Ordinance for admitting junior officers (sub-lieutenants and lieutenants and sometimes captains) of artillery and engineering and teaching them those subjects which form the foundation of the military knowledge particularly necessary to these departments of the army. The officers of the two departments just mentioned must obligatorily complete the course of study of this school.

This school contains two courses, namely the general course of one year and the higher course of the same duration. Out of those who have finished the general course, a number of students of excellent scholarship are selected and admitted into the higher course. Among those who have completed the higher course, a few of very excellent scholarship are selected from, and are allowed to pursue a post-graduate course of more than six months in mathematics, physics, chemistry, and languages, and then sent to the Faculties of Science and Technology in the Tokio Imperial University for further studies. Two tables follow; one shows the relations between this school and other military schools, and the other the corresponding classes in different schools with respect to mathematical instructions:—

Table I.

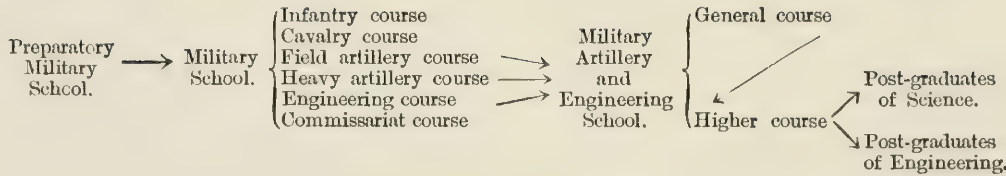


Table II.

Military Artillery and Engineering School

General course	Higher course	Post-graduates course
1 year	1 year	8 months

The 2nd Department of Higher School

1st year	2nd year	3rd year

College of Science or Engineering

1st year	2nd year	3rd year

The average ages of the students according to the inquiries made in December 1910 are as follows:—

27 years 7 months...... Post-graduate course
25 „ 0 „ ......Higher course of artillery
25 „ 0 „ ......Higher course of engineering
23 „ 11 „ ......General course of artillery
23 „ 8 „ ......General course of engineering.

CHAPTER II.

Aim and Subject-matter of Mathematical Instruction.

The principal object of teaching mathematics in this school is to give the students attainments necessary for understanding all the branches of study which are essential to military sciences. The subject-matter of mathematical instruction must change in accordance with the progress in the aforesaid branches of study. The general tendency is to go on higher and higher as civilization advances. In all cases, however, the object of mathematical instruction is to give logical training, and at the same time, ability for making practical applications.

The subjects of study and the number of hours assigned are given below. Hereby it is to be remarked that applied mathematics is included herein. The school year begins on the first of December and ends in the latter part of October next year. One lesson means one hour and a half.

General Course of Artillery.

Subjects	Details of lessons	Number of lessons	Number of hours
Analytical geometry	(Plane) Point, straight line, circle, ellipse, hyperbola, parabola, (poles and polars are omitted). (Solid) Straight line, plane.	30	45
Calculus	Limits, differentiation of simple functions, expansion of functions of	28	42

Subjects	Details of lessons	Number of lessons	Number of hours
	single variable, indeterminate forms, maxima and minima of functions of single variable. Integration, indefinite integrals, definite integrals, geometrical and dynamical applications (area and length of curves, surface and volume of solid of revolution, centre of gravity, moment of inertia), double integrals.		
Probability and theory of errors	Definition of probability, addition and multiplication of probability, probability of compound events, probability of causes, law and formula of probability of errors, various kinds of errors, probability integral, law of propagation of errors, general theorem relating to the deviation of projectiles.	15	22.5
Dynamics	Units, rest and motion, uniformly accelerated motion, circular motion, simple harmonic motion, force, momentum, laws of motion, gravity, work, energy, friction, equilibrium of forces, centre of gravity, simple machines, elastic bodies, motion of fluid, motion of particles, impulse, equations of motions of rigid bodies, moments of inertia.	36	54

Higher Course of Artillery.

Solid analytical geometry	Special kinds of quadrics, the discussion of general equations of the second degree.	5	7.5
Calculus	Mean value theorem, higher differential co-efficients of functions of many variables, expansions of functions of	29	43.5

Subjects	Details of lessons	Number of lessons	Number of hours
	many variables, maxima and minima of functions of two variables, geometrical applications (tangent, normal, asymptote, double point, curve tracing, curvature and radius of curvature, envelope). Integration of rational and irrational functions, successive integration, geometrical applications of double integrals (area of plane curves, area of curved surface and volume of solids). Ordinary differential equations, linear differential equations of the second order, integrations by series, general theorem on total and partial differential equations.		
Probability and theory of errors	Stirling's formula, Bernoulli's theorem and its inverse, probability of continuous quantity, method of least squares, direct observation of single quantity, indirect observations of several quantities, empirical formulae.	8	12
Rigid dynamics	Moments of inertia, principle of D'Alembert, motions in one plane, Euler's equations of motion, motion of top, equations of Lagrange.	26	39

General Course of Engineering.

Analytical geometry	(Plane) Same as in the general course of artillery.	46	69
	(Solid) Omitted.		
Calculus	Same as in the general course of artillery, but indeterminate forms and double integrations are omitted.		

Subjects	Details of lessons	Number of lessons	Number of hours
Dynamics	Same as in the general course of artillery.	30	45

Higher Course of Engineering.

Subjects	Details of lessons	Number of lessons	Number of hours
Solid analytical geometry	Straight lines, planes, special kinds of quadrics.	5	7.5
Calculus	Orders of infinitesimal, indeterminate forms, higher differential co-efficients of functions of many variables, expansion of functions of two variables, geometrical applications (curvature and radius of curvature, envelope, roulette). Integration of rational and irrational functions, successive integration, double integral and its geometrical applications (area of plane curves, area of curved surfaces, volume of solids). Ordinary differential equations, linear differential equations of the second order, simultaneous differential equations.	26	39
Spherical trigonometry	Fundamental formulae, conjugate formulae, formulae of D'Alembert and Napier, solutions of right-angled and oblique angled triangles.	5	7.5
Probability and theory of errors	Definition of probability, addition and multiplication of probability, probability of compound events, law and formula of errors, various kinds of errors, direct observation of single quantity, indirect observations of several quantities, conditional observations.	10	15
Dynamics	Motion of a rigid body in a plane, centrodes, friction wheels, toothed wheels, cam, belt, link motions, theorem of Bernoulli, water-dam and the quantity of water at the exit, motion of water taking friction into account.	14	21

Remark. In proving the principles of probability and dynamics, use is made of the calculus.

The mathematical subjects of the post-graduate course are higher algebra, analytical geometry, calculus, and dynamics. The object of teaching them is to supplement the knowledge the students have already acquired in the general and higher courses. The time given to these studies is from the first of December to the later part of July next year.

The subjects, text-books, and the number of lessons are, at present, as follows :—

Subjects	Text-books	Number of lessons	Number of hours
Algebra ..	Smith—A Treatise on Algebra. Burnside & Panton—Theory of Equations.	about 36	54
Analytical geometry	Puckle—Conic Sections. Aldis—Solid Geometry.	,,	,,
Calculus..	Todhunter—Differential and Integral Calculus. Williamson—Differential and Integral Calculus.	,,	,,
Dynamics..	Williamson and Tarlton—Dynamics.	,,	,,

Although students are expected to learn from text-books, yet teachers give lectures on certain topics in detail, in order to make the mathematical knowledge of the students sure and correct.

CHAPTER III.

Examinations.

The education of this school is compulsory, and no entrance examination is held.

The examination in mathematics is chiefly a written one and is commonly held once after every ten lessons, and as its object is to test the ability for practical applications,

students are chiefly examined in solving practical problems, rather than on the facts they have learned in their text-books, and consequently they are sometimes allowed to bring not only the text-books, the collection of formulae, and so on, but also their note-books.

CHAPTER IV.

The Methods of Teaching.

As regards the methods of teaching in this institution, each instructor writes the text-book for the subject which he is going to teach, and explains it to the students in the class-room, supplementing orally what is wanting in it. Sometimes the students are asked on some subjects or are requested to explain some matters in the text, and so on. Problems are generally given to them to be solved in the class-room, but sometimes they are given as home exercises. Reference books differ according to the foreign languages the students are studying, but are chiefly selected from among the books of mathematics in English, French, or German.

When an instructor teaches a text-book, he makes full explanations regarding what is hard to comprehend or liable to misunderstanding, never forgetting to give cautions by some appropriate means.

Those who study analytical geometry for the first time, are liable to consider an equation and its locus to be the same thing, and this brings about not a little difficulty to them. This may be owing to the fact that the ideas of elementary geometry, they have already learned, alone have taken the firm possession of their minds, and consequently they will have difficulties in connection with analytical problems of loci. Therefore, in solving the problems relating to straight lines or circles, there may be some students who solve them by an elementary geometrical method and then write down the equations for them, or in finding the equation of a locus, even it contains the variable besides the

current co-ordinates, they do not eliminate them and say at once that it is a certain kind of a curve, and so on.

As to calculus, there is a considerable difficulty in instilling its fundamental ideas into the minds of the students, and so the instructors should do their best to put away with this difficulty. It will be wrong to have recource to profound theories, but it will be equally wrong to try to accomplish this object only by calculations, and what should be done might be the mid-way between the above two, that is, combining theory with practice. Sometimes such students are met with, who make mistakes in finding the differential co-efficients of functions, such as,

$$\frac{d\cos\frac{x}{2}}{dx}=\sin\frac{x}{2} \qquad\qquad \frac{de^{2x}}{dx}=e^{2x}.$$

Similarly a few students may confound the variable (x) of integral and x of the limit with each other.

Regarding the theory of probability, many mistakes will be made in its addition. To give an example or two, there may occur cases where the value of probability, as the result of addition, becomes greater than 1 ; and in writing down the problems, great care should be taken, otherwise there can occur two or more solutions—it should not be forgotten to state that the formula of Gauss in the method of least squares is founded on mere supposition, for otherwise the result coming out of its application may be looked upon by many as a truth.

As to dynamics, the solutions of its problems will be made easier than otherwise, if we apply calculus in solving them ; but their meaning may, not in few cases, be obscured by doing so, and so it must be stated here that there is a necessity to give sufficient interpretations from that standpoint.

As models are indispensable in teaching, those concerning mathematics or machinery are collected, with which

explanations are given in order to let the students recognize by themselves the various ways of applying mathematics to practical uses.

The subject-matter of mathematics taught in this school should advance as military sciences develop, but, as the time allotted to the study is comparatively limited, a special method of teaching has been adopted. All mathematical subjects in one class being taught by one teacher, some of them which have intimate relations among themselves can be combined together into one. To cite an instance or two, in finding the equation of a parabola in analytical geometry, *i. e.*, $y=ax^2+bx+c$, the curve is explained as the path of a projectile in a vacuum in dynamics as well as in ballistics. When, as an example of double integral, $\int_0^\infty e^{-x^2}dx$ is computed, it is explained that it represents an important integral in the calculus of probability, *i. e.*, probability integral, and further, that it is very important to the study of the principle of firing in gunnery. Lastly, in the case of the motion of a top the attention of the students is called on to the equation of the motion of a projectile about the centre of gravity in ballistics, and so on.

As the teaching of all branches of applied mathematics is a special feature in this school, it is necessary to keep close connections with all branches of other applied sciences, or, in a word, whole energy should be directed towards this point. To enumerate here the applications of mathematics in this institution, analytical geometry is applied to ballistics, strength of materials, and a part of gunnery in the artillery course, and to a part of bridge and strength of materials in the engineering course; calculus is applied fully in dynamics, and consequently has close relations with ballistics, the theory of wheels, of guns, strength of materials, and the theory of machines in the artillery course; probability and the theory of errors have close connections with all the theories of

gunnery in the artillery course and the theories of land surveying in the engineering course.

SECOND PART

TENDENCIES IN THE TEACHING OF MATHEMATICS.

CHAPTER I.

Ideas concerning School Organization.

As the mathematical instruction in this school is organized with the object of applying it to military sciences, it can not be avoided that the kinds of subjects and the distribution of teaching hours differ from those of the schools under the direct control of the Department of Education, some subjects going earlier, some later, more time being allotted to one subject, less to another. Since this arrangement had first been made, more than ten years have elapsed and it has gradually been improved by experiences during the interval. "The standard Syllabus of the Subject-matter of Instruction in the Artillery and Engineering School" was fixed last year, as the result of labour of all the teachers who collected materials and consulted upon what are necessary and what unnecessary. It has been put into practice from this year, and though its satisfactory result has not yet been sufficiently realized, yet it is believed that there will be no more need of revision.

CHAPTER II.

Present Tendencies in the Aim and the Subject-matter of Instruction.

The subject-matter of instruction in this school mentioned in Chapter II of the first Part has been adopted according to the "Standard Syllabus of the Subject-matter of Instruction in the Artillery and Engineering School," as has been

stated in the preceding chapter. In consequence of this, the subjects which are comparatively unnecessary, or have few chances of application, have been omitted, and only those which are necessary to military sciences have been detained; and furthermore, the text-books of mathematics have been written by teachers of mathematics after consulting the opinions of the teachers of other branches of study which have close relations with mathematics. The connection between mathematics and applied sciences has, therefore, become closer, and this may be looked upon as the present tendencies.

CHAPTER III.

Examinations.

There is no idea whatever of either suppressing completely or remodelling the existing method of examinations.

CHAPTER IV.

Method of Teaching.

The method of teaching given in the first Part, is believed to be a suitable one to be adopted in a special kind of school like this, and what is to be regretted is the fact that the fundamental knowledge of elementary mathematics given to the students in schools of lower grade is too poor to be directly made use of in the study of higher mathematics. To state more fully, the students are very unskilful in—nay, they hate—calculations in arithmetic; they have almost forgotten the method of solving equations in algebra; they are very unskilful in logarithmic calculations; they make mistakes in applying important formulae in trigonometry, and so on. It is doubted if this were not the result of teaching all the branches of study too theoretically at the middle school, and especially of giving too difficult problems in geometry, inducing the students to despise the simple calculations in algebra and trigonometry. It is believed that

simple calculations in algebra and trigonometry should be taught as to let the pupils have sufficient interest with them by the proper choice of materials.

CHAPTER V.

Training of Instructors.

There is no accommodation to educate its own teachers as there is no necessity to do so. Notwithstanding it is hoped that its teachers shall be specialists who possess higher knowledge of mathematics and, at the same time, are skilful in applied mathematics, especially in dynamics and applied mechanics.

THE MILITARY COLLEGE

FIRST PART

Present State of the Organization and the Methods of Mathematical Instruction.

CHAPTER I.

The Nature of the Institution and its Organization.

This institution is under the direct control of the General Staff Office and has for its object the teaching of those sciences relating to the higher arts of war to the young promising officers (sub-lieutenants and lieutenants of all courses in the army) who have been selected, and at the same time improving their knowledge in all branches of study which are essential to the researches of military affairs.

Though there are three classes in the college, mathematics is taught only in the first two years, but not in the third.

The following table shows parallels in the classes of this institution and other schools :—

Military College.

1st year.	2nd year.

Officers of
Infantry Course
Cavalry Course
Field Artillery Course
Heavy Artillery Course
Engineering Course
Commissariat Course.

Higher Middle School
(2nd course).

1st year.	2nd year.	3rd year.

The average ages of the students according to the inquiries made in December 1910, are as follows :—

27 years 6 months..	First year
29 ,, 4 ,, ...	Second year.

CHAPTER II.

Aim and Subject-matter of Mathematical Instruction.

The object of teaching mathematics in this institution is to promote the knowledge of the students still further than they had already acquired previous to their being admitted into the college. In other words, the chief aim is to make the brains of the students clear and make them versed in logical conceptions. It is not necessary to teach them higher special subjects.

The school work begins in the earlier part of December and ends late in June next year. One lesson means one hour and half or sometimes a trifle more. The subjects of study and the number of hours are as follows:—

First year.

Subjects	Details of Lessons	Number of lessons	Number of hours
Probability	Definition of probability, addition and subtraction of probability, mathematical expectation, inverse probability, probability of testimony, geometrical probability, annuity, miscellaneous problems.	30	55

Second year.

Analytical geometry	(Plane) Point, straight line, circle, parabola, ellipse, hyperbola (pole and polar included), discussion of general equations of the second degree. (Solid) straight line, plane, sphere, special kinds of quadrics.	30	55

Text-books are compiled by teachers of mathematics every year, reference books being left at the students' choice.

As to the method of teaching, the students are taught both by developing and infusing the principles and laws of mathematics. The practical exercises are of two kinds, extemporary problems and home exercises. To the students who have finished the Artillery and Engineering School and have comparatively superior knowledge on mathematics, are given more or less difficult problems.

CHAPTER III.

Examinations.

The entrance-examination is subdivided into two parts, the preliminary and the second examinations.

The preliminary examination is a written one held to test the attainments of the candidates for admission. It is held at different places under the military jurisdiction of divisional headquaters of the Army, being fixed by their commanding Generals. The examination is everywhere held at the same time in the first ten days of April. The candidates are examined in algebra, geometry, and trigonometry at a standard higher than the graduation examination of the middle school. Those who have passed the preliminary examination are admitted into the Military College for trial, where they are once more examined, but orally, on the same subjects as in the preliminary examination.

Examinations in the college are of two kinds; one is the oral daily examination and the other is a written one. In the latter case, in which all the students of the same class are examined at the same time, they are permitted to use their text-books and refer to other manuals.

SECOND PART

Tendencies in the Teaching of Mathematics.

CHAPTER I.

Ideas concerning School Organization.

As stated in Chapter I of the first Part, the study of mathematics in this institution has no direct relations with military sciences and its primary object is to make the brains of the students clear, and therefore the choice of the subjects of study has been made free, but the present advanced state of sciences has brought its direct effects on military sciences and it will not at all be unnecessary to say that the scientific knowledge is important even to staff-officers, and consequently there will be the necessity of reforming what constitute the study of mathematics in this institution.

CHAPTER II.

Tendencies concerning the Aim and Subject-matter of Mathematical Instruction.

The subjects of study, which have been stated in Chapter I of the first Part, have been adopted since the change made in 1906. Before that time analytical geometry and calculus were taught, but, at present, probability has taken the place of calculus. As the knowledge which has a wide field of application will be useful to the students of this institution, it will be better to teach the elements of higher analysis without specifying any particular branch.

CHAPTER III.

Examinations.

There is no idea of reforming the present methods of examination.

CHAPTER IV.

Methods of Teaching.

Regarding the line of demarkation, order of precedence, and union among the different branches of mathematical study, it will be better to do as follows:—

It will be wrong to take away the line of demarkation between elementary algebra and elementary geometry, between algebra and calculus, between synthetic geometry and analytical geometry, and between elementary geometry and trigonometry.

The order of precedence between elementary geometry and elementary algebra will be proper as it is at present.

Plane geometry and solid geometry should not be mixed up.

It will be better to mix up differential calculus with integral calculus.

THE TRAINING SCHOOL FOR LAND SURVEYING

FIRST PART

Present State of the Organization and the Method of Mathematical Instruction.

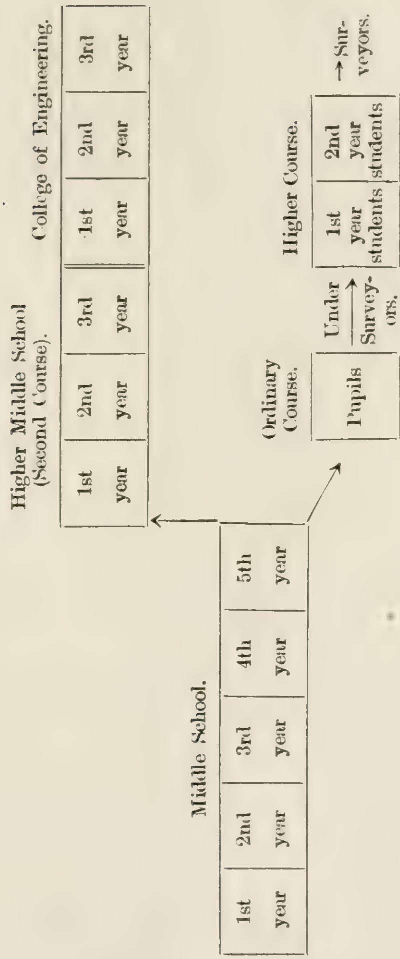
CHAPTER I.

Nature of the School.

This institution is for training specialists who are to be engaged in the work of land surveying.

The school contains two courses, the ordinary and higher. The former connects itself, in point of its graduation, to the middle school, or, in other words, the graduates of middle schools may be most conveniently admitted into it. The object of this course, which extends over one year, is to teach specially the art of surveying and to produce such youths as can be engaged in the practical service of surveying, mapping and lithography. The latter is a higher course of two years. The students who are admitted there are selected from among the surveyors who have had experience in practical services for some years and have proved themselves to be excellent. They are taught on higher special sciences with the aim of qualifying them as higher specialists who are able to guide common surveyors.

The following table shows the relations and parallels with different schools :—



The average ages of the students and the pupils according to the inquiries made in 1911 are as follows :—

25 years.. .. .. ..Pupils (Ordinary Course)

33 ,, ...Students (Higher Course).

CHAPTER II.

Aim and Subject-matter of Mathematical Instruction.

In the ordinary course, practice is chiefly aimed at, developing the mathematical instinct of the pupils as well. In the higher course, logical drill and practice constitute the objects.

The standard of the subjects of study and the number of hours devoted to the lessons are as follows:—

Ordinary Course.

Subjects	Contents	Number of hours
Algebra	Series, binomial theorem, partial fractions, logarithm, differential coefficient, maxima and minima.	35
Solid Geometry	Plane, dihedral and polyhedral angles, prism, cone, polyhedron, sphere, spherical triangle.	40
Plane Trigonometry	Trigonometrical functions, solutions of triangles, logarithmic table and its application.	45
Spherical Trigonometry	The relations between sides and angles, solution and application of spherical triangles.	25
Analytical Geometry	Co-ordinates, straight line, circle, ellipse, hyperbola, parabola, general equations of the second degree.	40
Method of least Squares	Kinds of observations, error, weight of observations, mean of observations.	40
Physics	Properties of matter, force, heat, sound, light, electricity, magnetism.	80

Higher Course.

Subjects	Contents	Number of hours
Higher Algebra	Series in general, determinant, probability, theory of equations.	60
Analytical Geometry	Plane and solid geometry.	110
Differential Calculus	Differentiation, expansions of functions, indeterminate forms, maxima and minima.	100
Integral Calculus	Integration, geometrical applications.	140
Differential Equations	Ordinary differential equations, partial differential equations.	120
Dynamics	Statics of rigid bodies, kinetics of rigid bodies.	90
Method of least Squares	Laws of errors, mean of observations, weight of observations, applications.	200
Physics	General physics, geometrical optics, electricity, magnetism.	240

CHAPTER III.

Examinations.

Extemporary questions and home exercises are given from time to time, and the marks thus obtained are reckoned every six months. At the time of graduation, there takes place examination in some of the more important subjects.

CHAPTER IV.

Methods of Teaching.

No text-book is used. Instruction is chiefly given by lectures. Sometimes some portion of lectures is printed for distribution to save the trouble of taking down notes.

CHAPTER V.

Training of Instructors.

Instructors are selected from among the surveyors and under-surveyors who have distinguished themselves in their knowledge. Other lecturers are either graduates of the Faculty of Science of the Imperial University or some specialists in mathematics.

SECOND PART

TENDENCIES IN THE TEACHING OF MATHEMATICS.

There is nothing particularly to be mentioned under this heading.

DIVISIONAL REPORTS

ON

THE TEACHING OF MATHEMATICS

IN

JAPAN

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Article XIV.—The Teaching of Mathematics in Schools under the Navy Department. By S. Ōba and S. Shimizu, *Professors of the Navy Department.*

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CONTENTS.

REPORT ON THE TEACHING OF MATHEMATICS IN SCHOOLS UNDER THE NAVY DEPARTMENT

CHAPTER I.

General Remarks.

The following three schools are directly under the Department of Navy: the Naval College, the Naval Engineering College, and the Higher Naval College. The system of education in these schools will be each dealt with in a separate chapter. The Naval College has a single undivided course, and the same is true of the Naval Engineering College, while the Higher Naval College contains seven distinct courses. Those who are admitted into the Higher Naval College are chosen from among the graduates of the two other colleges. For the students of the Higher Naval College who come from the Naval College, there are two courses (A) and (B); course (A) is for those who study naval tactics only, and course (B) is for all others. Those who pursue course (B), after finishing it, split up into three sections; one section continues to remain in the college and goes to the course (C) for navigation, the second section goes to the Gunnery School, and the third section goes to the Torpedo School. For the students of the Higher Naval College who come from the Naval Engineering College, there is the engineering course, which divides itself into course (D) for marine engineering and course (E) for electric engineering. Besides there are elective courses for those coming from the Naval College as well as for those coming from the Naval Engineering College.

CHAPTER II.

The Naval College

(1) The Nature of the College.

The object of this college is to give instruction in those subjects which are necessary for future naval officers.

The cadets are taken by competitive examination which is of the same standard as the graduation examination at middle schools. The course extends over three years, and at present the average age of the cadets is twenty years and four months.

(2) The Object and Syllabus of the course of Instruction in Mathematics.

The special subjects of study comprise Gunnery, Torpedo, Seamanship, Navigation, and Engineering, and the general subjects of study include Mathematics, Physics, Chemistry, Dynamics, and Foreign Language, all of which, except foreign language, require directly or indirectly the knowledge of mathematics.

The object of teaching mathematics at this college is, therefore, to train the cadets in the art of calculation and at the same time to give them preparatory mathematical knowledge for the study of other subjects, which they will find necessary for the study of special subjects pertaining to naval affairs.

The branches of mathematics taught at the college are algebra, trigonometry (both plane and spherical), geometry, analytical geometry, and calculus. The syllabuses of these subjects are as follows :—

Algebra —— Numbers, Kinds of Functions, Polynomials, Miscellaneous Theorems on Equations, Elimination, Limiting values, Proportion, Series, Permutations and Combinations, Probability.

Plane Trigonometry —— Circular Measure, Rules of Signs, Trigonometric Functions, Trigonometric Formulae, Inverse Trigonometric Functions, Trigonometric Functions of a Small Angle, Logarithms, Use of Tables, Solution of Triangles, Application to Surveying and Navigation, Summation of Series, De Moivre's Theorem and its Applications.

Spherical Trigonometry —— Spherical Triangles, Area of a Spherical Triangle, General Formulae, Formulae for Right-angled and Quadrantal Spherical Triangles, Application to the Celestial Sphere, Solution of Spherical Triangles.

Geometry —— Polyhedra, Cylinder, Cone, Sphere, Mensuration.

Analytical Geometry —— Co-ordinates, Equations of Plane Curves, Straight Lines, Transformation of Co-ordinates, Principal Properties of Plane Curves of the Second Degree, General Equation of the Second Degree, Empirical Formulae, Graphical Method for Definite Integration, Co-ordinates of a Point in Space, Equation of Surfaces of Revolution.

Infinitesimal Calculus —— Differentiation of Functions, Simple Indefinite Integrals, Geometrical and Physical Applications, Successive Differentiation, Partial Differentiation, Differentiation of Implicit Functions, Taylor's and Maclaurin's Theorems, Convergency of Series, Maxima and Minima, Indeterminate Forms, Definite Integrals, Area, Volume, Length of Arc, Curvature.

Remarks :—A part of spherical astronomy is given as an application of spherical trigonometry in order to show the close relation between the two.

(3) Distribution of the Subject-matter and Number of Hours allotted.

First Year.

Algebra and Infinitesimal Calculus (up to Geometrical and Physical Applications

in the above Syllabus) .. .. 60 hours
Plane Trigonometry .. .. 30 „
Geometry and Analytical Geometry .. .. 92 „

Second Year.

Infinitesimal Calculus (the remaining part). . 73 hours
Spherical Trigonometry .. .. 50 „

(4) Examination.

One school year is divided into three terms, and at the ends of the first and the second term the cadets are examined in the portion they have learned during that term and at the end of a year a final examination covering the whole year's course is held. The computation table compiled by the college is allowed to be used in the examination.

(5) Method of Instruction.

The cadets are taught by means of text-books compiled by the college, and as they have little time at their disposal, they are required to solve problems in the class-room, home exercises being seldom given.

As for the exercise of analytical geometry and curve tracing, squared and circled papers especially prepared for the purpose are given to the cadets as the case may require; besides, vertical and horizontal lines are engraved on the surface of the blackboard for the convenience of explanation.

In practical calculation the cadets are supplied with the computation table compiled by the college to familiarize them in the application of formulae and the use of table of logarithms.

CHAPTER III.

The Naval Engineering College

(1) The Nature of the College.

The object of this college is to give instruction in those subjects which are necessary for future naval engineer officers.

The cadets are taken by competitive examination which is of the same standard as the graduation examination at middle schools.

The course at this college covers three years, and at present the average age of the cadets is twenty years and five months.

(2) The Object and Syllabus of the Course of Instruction in Mathematics.

Everything that is taught at this college, except foreign language, requires directly or indirectly the knowledge of mathematics, and the branches of mathematics which are taught at this college are algebra, trigonometry, geometry, plane analytical geometry and calculus.

The chief object of teaching the above subjects at this college is to enable the cadets to understand the mathematics they may come across in their study of engineering, ship-building, physics, chemistry, dynamics and applied dynamics, and drawing, and to familiarize them in practical calculation, and at the same time to train them in exact thinking.

The following is the syllabus of the course of instruction in mathematics :—

Algebra —— Variation, Permutations and Combinations, Binomial Theorem, Series, Logarithms, Partial Fractions, Discussion of Roots of Equations, Approximate Graphic Solution of Equations, Separation of Roots of Equations, Determinants.

Trigonometry —— Angles, Trigonometric Ratios, Use of

Logarithmic Tables, Solution of Triangles, Application to Surveying, De Moivre's Theorem, Expansion of Trigonometric Functions, Construction of Trigonometric Tables.

Geometry —— Recapitulation of Elementary Geometry, Loci, Maxima and Minima, Volume.

Plane Analytical Geometry —— Co-ordinates, Transformation of Co-ordinates, Straight Lines, Curves of the Second Degree, General Equation of the Second Degree, Polar Equations.

Infinitesimal Calculus —— Functions, Continuity, Limit, Differentials, Geometrical and Physical Applications, Differentiation of Simple Functions, Successive Differentiation, Mean Value Theorem, Maxima and Minima, Calculation of Errors, Partial Differentiation, Exact Differentials, Indefinite and Definite Integrals, Integration of Simple Functions, Simple Definite Integration, Geometrical and Physical Applications, Expansion of Functions, Taylor's and Maclaurin's Theorem.

Remarks :—Integral calculus is put before expansion of functions, as it is necessary to do so in view of its application to physics. Elementary dynamics is taught without using calculus.

The table of logarithms and slide-rule are used in physics, applied dynamics and engineering. The theory of logarithms is taught in algebra, and the logarithmic calculation in trigonometry; a table of fouror five figures is employed in exercise while a table of four figures is used in practical calculation. The use of slide-rule is taught in physics.

(3) Distribution of the Branches of Mathematics and their Hours.

First Year

Algebra	72 hours
Trigonometry	63 ,,
Geometry	36 ,,

Analytical Geometry	.. ..	72 hours
Infinitesimal Calculus	.. ..	36 ,,
Second Year		
Analytical Geometry	.. ..	27 hours
Infinitesimal Calculus	.. ..	90 ,,

(4) Examination.

The examination in mathematics is of two kinds, viz. review and annual examinations. A year's course is divided into portions and cadets are examined in each portion from time to time, and at the end of a year they go through an examination covering the whole of the year's course. Both the review and annual examinations are written ones.

(5) Method of Instruction.

The cadets are supplied with text-books and are required to read them thoroughly before they come to the class-room so that they may better understand the lectures. There being so many subjects to be learned, cadets have very little time; so home exercises are seldom given.

Comparatively greater portion of the hours allotted to the instruction of mathematics is used in solving problems, preference being given to applications rather than to the theory.

Reference books are given by the instructors as the case may require.

CHAPTER IV.

The Higher Naval College

(1) The Nature of the College.

As was stated in the First Chapter, there are seven different courses in this college, among which there are only two, which contain instruction in mathematics. They are the (*B*) course and the Engineer course.

(2) Students of (*B*) Course.

Students of this course are lieutenants and sublieutenants. They are selected by a specially appointed committee consisting of superior officers.

Officers of this course enter the college in two groups every year, about thirty or forty officers forming one group being admitted at a time, and they study in view of later becoming students pursuing the navigation course at the Higher Naval College, the students of the Advanced Class of the Gunnery School, and the students of the Advanced Class of the Torpedo School. So, they study, besides naval sciences, such subjects as mathematics, dynamics, applied electricity, and applied chemistry which are necessary for their respective study of navigation, gunnery, and torpedoes.

The course extends over six months, and at present officers enter the college about six years after graduating the Naval College.

This course includes two branches of mathematics; namely, infinitesimal calculus and analytical geometry of space. The main object to be aimed at, is besides developing their capacity for mathematics, to give the students sufficient knowledge of mathematics, so as to enable them to understand the following subjects:

For (*B*) Course cadets —— Applied Electricity, Dynamics, Compass Adjustment.

For (C) Course cadets —— Applied Astronomy, Compass Adjustment.

For cadets of the Advanced Class of the Gunnery School —— Ballistics.

For cadets of the Advanced Class of the Torpedo School —— Wireless Telegraphy and Telephony.

The following is the syllabus of the course of instruction in mathematics:

Infinitesimal Calculus (59 lectures, one lecture $1\frac{1}{2}$ hour).

I. Differential Calculus —— Theory *versus* Practice, Remarks on Practical Mathematics, Interpolation. Elements of Theory of Errors, Differentials, Subject-matters of Differential Calculus, Fundamental Problems of Differential Calculus, Mean Value Theorem, Hyperbolic Functions, Geometrical Application of Derived Functions, Maxima and Minima, Partial Differentials, Total Differentials.

II. Integral Calculus —— Subject-matters of Integral Calculus, Subject-matters of Differential Equation, Fundamental Problems of Integral Calculus, Calculation of Integrals, Geometrical Application of Integration, Elementary Differential Equations, Definite Integrals, Differentiation and Integration under the Sign of Integration, Double Integrals.

Analytical Geometry of Three Dimensions (20 lectures, one lecture $1\frac{1}{2}$ hours.) Co-ordinates, Distance between two Points, Direction Cosines, Projection, Angle between two Straight Lines, Relations between Rectangular and Polar Co-ordinates, Planes, Straight Lines, Equations of the Second Degree, Cylindrical Surfaces, Conical Surfaces, Ellipsoids, Hyperboloids, Paraboloids, Generating Lines, Tangent Planes and Normals, General Equation of the Second Degree.

Remarks:—The above are distributed to differential calculus, integral calculus, and geometry, and are instructed

at the same time.

The students of this course having already learned elementary calculus at the Naval College, there is no necessity of proceeding in the usual order, and it is even advisable to change the order adopted in the text-book of the Naval College for the purpose of attracting the officers' attention ; for this reason, in deciding the arrangement of the contents the order of various other subjects of study which are to be taught at the same time is chiefly taken into consideration.

Useful as the theoretical investigation is, it is to some extent dispenced with in order to make the instruction as much practical as possible, and many practical problems concerning navigation, gunnery, and torpedo are given instead. Besides, the instructor gives many problems and exercises, and makes each officer solve them in the class-room under his guidance, thus making the officers familiar with practical calculation.

As the officers have many subjects to learn and have little time at their disposal, no home exercise is given except when they have long vacations such as summer holidays.

A brief text-book is given to the officers, but the instructor delivers lectures, for the most part, independently of it, and makes the officers take notes of them.

(3) Students of Engineer Course.

Students of this class are engineer lieutenants. Every one and half years a number of candidates are recommended by commanding officers from among the best lieutenants under them, and afterwards they are examined by a specially appointed committee in their knowledge of plane analytical geometry, calculus, engineering, and English language, and about ten best ones are admitted at a time.

Instruction is given at the college in engineering, electrical engineering, and such preparatory subjects as are necessary for understanding the special subjects just men-

tioned, and after the completion of the course they are placed in important positions as engineer officers.

The course covers one and half years, and officers enter the college nine or ten years after graduation at the Naval Engineering College.

The branches of mathematics taught are analytical geometry of space, differential equations, and calculus, the chief object being to enable the officers to get intelligent grip of the following subjects:—

Dynamics, especially Hydrodynamics, Resistance of Ship and Propulsion, Strength of Materials, Conduction of Heat, Thermo-dynamics, Treatise on Steam, Electricity, Applied Electricity.

The syllabus of the course of instruction in mathematics is as follows:—

Analytical Geometry of Three Dimensions (20 lectures, one lecture $1\frac{1}{2}$ hours).

Co-ordinates, Direction Cosines, Projection, Transformation of Co-ordinates, Planes, Straight Lines, Surfaces of the Second Degree, Diametral Planes, Diameters, Tangent Planes, Normals, Elementary Theory on Surfaces in general and on Curves in Space.

Differential Equations (17 lectures, one lecture $1\frac{1}{2}$ hours).

Formation and Solution of Differential Equations, Exact Equations, Integrating Factors, Equations of the First Order and First Degree, Equations of the First Order and Higher Degree, Equations of the Second Order, General Linear Equations with Constant Coefficients, Simultaneous Equations, Solution of Linear Equations in Series.

Infinitesimal Calculus. (65 lectures, one lecture $1\frac{1}{2}$ hours).

Limiting Values, Differential Coefficients, Tangents and Normals, Maxima and Minima, Concavity, Change of Variables, Differentials, Involutes and Evolutes, Mean Value Theorem, Indeterminate Forms, Convergency of Series,

Expansion, Asymptotes, Total Differentials, Exact Differentials, Mean Value Theorem, Expansion, Thermodynamical Applications, Laplacian Equation, Change of Variables, Differential Equations of Waves and of Conduction, Envelopes, Singular Points, Curve Tracing, Indefinite and Definite Integrals, Summation, Methods of Integration, Gamma Function, Elliptic Integrals, Elliptic Functions, Line Integrals, Surface Integrals, Volume Integrals, Geometrical and Physical Applications, Theory of Errors, Differentiation and Integration under the Sign of Integration, Fourier's Series and Integrals, Partial Differential Equations, Waves and Conduction.

As the students of this course have learned plane analytical geometry and elementary calculus at the Naval Engineering College, instruction in analytical geometry of space and differential equations is given at the college for the furtherance of their knowledge in mathematics, and is completed in the first six months in order to meet the requirements of other subjects of study.

Calculus is also taught at this college, supplementing what the officers have already learned at the Naval Engineering College. The chief object to be aimed at is to give them the firm theoretical knowledge of infinitesimal calculus and at the same time to familiarize them with applications to physics, mechanical engineering, electrical engineering, etc.

Text-books which contain nearly all of what is to be taught, are given to the officers and verbal lectures are made as brief as possible. About one third of the whole time assigned to mathematics is used in solving problems, and many home exercises are given also.

(4) Examination.

All examinations are written ones, and every one of the officers is required to answer the same set of questions.

Sometimes the officers are allowed to bring text-books and sometimes not. There are three or four review-examinations and one final examination. The results of these examinations combined with daily marks determine the status of an individual officer.

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CONTENTS.

REPORT CONCERNING

THE SHŌSEN-GAKKŌ (Nautical College).

THE TEACHING OF MATHEMATICS IN THE SHŌSEN-GAKKŌ.

1. GENERAL INFORMATION CONCERNING THE SHŌSEN-GAKKŌ.

The Shōsen-Gakkō is the institution in which the cadets are trained to become higher seamen, and the course is twofold, namely a department of navigation and one of engineering. In the navigation department cadets are fitted for becoming mates and captains, and in the engineering department they are fitted to become engineers.

The navigation department requires two years and a half of theoretical study at the college, and moreover three years for gunnery and practical apprenticeship.

The engineering department requires two years of theoretical study and three years of practical apprenticeship, so that a full course in the navigation department covers five years and a half, whereas in the engineering department five years are required.

Each college year consists of two terms, and in the beginning of each term forty students who have passed the special or common competitive entrance examination are allowed to enter into each department.

The special competitive entrance examination is for those applicants who are graduates from some middle schools, and the common competitive examination is for general applicants.

The special competitive entrance examination consists of translations from English into Japanese and from Japanese

into English, Japanese composition, Chinese classics, mathematics (arithmetic, algebra, plane geometry and trigonometry) ; and the common competitive entrance examination consists of physics, chemistry, geography, history and drawing in addition to the branches mentioned for the special competitive entrance examination. As the status of these examinations corresponds to that of graduates of middle schools, the proficiency of an applicant must be equal at least to that of a middle school graduate.

Though the age of applicants may range from sixteen to twenty-one, the average age of beginners in the navigation department is nineteen years and four months and in the engineering department nineteen years and seven months. The reason for the seniority of engineering applicants is the fact that applicants are generally inclined to apply first for the navigation department and for the engineering department only when they find themselves unable to enter the former department.

The Shōsen-Gakkō is under the direct control of the Department of Communications, but the cadets are enlisted in the Imperial Navy on the day of their matriculation and after their graduation they receive the privilege of becoming members of the Imperial Naval Reserve with the rank of midshipmen, and are exempt from the governmental examination required for becoming higher seamen.

The students attending the local Shōsen-Gakkō (mercantile marine schools) established in accordance with the Special School Ordinance of the Mombusho (the Department of Education) are allowed to defer their military service whilst they are at school and after their graduation, though they may be appointed as navigating officers or engineering officers of the third rank of the naval reserve, they have no privilege of exemption from the higher seamen's examination. Another characteristic of the Imperial Shōsen-Gakkō distinguishing it from ordinary local mercantile marine schools is, that the

grade of the entrance examination for the former is that of middle school graduates while for the latter it is only of the higher elementary school grade.

2. OBJECT AND SUBJECT-MATTER OF THE MATHEMATICAL INSTRUCTION.

Although the majority of the cadets are graduates of middle schools, yet a few of them are not, so that on their entrance all cadets are taught in algebra, solid geometry and plane trigonometry in order to make them review those studies as well as to supplement their deficiencies. Thus, the endeavour is made to give them the necessary knowledge for the study of higher mathematics, navigation and engineering.

These principles are always borne in mind by the instructors even when the cadets advance to the higher classes and are instructed in spherical trigonometry, analytical geometry and calculus. Moreover, great attention is continually bestowed upon the system of mathematical instruction in order to create and always to maintain in the minds of the cadets a clear course of mathematical reasoning.

The schedule of the mathematical course is as follows:

Mathematical instruction in the navigation department.

Algebra: 3 hours per week for one term. Ratio, proportion, variation, permutation, combination, probability, binomial theorem, interpolation, logarithms, use of slide-rule, series, determinants, variation of algebraic function, ordinary and graphical solutions of equations, limiting values, calculation of area and volume, derived functions.

Plane trigonometry: 3 hours per week for one term. Trigonometrical ratio, use of logarithmic tables, theorems on plane figures, solution and calculation of triangles, theory of proportional parts, solution of trigonometrical equations, trigonometrical series, inverse trigonometrical functions, De

Moivre's theorem, explanation and application of traverse tables, parallax, dip, and other practical problems.

Solid geometry: 2 hours per week for one term. Lines, planes, and their relations, projection, polyhedral angles, properties of polyhedrons, properties of spherical polygons, latitude and longitude, definition of astronomical terms.

Spherical trigonometry: 2 hours per week for two terms.

Formulae, solution and calculation of various triangles, approximate values and small variations in the elements of triangles, elements of astronomy, azimuths and altitudes of heavenly bodies, explanation of various methods of finding latitude and longitude, and other practical problems.

Analytical geometry: 2 hours per week for one term. Co-ordinates, straight lines and planes, transformation of co-ordinates; properties of curves of second degree and their tangents and normals; co-ordinates of three dimensions, properties of surfaces of second degree, Simpson's rule, orbit of earth, and other practical problems.

Differential and integral calculus: 2 hours per week for one term.

Differentiation, theory of curves, elements of integration, quadrature, centre of gravity, ship's stability, successive differentiation, partial differentiation, relations of small errors, expansion of series, maxima and minima, method of least squares, deviation of compass, meridional parts, and other practical problems.

Mathematical instruction in the engineering department.

Algebra: 3 hours per week for one term.

The syllabus of this study is the same as in the navigation department.

Plane trigonometry: 2 hours per week for one term.

Trigonometrical ratio, use of logarithmic tables, theorems on plane figures, solution and calculation of triangles, theory of proportional parts, solution of trigonometrical equa-

tions, trigonometrical series, inverse trigonometrical functions, De Moivre's theorem, calculation of length, area, volume etc.

Solid geometry: 2 hours per week for one term. Lines, planes, and their relations, projection, properties of polyhedrons, and their areas and volumes, similar bodies, conic sections, solid of revolution, theory of projection and perspective.

Mensuration: one hour per week for one term.

Contracted methods, short methods, conversion of measures; perimeter and area of plane figures; surface and volume of prism, cylinder, pyramid, cone and sphere; weights of timber and metal, ship's displacement, and other practical problems.

Analytical geometry: 2 hours per week for one term.

Co-ordinates, straight lines and planes, transformation of co-ordinates, properties of curves of second degree and their tangents and normals; co-ordinates of three dimensions, properties of surfaces of second degree, determination of empirical formulae, Simpson's rule, principle of Bourdon's steam-gauge, and other practical problems.

Differential and integral calculus: 2 hours per week in the first term, and one hour per week in the second term.

The first term: Differentiation, theory of curves, maximum and minimum, integration, and other practical problems.

The second term: Successive differentiation, partial differentiation, expansion of series, multiple integration, solution of differential equations, and other problems.

In algebra, instruction begins with ratio and proportion and ends with determinants and derived functions. In the mean time, for the variation of algebraic functions and the graphical solution of equations and so forth, the use of section paper is taught, and the application of limiting values in the calculation of areas, volumes or moments of inertia and so forth is explained. Thus the fundamental principle of calculus is impressed upon the cadets, and the materials

necessary for the study of other branches of science are supplied as well.

In this course the study of probability, interpolation and so forth is chiefly insisted upon in the navigation department, while the use of section paper, the application of limiting values and so forth are chiefly taught in the engineering department.

Plane trigonometry is commenced at a point somewhat in advance of what is taught in the middle school, and ends with the theorem of De Moivre and its application. In the navigation department the solution and calculation of triangles are especially considered and in the engineering department problems in area and volume of figures are solved by the use of logarithmic tables.

Spherical trigonometry begins with the right-angled triangle, goes on to the oblique-angled triangle, explains approximate values of the sides of triangle and relations of small variations of its sides and angles, and the principles of the formulae used in navigation. Two terms being devoted to spherical trigonometry, in the first term one hour or two per week is always used for solutions and calculations of triangles.

At present the ordinary method of instruction in spherical trigonometry usually commences with the oblique-angled triangle and then goes on to the right-angled triangle but in the Shōsen-Gakkō the opposite is the case. This reverse method of instruction resembles the one followed in plane trigonometry and as it leads gradually from simple cases to more complicated ones, it helps the cadets to grasp the general idea, but on the other hand it may lead to some repetition in the course of instruction.

The status of instruction in solid geometry is somewhat higher than that in the middle school. It commences at the very beginning and following the usual order proceeds to the spherical and descriptive geometry. In the navigation depart-

ment, the properties of spherical triangles are chiefly considered, but in the engineering department greatest stress is laid on the solid and descriptive geometry.

In mensuration, the conversion of measures, the area and volume of figures, the weights of timber and metal, and other practical problems are discussed. Thus the cadets are taught in the practical side of engineering and they are drilled as well in actual calculations, for though any one may have a superior knowledge of engineering, yet he will fail to discharge his duties properly unless he be skilled in general calculation.

In analytical geometry the instruction is given in the usual order; beginning with lines and planes it proceeds to curves and surfaces of the second degree; and by way of application, the navigation department takes up Simpson's rule, difference of geographical and geocentric latitudes, central orbit; and the engineering department takes up expansion of steam, computation of empirical formulae and the principle of Bourdon's steam gauge.

In calculus the method of explanation in the navigation department differ from that in the engineering department. In the navigation department differentiation of simple functions is followed by its inverse integration as the differentiation of monomial integral expressions is followed by its inverse integration, and practical problems are introduced at appropriate intervals. In the engineering department integral calculus is taught after the differential in the ordinary way, but complicated or unpractical problems are sometimes omitted or taken up after finishing the general course of integral calculus.

To sum up the whole, the calculus taught in the Shōsen-Gakkō is thoroughly practical, and appears to be treated as the necessary means of solving practical problems; else it would be impossible to give any useful knowledge of calculus in the short time of thirty two hours in the navigation de-

partment, and of forty eight hours in the engineering department.

The following lists are added for reference in order to show the difference in methods of instruction in algebra, spherical trigonometry, analytical geometry and calculus followed at the Shōsen-Gakkō from those of ordinary institutions.

3. EXAMINATIONS.

Examinations are in writing, but if they were to take place only once at the end of each term, the result would often be somewhat unjust, therefore one hour per week out of three is always used for the solution of practical problems and in such studies as spherical trigonometry and mensurations, which require skill in calculation, the cadets are drilled in practical problems at least once a week notwithstanding the limited number of hours devoted to the subject. To the marks obtained in this way, the examination marks are added and the final result is computed.

In the Shōsen-Gakkō in accordance with the fourth article of chapter five of the "College Regulation," the marks of the examination may be altered according to the average of the marks of the special examinations as follows; if for instance, a cadet obtains an average of eighty per cent in his special examinations and seventy per cent in the term examination, five per cent is added and his resulting marks will be seventy five; and if he obtain an average of forty per cent in his special examinations and eighty per cent in his term examination, four per cent being subtracted from the latter, his final standing will be seventy six per cent.

(REFERENCE)

COLLEGE REGULATIONS OF THE SHŌSEN-GAKKŌ.

CHAPTER V.

EXAMINATIONS.

Art. 4. The final calculation of the examination marks for a term is made in accordance with the following provisions, provided the marks allowed to be added to or subtracted from the term examination marks are limited to the extent of eight per cent in all.

From	100%	to	95%	8%	is	added
,,	94	,,	90	7	,,	,,
,,	89	,,	85	6	,,	,,
,,	84	,,	80	5	,,	,,
,,	79	,,	75	4	,,	,,
,,	74	,,	70	3	,,	,,
,,	69	,,	65	2	,,	,,
,,	64	,,	60	1	,,	,,
,,	59	,,	55	1	,,	subtracted
,,	54	,,	50	2	,,	,,
,,	49	,,	45	3	,,	,,
,,	44	,,	40	4	,,	,,
,,	39	,,	35	5	,,	,,
,,	34	,,	30	6	,,	,,
,,	29	,,	25	7	,,	,,
,,	24	,,	20	8	,,	,,

4. METHOD OF INSTRUCTION.

That branch for which an appropriate text-book exists, is taught according to it; and in those branches for which

no text-book is used, instruction is given by lectures; provided each cadet is supplied beforehand with papyrographic summary notes of the said lectures. In some studies models are used for illustration. All the problems for exercise worked out and presented by the cadets must be entered in their own note-books which are strictly and minutely examined by the instructors in charge, and moreover, the work and answer for the same problem must be written by the instructor and printed and distributed among the cadets, in order to give them a chance of comparison with their own work.

5. FUTURE PROSPECTS.

It is well known fact that the mathematics required both at home and abroad for the higher seamen's examination are of a comparatively very elementary kind and that even the attainments required of candidates for the post of captain or chief engineer do not much exceed advanced primary school arithmetic, and in those daily calculations which are performed in a mechanical manner ordinary navigators rarely understand the principles upon which their calculations are based. This being the case with mariners generally, many mercantile marine institutions abroad are said to be of similar standing with our Shōsen-Gakkō, and the candidates for the higher seamen's examination may easily achieve success after their own respective apprenticeship on board or in machine shops if they have some scientific knowledge.

But on the other hand the Shōsen-Gakkō admits only those applicants who have passed an entrance examination of middle school grade and after their entrance the institution gives the cadets a higher knowledge of science during a space of from two years to two years and a half, so that in their scientific attainments they ought easily to equal the candidates for extra master or extra chief engineer in England. Therefore, though from a mercantile marine institution

no one would expect still higher instructions, yet if the curriculum of this institution is compared with those of the naval educational institutions of similar grade, the attainments in mathematics are found to be much below the latter, and in order to supplement this defect, either the grade of the entrance examination must be raised and admission restricted to the graduates of middle schools, or else the number of daily lessons must be increased in favor of mathematical studies and greater exertion must be made to drill the cadets in practical exercises, or perhaps some other technical institutions of learning must be provided with more advanced standing and transferring the mathematical department thither, it must be allowed to follow its own natural development.

The following quotations are added for reference to serve for comparing the number of hours devoted to the instruction in mathematics in the Shōsen-Gakkō, with that of other institutions of similar nature.

In this institution cadets recite their lessons under the charge of the instructors for the same number of hours as those devoted to lectures.

APPENDIX.

DETAILED TOPICS OF INSTRUCTION IN ALGEBRA.

NUMBERS.

Integers and fractions—Positive and negative numbers—Rational and irrational numbers—Surds, real and complex quantities—Approximate calculation of square root and cube root—Theory of surds—Square root of $a+\sqrt{b}$—Square root of a complex number—Cube root of unity—Conjugate surds—Cube root of $a+\sqrt{b}$—Surd root of an equation.

INDICES AND LOGARITHMS.

Laws of indices—Logarithms—Theory of logarithms—Characteristic and Mantissa—Logarithmic tables—Slide rule—Change of logarithmic base—Exponential equation—Entropy of water—Adiabatic expansion of saturated steam.

RATIO AND PROPORTION.

Ratio—Theory of ratio—Proportion—Variation—Graph.

PROGRESSIONS.

Arithmetical progressions—Geometrical progressions—Harmonical progressions—Series formed by squares of natural numbers—Series formed by cubes of natural numbers—Finding logarithms by geometrical progression.

CHOICE AND CHANCE.

Choice—Permutations—Combinations—Theory of combinations—Probability—Probability of compound events.

Binomial and multinomial Theorem.

Binomial theorem for positive integral index—Euler's generalization of binomial theorem—Approximate value of $\sqrt[n]{a \pm b}$—Multinomial theorem—Terrestial magnetism.

Interpolation.

Newton's formula—Stirling's formula—Bessel's formula—Simple interpolation—Summation of series.

Variation of Algebraic Functions.

Quadratic expressions—Cubic expressions—Maxima and minima.

Solution of Equations.

Graphical solution—Newton's method—Horner's method —n^{th} root of any number—Solution of equation by interpolation—Solution of quadratic equation—Cardan's method of solving cubic equation—Euler's method of solving biquadratic equation—Another method of solving cubic equation—Another method of solving of biquadratic equation.

Indeterminate Coefficients.

Rational integral function—Comparison of two rationa integral functions—Partial fraction—Summation of series—Solution of equation by series.

Limiting Values.

Limiting values—$\mathrm{Lim}_{x=a} \dfrac{x^n - a^n}{x - a}$ — $\mathrm{Lim}_{n=\infty} \dfrac{\Sigma n^{m-1}}{n^m}$ —— Base of Napierian logarithm—Area and volume—Area of parabola—Volume of paraboloid—Volume of ellipsoid—Prismoidal formula—Moment of inertia—Statical moment—Binomial theorem—Expansion of $\sin \theta$ and $\cos \theta$.

Exponential and Logarithmic Series.

Exponential series—Logarithmic series—Calculation of logarithms—Relation between common logarithm and Naperian logarithm—Theory of proportional parts—Expansion of solid—Atmospheric pressure—Isothermal expansion of steam.

Divergency and Convergency of Series.

Preamble—Divergency and convergency of series—Theory of divergency and convergency of series—Application.

Determinant.

Definition—Theorems—Expansion of determinant—Multiplication of diterminants—Solution of simultaneous linear equations—Elimination—Resolution of homogeneous quadratic expression of x, y, z into two linear factors—Solution of simultaneous quadratic equations.

Derived Function.

Definition of a function and its derived function—Theorem concerning functions and their derived functions—Derived functions of simple functions—Prime functions.

DETAILED TOPICS OF INSTRUCTION IN SPHERICAL TRIGONOMETRY.

Relation between the sides and angles of a sphericalrl right-angled triangle.

Object of spherical trigonometry—Formulae relating to the sides and angles of a spherical triangle—Complementary triangle—Napier's circular parts—Formulae for quadrantal triangle.

Solution of right-angled Triangle.

Classification of solutions of right-angled triangle—Precautions to be taken for solving right-angled triangle—Two

sides given—Two angles given—Hypotenuse and a side given—Hypotenuse and an angle given—A side and an adjacent angle given—A side and the opposite angle given —Solution of quadrantal triangle.

Relations between the Sides and Angles of a Spherical Triangles.

Relation between three sides and an angle—Relation between three angles and a side—Relation between sides and the angles opposite to them—Relation between three sides and two angles or two sides and three angles—Relation between two sides, the included angle and an opposite angle —Formulae of a spherical triangle found by orthogonal projection—Formulae of a spherical triangle found by transformation of axes—Napier's analogies—Delambre's analogies—Formulae for spherical triangles—Similarity of plane and spherical figures—Three points on a great circle.

Solution of oblique-angled triangles.

Classification of solutions of an oblique-angled triangle—Three sides given—Three angles given—Two sides and the included angle given—Two angles and the included side given—Two sides and an opposite angle given—Two angles and a opposite side given—Cauchy's solution—Spherical traverse tables.

Approximate Value of an Element and small Variations in the Elements of a Spherical Triangle.

Sine and cosine of a small angle and small variations of sine and cosine of any angle—Approximate value of the opposite side of a small angle in a spherical triangle—Approximate values of sides of a spherical triangle, one of which is small—Relation of small variations of a side and the opposite angle of a spherical triangle having the two remaining sides constant—Relation of small variations of an

angle and the opposite side of a spherical triangle having the two remaining sides constant—Relation of small variations of two sides of a spherical triangle having the third side and its opposite angle constant—Relation of small variations of a side and the opposite angle to quadrant of a quadrantal triangle having the other sides constant.

Ship's Course and Distance.

Object of navigation—Length of parallel of latitude—Parallel sailing—Middle-latitude sailing—Mercator's chart and gnomonic chart—Mercator's sailing—Great circle sailing.

Position of Heavenly Bodies and Apparent Motion of the Sun.

Apparent position of heavenly bodies—Celestial sphere and parallax—Horizon, vertex, nadir, and horizontal parallax—Axis and poles of the heaven—Vertical circles and meridian—Altitude and zenith distance—Corrections of altitude—Azimuth and amplitude—Celestial equator—Declination and polar distance—Hour angle—Geographical position of heavenly bodies—Line of position—Correction for sun—Equinoxes and solstices—Right ascension—Celestial latitude and longitude—Retardation of vernal equinox—Different kinds of years, days and time—Mean sun—Equation of time—Conversion of different measures of time.

Azimuth, Hour-Angle and Altitude of Heavenly Bodies

Azimuth and hour angle of heavenly bodies found by altitudes—Lecky's altitude-azimuth tables—Mean time found by hour angle—Azimuth of heavenly body found by hour angle—Time-azimuth tables—Hour angle of heavenly bodies found by azimuth—Hour angle of heavenly bodies found by altitude and azimuth—Error of hour angle—Lecky's A B C

tables—Hour angle of rising or setting and amplitude of heavenly bodies—Hour angle of heavenly bodies in prime vertical—Abbreviated method of time-altitude—Equation of equal altitudes—Altitude found by hour angle—Variation of altitude in short time—Azimuth of heavenly bodies found by variation of altitude—Hour angle of heavenly bodies found by variation of altitude—Hour angle and altitude of heavenly bodies found by stereographic chart.

POSITION OF SHIP.

Latitude—Latitude found by merictian altitude of heavenly bodies—Latitude found by exmeridian altitude of heavenly bodies—Latitude found by altitude of pole star—Longitude—Longitude found by lunar distance—Position of ship found by double altitudes; I. Pvoly's method, II. Lalande's method, III. Pohnson's method, IV. Summer's method.

MISCELLANEOUS PROBLEMS.

Maximum azimuth of heaventiy bodies—The most favourable time for determining hour angle by altitude of heavenly bodies—celestial latitude and longitude found by declination and right ascension—Twilight—Hour angle at maximum altitude of heavenly bodies—Convergency of meridian—Difference in rhumb bearing found in Mercator's chart and true bearing of a certain object—A few practical problems—Hall's nautical slide-rule—Application of Lecky's tables—Errors of sextant.

DETAILED TOPICS OF INSTRUCTION IN ANALYTICAL GEOMETRY.

CURVE TRACING AND POSITIONS OF POINTS.

Curve tracing—Object of analytical geometry—Axes and co-ordinates of a point—Co-ordinates of a point in a straight

line joining two points divided in a given ratio—Distance of two points.

Equation of Straight Line and Curve.

Equations of loci—Equation of a straight line—Equation of a circle—Equation of an ellipse—Equation of a hyperbola—Equation of a parabola.

Straight Line.

Equation of a straight line—Friction—Logarithmic co-ordinates—Rope friction—Saturated steam—Perpendicular on a given straight line from a given point—Intersection of two straight lines—Equation of straight line through the intersection of two straight lines—Condition that three straight lines are concurrent—Problems in triangle—Problems in parallel lines—Problems in loci.

Transformation of Axes and Classification of Quadratics.

Transformation of axes—Classification of quadratics by the form of their equations—Classification of quadratics by the value of their eccentricity.

Polar Equation.

Polar co-ordinates—Relation between polar and Cartesian co-ordinates—Polar equation of a straight line—Polar equation of a cirle—Polar equation of a conic—Problems involving focal chords of a conic.

Tangent and Normal of a Conic.

Tangent line—Equation of tangent to a parabola—Equation of tangent to a central conic—Equation of normal—Subtangent of a parabola—Subnormal of a parabola—Direction of tangent of a parabola—Search light—Intersection of a normal

of an ellipse with the axes—Figure of earth—Reduced latitude—Perpendiculars to a tangent from the foci of an ellipse—Central orbit—Orbit of the earth—Anomalies—Two tangents from a given point to a conic.

Miscellaneous Theorems on Conics.

Pole and polar—Diameters—Equation of a parabola, referred to any diameter and the tangent at its vertex—Secants of a conic passing through a given point—Conjugate diameters—Eccentric angle of an ellipse—Relation between eccentric angles at the extremities of conjugate diameters—Length of conjugate diameters—Parallelograms formed by conjugate diameters—Equation of an ellipse, referred to conjugate diameters—Conjugate hyperbola—Asymptotes—Rectangular hyperbola—Equation of hyperbola, referred to asymptotes—Asymptotes and tangents—Area of a parabola of the n^{th} degree—Area of a parabola —Adiabatics—Area of a hyperbola—Isothermals—Steam jacket—Radius of curvature—Equation of a conic, referred to a tangent and the normal.

Transendental Curves.

Cycloid—Trochoid—Epicycloid and Hypocycloid—Logarithmic spiral—Catenary.

Straight Lines and Planes in Space.

Co-ordinates of a point—Distance to a given point from the origin—Direction cosines of a straight line—Distance of two points—Equation of a straight line passing through a given point and drawn in a given direction—Equation of a straight line through two given points—Angle between two straight lines—Equation of a plane in terms of the length and the direction of the perpendicular upon it from the origin—Equation of a plane in terms of the intercepts which it makes on the axes—Angle between two intersecting planes

—Parallels and perpendiculars to a given plane or straight line—Distance from a given point to a given plane—Condition that a straight line lies on a plane—Shortest distance between two straight lines—Intersection of two straight lines.

SURFACES OF THE SECOND DEGREE.

Classification of surfaces of the second degree—Sphere—Cylindrical surface—Conical surface—Ellipsoid—Hyperboloid—Paraboloid—Equation of tangential plane to a surface of the second degree—Principal radii of curvature at any point of a surface of the second degree—Principle of Bourdon's pressure gauge—Helix.

DETAILED TOPICS OF INSTRUCTION IN CALCULUS.

[Navigation Department]

Object of calculus—Differentiation of x^n—Variation of algeblaic expressions—Integration of $x^n dx$ and its application—Center of buoyancy and ship's stability—Strength of beam and shaft—Planimeter—Differentiation of e^x and $\log x$, and the inverse operation—Expansion of solids—Relation between atmospheric pressure and height—Differentiation of complicated functions—Differentiation of trigonometrical functions—Problems on tangents of a curve—Problems in maxima and minima—Order of infinitesimals, differentials and differential co-efficients—Taylor's theorem, Maclaurin's theorem; and their applications—Fourier's theorem and compass deviation—Curvature of a curve—Nautical mile and the length of a minute of meridian—Errors of relative quantities—Introduction to the method of least squares—Magnetic coefficients found by the method of least squares—Integration of $\frac{dx}{\sqrt{a^2-x^2}}$, $\frac{dx}{a^2+x^2}$, etc.—Arrangement of needle on the card—Integration—Meridional parts—Refraction—Central orbits.

DETAILED TOPICS OF INSTRUCTION IN CALCULUS.

[Engineering Department]

DIFFERENTIATION.

Object of calculus—Functions and their continuity—Independent and dependent variables—Derived functions and differential coefficients—Increments and differentials—Differentiation of simple functions—Differentiation of the sum—Differentiation of the product—Differentiation of the quotient—Differentiation of a function of a function—Logarithmic differentiation.

APPLICATION OF DIFFERENTIAL CALCULUS.

Watt's parallel motion—Parallel motion of beam engine—Coefficients of expansion—Equations of tangent and normal—Subtangent and subnormal—Angle between a tangent and the radius vector—Relation between the speed of piston and that of crank—Connecting rod for transmission of power—Maxima and minima of a function—Maximum speed of piston—Peclet's theory of chimney draught.

INTEGRATION.

Integration—Precaution to be taken in integrating—Integration of simple functions—Integration by substitution—Integration by parts—Integration of rational fractions—Reduction formulae for $x^m(a+bx^n)^p$—Integration of algebraic irrational functions—Integration of trigonometrical functions—Reduction formulae for trigonometrical functions—Definite integration—Integration regarded as summation—Theorems on definite integrals—Definite integrals.

APPLICATIONS OF INTEGRATION.

Length of a curve—Area of a plane figure—Simpson's Rule—Planimeter—Integral curve and integraph—Volume of a solid—Prismoidal formula—Surface of revolution—Statical

moment—Moment of inertia—Center of gravity—Guldinus's theorem—Moments of inertia of an area with respect to parallel axes—The total pressure of water and its center—Ship's stability—Velocity of a body moving with constant acceleration and the space described by it—Discharge of water at constant level—Discharge of water from a tank—Rushing in of sea water into a ship through a hole—Bursting force of steam against the boiler—Isothermal expansion of steam—Adiabatic expansion of steam—Damage of fly-wheel due to centrifugal force—Friction on belt or rope—Friction on flat pivot—Friction on journal—Newton's law on cooling-efficiency of heating surface of a boiler—Efflux of gases—Bernouilli's theorem—Law of comparison with referrence to ship's resistance.

Successive and Partial Differentials.

Successive differentials and successive differential coefficients—Velocity and acceleration of a piston—Concavity and convexity of a curve—Curvature of a curve and the radius of curvature—Points of inflexion on a curve—Leibnitz's theorem—Maclaurin's theorem—Gradient of numerical values of functions obtained from experiment—Taylor's Theorem—Maxima and minima of a function—Indeterminate forms—Asymptotes of a curve—Partial differentials and total differential—Theorems on partial differential coefficients—Extension of Taylor's theorem—Implicit functions—Envelope—Tangent plane of a surface—Tangent line and osculating plane of a tortuous curve—Curvature of tortuous curve—Change of state of perfect gases according to the change of quantity of heat—Entropy of gases—Entropy of liquid—Entropy of steam—Temperature—Entropy diagram of water and steam.

Multiple Integration.

Successive integrations—Accelerating motion—Rectilinear

motion in resisting medium—Maximum bending moment and deflection of a fixed beam—Ship's rolling—Clapeyron's theorem on three moments of a continuous beam—Multiple integration.

Differential Equations.

Classification of differential equations—Differential equations of the first order and the first degree—Homogeneous equations—Total differential equations—Integrating factors—Differential equations of the first order and higher degrees—Singular solutions—Differential equations of the second order—Differential equations of higher order—Differential equations with constant coefficients—Solution of differential equations in series—Simultaneous differential equations—Simultaneous differential equations with three variables—Partial differential equations—Problems on tangents and normals—Variation of electric current due to self-induction—Euler's formula on the strength of column—Lame's formula on the strength of thick cylinder—Periodic waves in deep sea.

THE TEACHING OF MATHEMATICS

IN

The Training School for Officials in the Department of Communications.

1. SYSTEM OF THE TRAINING SCHOOL FOR OFFICIALS IN THE DEPARTMENT OF COMMUNICATIONS.

The training school for officials in the department of communications is the place where technical instruction is given to the officials employed in the department of communications. It consists of a school of administration, of a technical and an electric communication department. The chief object of the school of administration is to teach its pupils how the administration of the department of communications is conducted. The technical department is intended chiefly to teach the science of telegraphy, telephony and electric power, and the main purpose of the electric communication department is to give advanced instruction in the technique of communication by electricity.

The complete course of instruction in the school of administration and in the technical department comprises two years each, and in the electric communication department one year. The course of instruction in the school of administration and in the technical department is divided into a theoretical and a practical term. One year and a half are devoted to the former, and the remaining half year is assigned to the latter. In the school of administration the practical course is taken up first, and the theoretical studies later on; whereas, in the technical department this order is reversed; moreover, the theoretical period is divided into three terms of six months each.

In the school of administration only students who by their good record during the practical term are deemed fit for further instruction are allowed to pursue their theorectiel course ; and in the technical department only those are allowed to graduate, who are making a good showing throughout their experimental term.

Candidates for the school of administration and the technical course must be not less than seventeen and not more than twenty five years of age, employes of the department of communications as well as other young men are eligible to undergo an entrance examination of middle school grade in the following branches.

1. Japanese language.
2. English.
3. Mathematics (Arithmetic, Algebra and Geometry).
4. Geography and History.
5. Natural science.

The graduates of the governmental or public middle schools or of schools which are deemed by the department of education to be of equal standing, may be exempted from the examination in branch No. 5 of the above schedule for entrance into the school of administration and in branch No. 4 for entrance into the technical department.

Candidates for the course in electric communications must be not less than sixteen and not more than twenty four years of age and must have had experience in the actual service, of some branch of electric communications either during six consecutive months immediately preceding the time of application or for more than a year at some previous time.

The entrance examination for the course in electric communications comprises the following studies.

1. Japanese language.
2. English.
3. Mathematics.

4. The technique of electric communication.

The graduates of the governmental and public middle schools and of those schools which are deemed by the department of education to be of equal standing may be exempted from the examination in the study mentioned in No. 3 of the above schedule.

Those who by their entrance examination are qualified as competent, whether they be officials in the department of communications or not, are engaged as non-salaried employes.

These apprentices receive a monthly payment of ten yen to cover their expenses, except that such as are selected from among the officials of the communitions department may have this amount increased up to fifty per cent.

Graduates of this institution are bound to serve the Department of Communications for five years.

2. PURPOSE AND SUBJECT MATTER OF THE INSTRUCTION IN MATHEMATICS.

The purpose of the mathematical instruction in the school of administration at this institution is to train the pupil in the use of the soroban. In the technical course it is to teach them the various calculations required in telegraphy, telephony, electric traction etc. and to give them a knowledge of mathematics sufficient to enable them to comprehend the principles of the various formulae; and in the electric communication course to teach the mathematics necessary in the study of electric communications; therefore, the object of the mathematical instruction given in this institution is chiefly of a practical nature rather than strictly scientific.

The schedule of the courses of study in mathematics and mechanics, and of the distribution of periods is as follows:

The course of mathematical instruction in the school of administration.

Calculation: 15 hours.

Soroban-calculation.

Arithmetical short-cuts.

The Course of mathematical Instruction in the Technical Department.

Algebra and geometry : 66 hours.

Practical drill in algebra, from the beginning to quadratic equations inclusive ; the whole of plane geometry ; both these branches being of the middle school grade ; moreover in algebra : ratio, proportion, permutation, combination, binomial theorem, series, convergency and divergency of series, undetermined coefficients, partial fractions, interpolation, determinants, solution of simultaneous quadratic equations, algebraic solution of equations of the third and fourth degree, approximation, numerical solution of equations of a higher degree, maximum and minimum of algebraic expressions are lectured upon and practiced. In geometry, the practical problems relating to lines, area, volume and so forth are given.

Trigonometry : 53 hours.

Introduction, measurements of angles, angles, circular measurements, trigonometrical functions, relation of trigonometrical functions, trigonometrical functions of double and half angles, value of trigonometrical functions of particular angles, trigonometrical functions of angles of any size, variation of values of trigonometrical functions, trigonometrical functions of the sum and difference of two angles and theorems derived therefrom, use of tables of logarithms and of trigonometrical functions, properties of triangles, solution of triangles, heights and distances, inverse trigonometrical functions, De Moivre's theorem, evolution of numbers, summation of series of trigonometrical functions, vectors.

Analytical geometry : 20 hours.

Descartes' co-ordinates, problems in distances, area of rectilinear figures, polar co-ordinates, loci (straight line,

circle, parabola, hyperbola, spiral, logarithmic spiral etc.), transformation of co-ordinates, equation of straight lines, theorems on straight lines, equation of circle, tangents and normals on circle, equation of parabola, simple properties of parabola, area of parabola, Simpson's rule, deflection of suspended wire, equation of ellipse and hyperbola, equation of tangents and their normals, conjugate diameters, equation of ellipse and hyperbola referred to conjugate diameters, asymptotes, rectangular hyperbola, three dimension co-ordinates, direction cosines, angle between two intersecting lines.

Differential and integral calculus: 43 hours.

Function, continuity and discontinuity of function, definition of differentials, differential coefficient and derived function of function, differential coefficients of sums, product and quotient of functions, differentiation of function of function, differentiation of inverse function, differential coefficient of power of function, exponential function, logarithmic function, circular function, inverse circular function etc., exercise in calculating differential coefficients, integration, integration of rational expressions, integration of expressions containing the square root of quadratic expressions, integration of ordinary transcendental functions, integration by substitution, integration by parts, exercise in integration, definite integral, successive differentiation, Taylor's theorem, Maclaurin's theorem, expansion of series, exponential series, Euler's theorem, logarithmic series, calculation of logarithms, expansion of circular functions, inverse circular functions and other functions, calculation of π, properties of curves, calculation of line and area, use of planimeter, volume, center of gravity, moment of intertia, radius of gyration, maximum and minimum of function, solution of simple differential equations, Fourier's series; application to potential, attraction, oscillation, electric capacity, coefficient of induction, voltage, current etc.

Dynamics: two hours per week during the second term.

Laws of motion, rectilinear motion, circular motion, centrifugal force, simple harmonic motion, simple pendulum, concurrent forces, composition of parallel forces, centroid of parallel forces, centre of gravity, equilibrium of co-planar forces, equilibrium of rigid bodies, motion of rigid bodies, translation, rotation, moment of inertia, radius of gyration, compound pendulum, work and laws of energy; stable, neutral and unstable equilibrium, equilibrium and motion of bodies in friction, simple machines, strain, stress and elasticity etc.; catenary, deflection of suspended wire, strength of structures, deflection of beams, shear, torsion and wind pressure etc.; flow of water, pressure of moving water.

The Course of Mathematical Instruction in the Electric Communications Department.

The mathematical standing of pupils admitted to this department comprises the entire arithmetic of the middle school grade up to the solution of equations of the first degree in algebra.

Algebra: 37 hours.

Explanation of terms, addition, subtraction, multiplication and division of integral expressions, factoring, solution of simultaneous equations of the first degree, and of quadratic equations and fractional calculations.

Geometry: 32 hours.

Angles, straight lines, parallel lines, triangles, parallelograms, theorems on areas of rectilinear figures, properties of circle and tangent, problems, properties of circle concerning areas, co-relation of circles, construction of regular polygons, ratio, proportion, similar figures.

Trigonometry: 19 hours.

Measurement of angle, angles, circular measurements, definition of circular functions, relation of circular functions,

circular functions of 30°, 45°, 60° etc., circular functions of double and half angles, use of tables of logarithms and of circular functions, solution of triangles, simple surveying.

To sum up; in the electric communications department the purpose is to give the pupils the necessary mathematical knowledge required for simple calculations concerning electric communications, and for the understanding of the underlying principles.

3. EXAMINATIONS

Though mid-year and final examinations take place at the end of each term or of the entire course, now and then special examinations are given and their rating is taken into consideration, in computing the results of the term and of the final examinations.

4. METHOD OF INSTRUCTION.

The method of instruction consists in first explaining the principles and then giving practical exercises in order to let the pupils acquire as much practical knowledge as possible.

5. FUTURE PROSPECTS.

The general trend of technical schools is to lay greater stress upon strict technicalities and their direct accessories, so that the fundamental principles upon which those technical problems are based are often slighted. Although in this institution there is no defect of this nature, yet the scarcity of time is felt in some respects, and especially in analytic geometry. Therefore if efforts be made to increase the number of hours as far as possible it is believed that better results are very likely to be secured.

明治四十五年六月二日印刷
明治四十五年六月五日發行

非賣品

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